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GPU Accelerated Hall Magnetohydrodynamics

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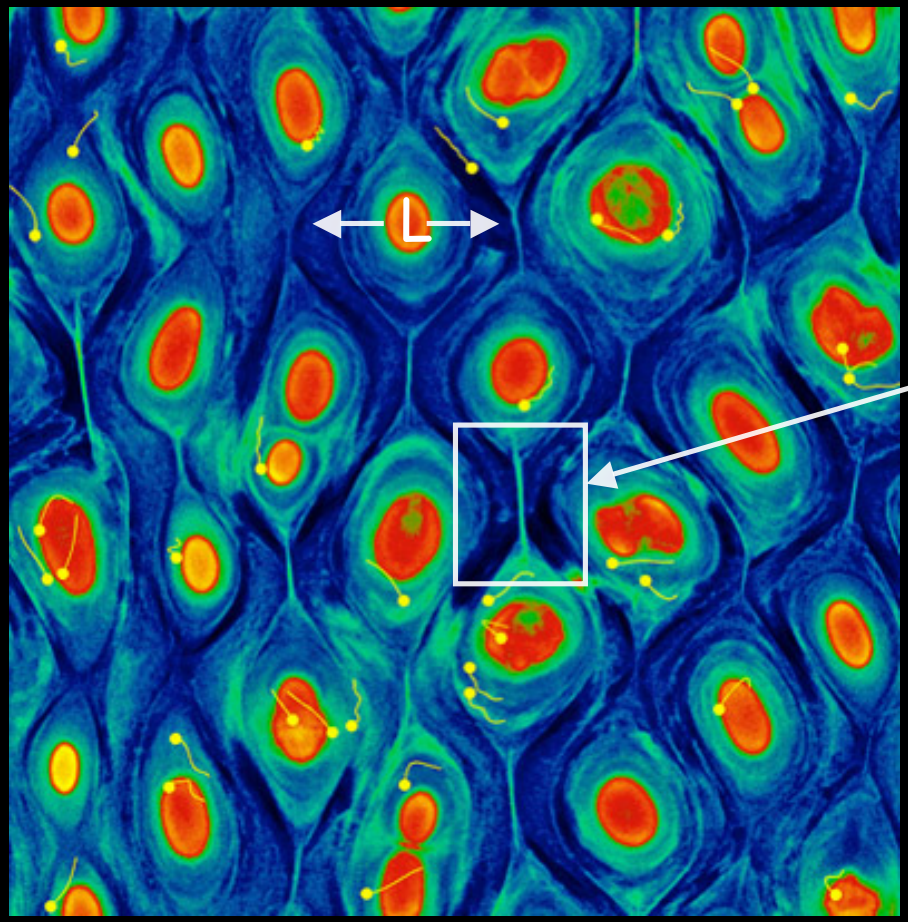
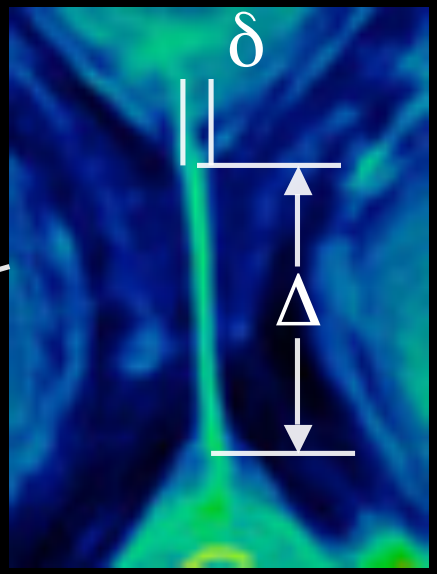
The Hall effect plays an essential role in fast (dissipation-insensitive) magnetic reconnection in collisionless plasmas.

Graphics Processing Units make large scale Hall MHD simulations computationally feasible and cost effective.

What happens when two current filaments meet?



