

E, B. fields

We skip the derivation of E & B transformations, & just state their Lorentz transforms:

$$\begin{aligned} E'_x &= E_x \\ E'_y &= \gamma(E_y - \beta c B_z) \\ E'_z &= \gamma(E_z + \beta c B_y) \\ B'_x &= B_x \\ B'_y &= \gamma(B_y + \beta E_z/c) \\ B'_z &= \gamma(B_z - \beta E_y/c) \end{aligned}$$

where we again assume S' moves at $v (= \beta c)$ in x-direction compared to S. Useful for synchrotron especially.

Thomson scattering

Before addressing Compton scattering, consider the nonrelativistic case, Thomson scattering.

Appropriate for low photon energies, $h\nu \ll m_e c^2$.

Force on electron from EM wave incident on it

$$(\vec{F} = q\vec{E}) \rightarrow \vec{F} = q\vec{E}_0 \sin(\omega t) = m\vec{a}$$

so the acceleration $\vec{a} = (qE_0/m_e) \sin(\omega t)$,

& the time-averaged acceleration is $\langle \vec{a} \rangle = (qE_0/m_e)/2^{1/2}$.

We put this into the Larmor formula, in 2 forms:

$$\text{First, } \frac{dE}{dt} = \frac{dP}{dz} = \frac{q^2 q^2 \sin^2 \theta}{(4\pi)^2 \epsilon_0 c^3} = \boxed{\frac{e^4 E_0^2 \sin^2 \theta}{32\pi^2 m_e^2 \epsilon_0 c^3} = \frac{dE}{dt}}$$

where θ is measured relative to the E field (=& thus acceleration) direction.

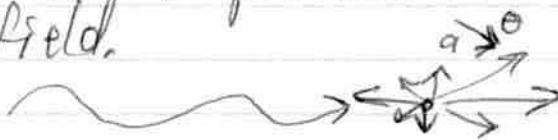
Integrating over all angles, $\int \sin^2 \theta = \frac{8\pi}{3}$, so

$$\boxed{\frac{dE}{dt} = \frac{e^4 E_0^2}{12\pi \epsilon_0 m_e^2 c^3}}$$

The energy density $U_{\text{rad}} = \frac{E_0^2 \epsilon_0}{2}$, so

$$\frac{dE}{dt} = \frac{e^4 U_{\text{rad}}}{6\pi \epsilon_0^2 m_e^2 c^3}$$

The emitted radiation is polarized in the plane of the electric field.



We can figure out the incident flux,

$$\text{Poynting flux } \langle S \rangle = \left\langle \frac{1}{\mu_0 c} |\vec{E}|^2 \right\rangle = \frac{\epsilon_0 c E_0^2}{2} \quad (c^2 = \frac{1}{\mu_0 \epsilon_0})$$

If we divide the output power by the incident flux, we get the cross-section for scattering.

First the angle-dependent version,

$$\frac{dE}{dt d\Omega} = \langle S \rangle \frac{d\sigma}{d\Omega} = \frac{\epsilon_0 c E_0^2}{2} \frac{d\sigma}{d\Omega}, \text{ so}$$

$$\frac{d\sigma}{d\Omega} = \frac{e^4}{16\pi^2 \epsilon_0^2 m_e^2 c^4} \sin^2 \theta = r_0^2 \sin^2 \theta$$

where $r_0 \left(\frac{e^2}{4\pi \epsilon_0 m_e c^2} \right)$ is the classical electron radius.

Comparing the angle-integrated $\langle S \rangle$ and dE/dt , we see

$$\alpha_T = \frac{8\pi}{3} r_0^2 = 6.65 \times 10^{-29} \text{ m}^2 \quad \& \quad \frac{dE}{dt} = \alpha_T c U_{\text{red}}$$

Any unpolarized radiation can be regarded as the superposition of polarized waves.

If the radiation is scattered through an angle α (Θ is measured between the electron's acceleration & the radiation, thus $\alpha = \pi/2 - \Theta$), we see that the incoming wave can be seen as

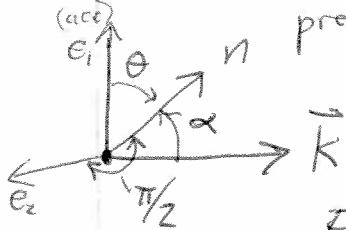
two waves with perpendicular axes. One is scattered by α , & the other by ~~10°~~. So we get the angular cross-section by averaging them;

$$\frac{d\sigma}{d\Omega} = \frac{1}{2} \left(\frac{d\sigma(\theta)}{d\Omega} + \frac{d\sigma(\pi/2)}{d\Omega} \right) = \frac{r_0^2}{2} (1 + \cos^2 \alpha)$$

which depends only on α , appropriate for unpolarized light.

Thomson scattering is symmetric forwards vs. backwards; & vs. $\pi - \alpha$.

The two terms in last equation are intensities in 2 perpendicular directions, both normal to the direction of ~~acceleration of electron~~ propagation of radiation \vec{k} . ~~Call these directions~~ $e_1^{(acc)}$, $e_2^{(acc)}$.



Electron scattering of even unpolarized light gives some degree of polarization, depending on viewing angle.

If we look at radiation scattered 180° (straight backwards), we see no net polarization.

If we look at radiation scattered 90° , we see 100% polarization, since the electron's motion must be \perp to the incoming wave, & for us to see any radiation its motion must be \perp to our line of sight,

Reflection from the surface of metals is Thomson scattering (of essentially free electrons in the metal). As the boundary gives a preferred direction of reflection, these reflections are (generally) polarized.

Polarized radiation is common in radio, & well-studied. Also seen in optics, etc.

New US NASA space satellite, GEMS, to perform X-ray polarization studies.

Will constrain the geometry of X-ray sources, such as the scattering geometry of disks around black holes,