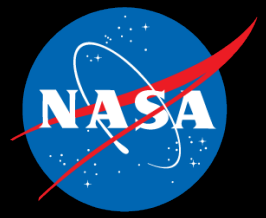


δ = current sheet thickness

Δ = current sheet length

L = "system size"

Princeton "Art of Science" Gallery, 2010.
Lorenzo Sironi and Anatoly Spitkovsky
Dept. of Astrophysical Sciences



The GEM Reconnection Challenge

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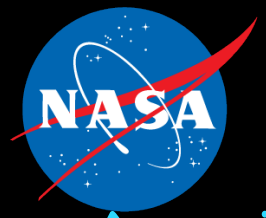
Reference	Physical Model
Hesse et al., JGR, 106, 3721-3735, 2001	Electromagnetic PIC, Resistive Hall MHD
Birn, J. and M. Hesse, JGR, 106, 3737-3750, 2001	Resistive MHD, Resistive Hall MHD
Otto, A., JGR, 106, 3751-3757, 2001	Resistive MHD, Resistive Hall MHD
Shay et al., JGR, 106, 3759-3772, 2001	Collisionless Hall MHD Collisionless Hybrid Electromagnetic PIC
Ma, Z. W. and A. Bhattacharjee, JGR, 106, 3773-3782, 2001	Resistive Hall MHD
P. L. Pritchett, JGR, 106, 3783-3798, 2001	Electromagnetic PIC

A number of independent codes were run with identical initial/boundary conditions. All of the codes (save resistive MHD with constant resistivity) produced fast reconnection. What did the "fast" codes have in common? The Hall term.



Generalized Ohm's Law

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Assuming quasineutrality, and neglecting terms proportional to m_e/m_i, one can derive the following expression for the electric field.

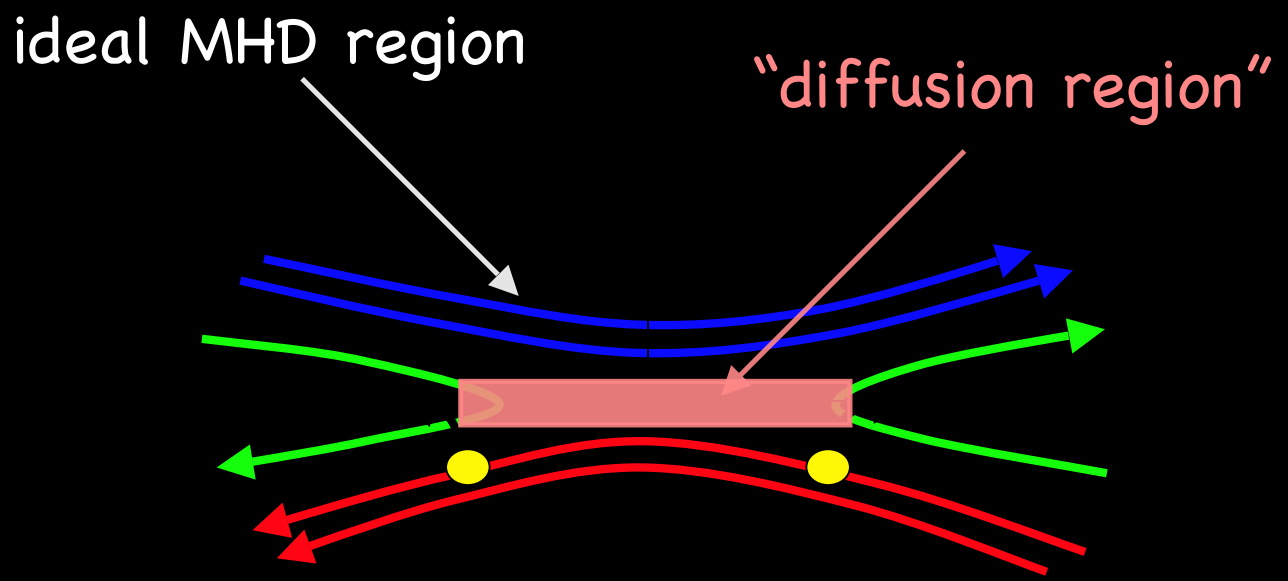
$$E = -\frac{1}{c}U \times B + \frac{1}{nec}J \times B - \frac{1}{ne}\nabla \cdot P_e^{CM}$$
$$- \frac{m_e}{ne^2}\frac{\partial J}{\partial t} - \frac{m_e}{ne^2}\nabla \cdot (UJ + JU)$$
$$+ \sum_\alpha \int dv\, q_\alpha n_\alpha v \left(\frac{\partial f_\alpha}{\partial t}\right)_c$$

Ideal MHD term (Frozen Flux Theorem)
Hall term (Electron Frozen Flux Theorem) [ION INERTIAL SCALE]
Electron Pressure Anisotropy (thaws magnetic flux) [ION INERTIAL SCALE]
Electron Inertia (thaws magnetic flux) [ELECTRON INERTIAL SCALE]
Particle Scattering (thaws magnetic flux - e.g., resistivity)

Resistive MHD

"External" perfectly conducting fluid + "internal" resistive boundary layers....

$$E = -\frac{1}{c} U \times B \qquad + \sum_{\alpha} \int dv\, q_\alpha n_\alpha v \left(\frac{\partial f_\alpha}{\partial t} \right)_c \longleftarrow \eta J$$



Sweet-Parker Current Sheets

P. A. Sweet

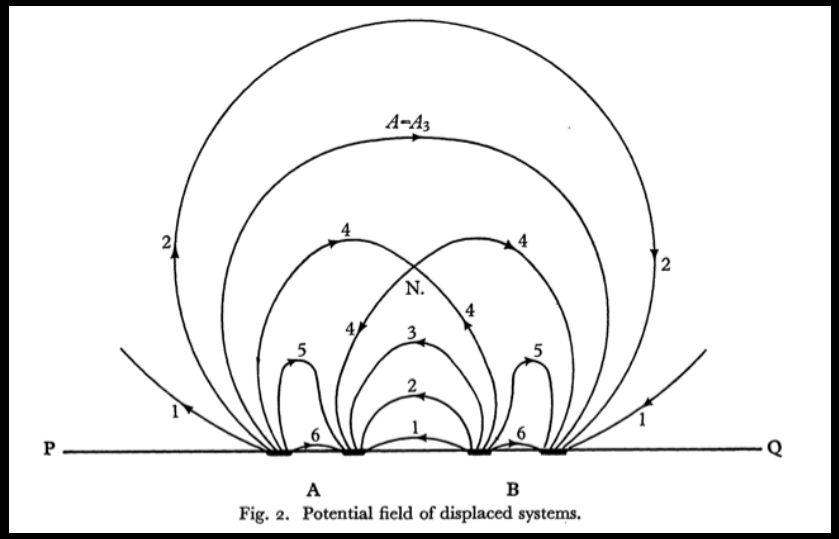
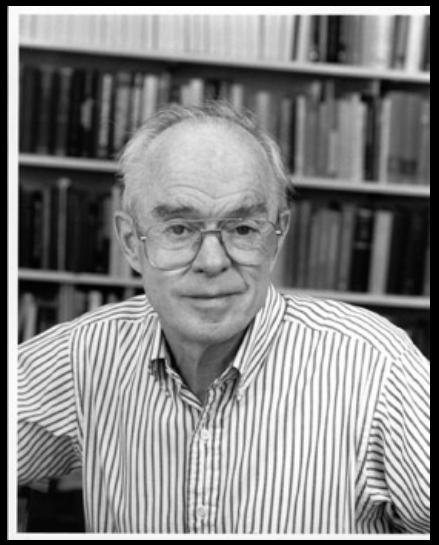


Fig. 2. Potential field of displaced systems.



E. N. Parker

What happens when two magnetic dipoles are forced together by external plasma motions?

Parker, E. N., Sweet's mechanism for merging magnetic fields in conducting fluids, J. Geophys. Res., 62, 1957.

Sweet, P. A., The neutral point theory of solar flares, Electromagnetic phenomena in cosmical plasmas, Proceedings from the IAU Symposium No. 6, 1958.

Sweet-Parker Reconnection

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P. A. Sweet

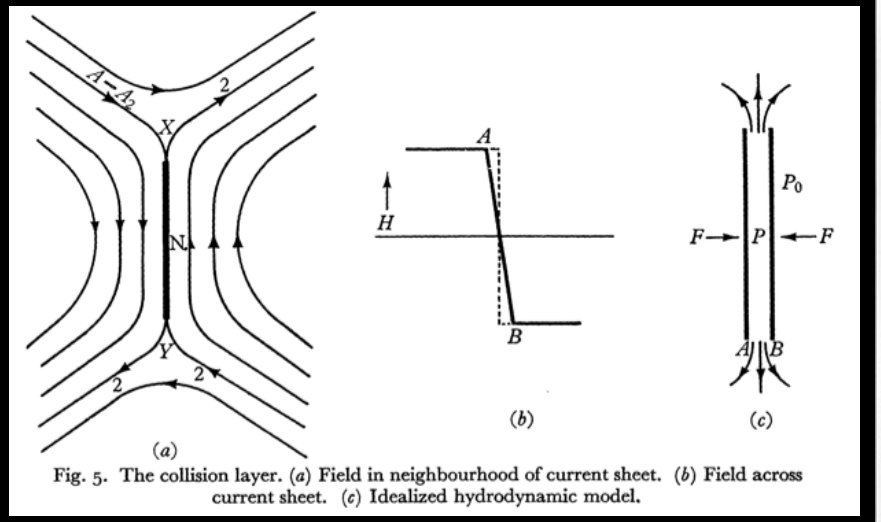
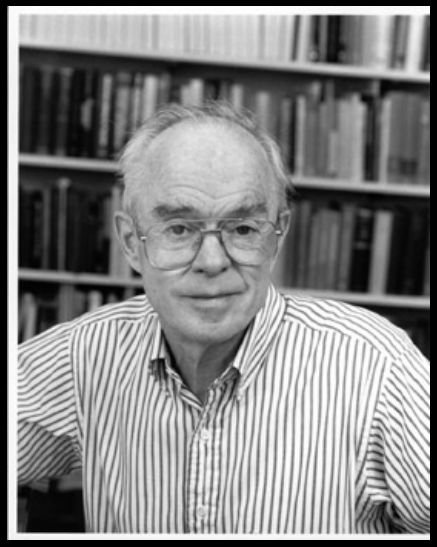
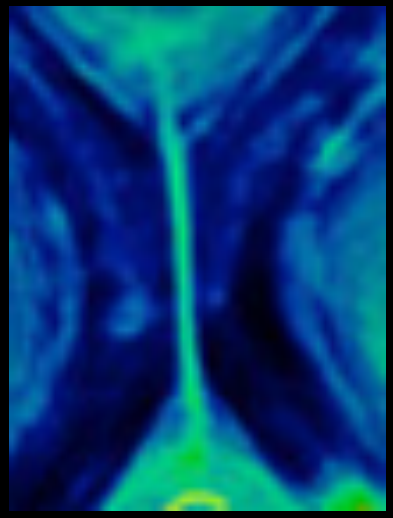


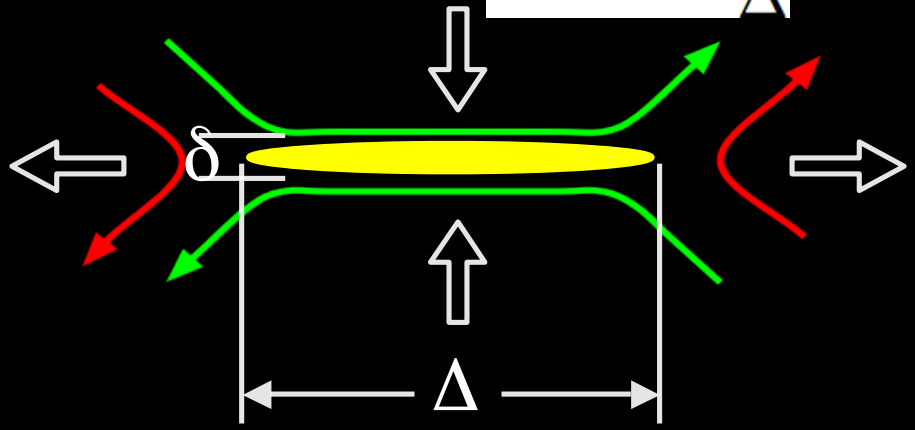
Fig. 5. The collision layer. (*a*) Field in neighbourhood of current sheet. (*b*) Field across current sheet. (*c*) Idealized hydrodynamic model.



E. N. Parker



$$U_{in} = V_A \frac{\delta}{\Delta}$$

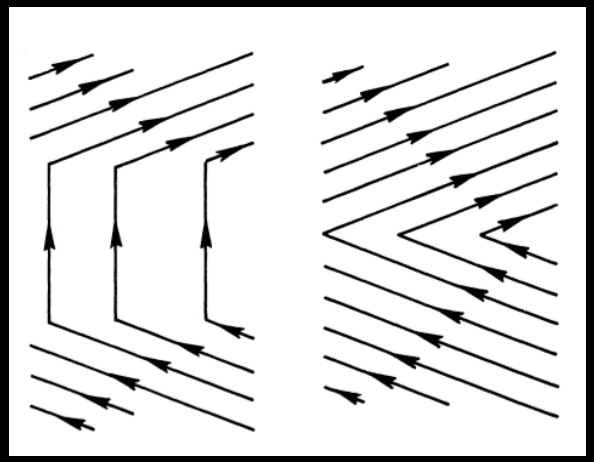


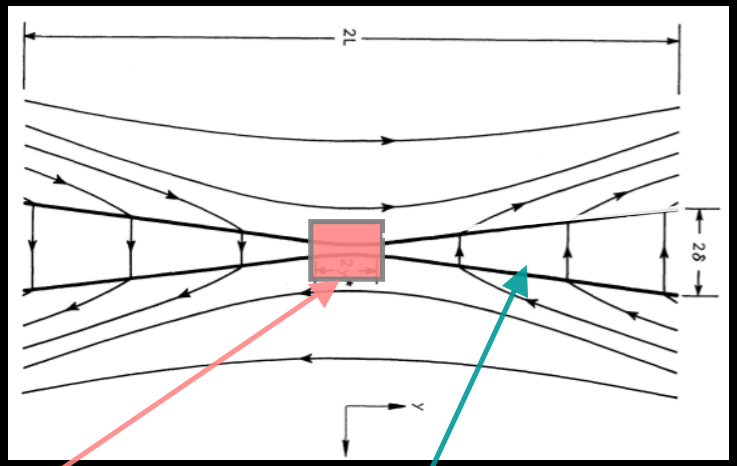
$$V_{out} = V_A$$

$$\delta \propto \sqrt{\eta}$$

The Importance of Plasma Waves

Harry Petschek





Diffusion

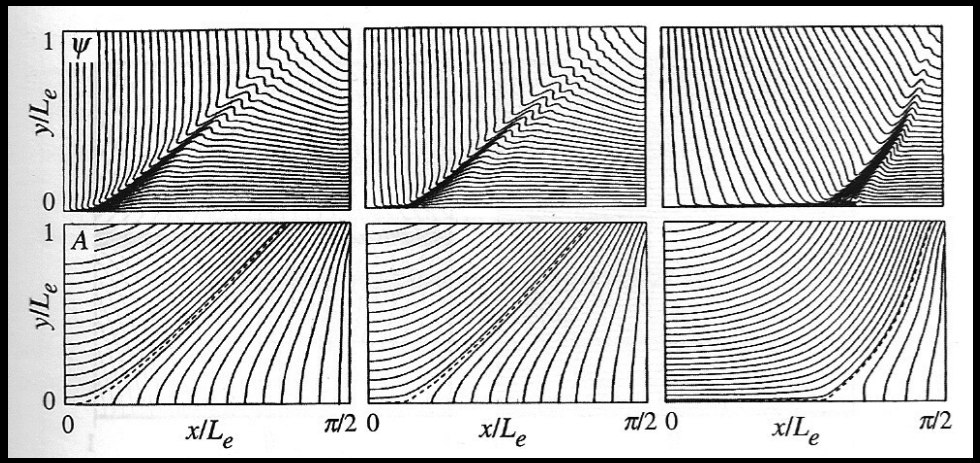
Waves

Waves play the dominant role (rather than diffusion) in converting magnetic energy to plasma energy.
Reconnection becomes insensitive to the plasma resistivity!

Dr. Sweet: I would like to make two points: One is that I am in favor of your theory, which I thoroughly approve. Dr. Parker and I have been living with this problem for several years and have got the feel of it. Your solution struck me at once as the solution for which we have been seeking.

Petschek, H., Magnetic field annihilation, in Physics of Solar Flares, et. W. N. Ness, NASA SP-50, 425, 1964.

Did Petschek correctly match the diffusion and wave solutions?



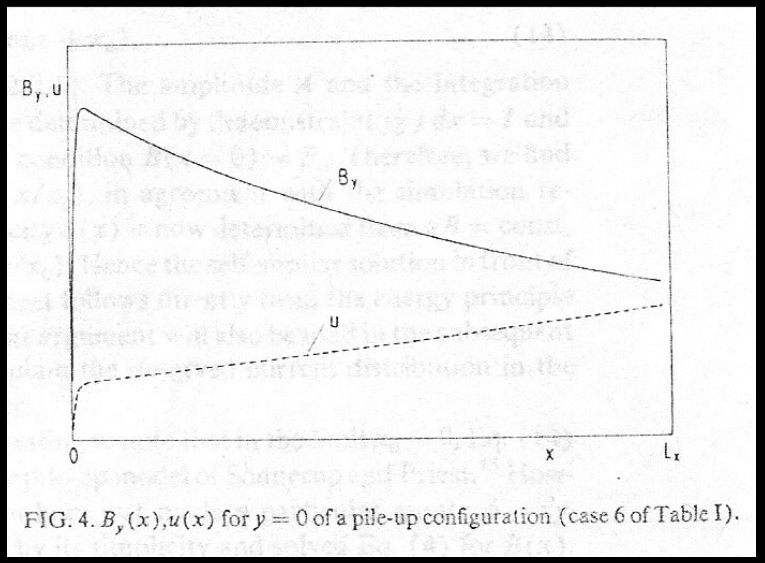


FIG. 4. $B_y(x), u(x)$ for $y = 0$ of a pile-up configuration (case 6 of Table I).

Biskamp, D., Magnetic reconnection via current sheets, *Phys. Fluids,* 29, 1520-1531, 1986.

Δ increases with decreasing resistivity; this directly contradicts the Petschek model, which requires $\Delta \sim \delta \sim \eta$.

the particular choice of boundary conditions. Because of the presence of a current sheet, the overall reconnection process is quite slow. This picture essentially agrees with Syrovatsky's [Sov. Phys. JETP **33**, 933 (1971)] theory and disproves Petschek's [*AAS/NASA Symposium on the Physics of Solar Flares*, (NASA, Washington, DC, 1964) p. 425] mechanism of fast magnetic

The Hall Effect (Hall MHD)

Ion fluid elements are no longer tied to field lines; electron fluid elements remain frozen….

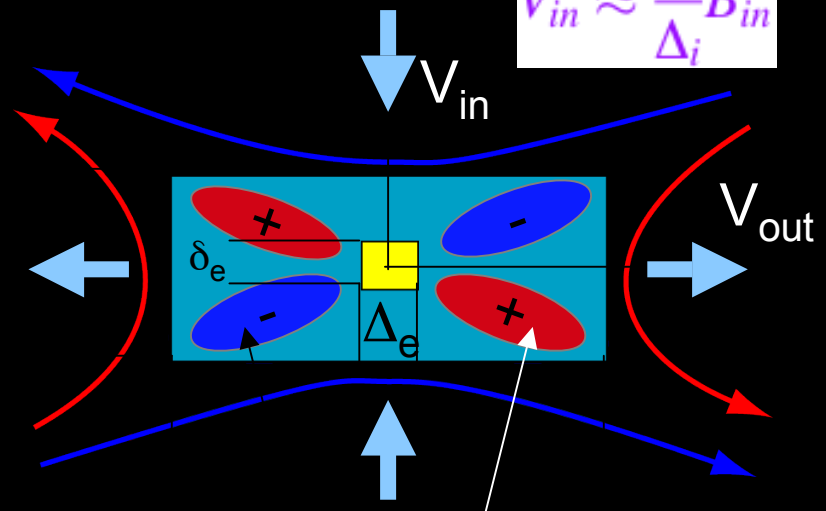
$$E = -\frac{1}{c}U \times B + \frac{1}{nec}J \times B - \frac{1}{ne}\nabla \cdot P_e^{CM}$$

$$J = ne(U_i - U_e)$$

$$E = -\frac{1}{c}U_e \times B$$

Petschek resurrected?

$$V_{in} \approx \frac{d_i}{\Delta_i} B_{in}$$



out-of-plane magnetic field, B_z

Whistler waves?

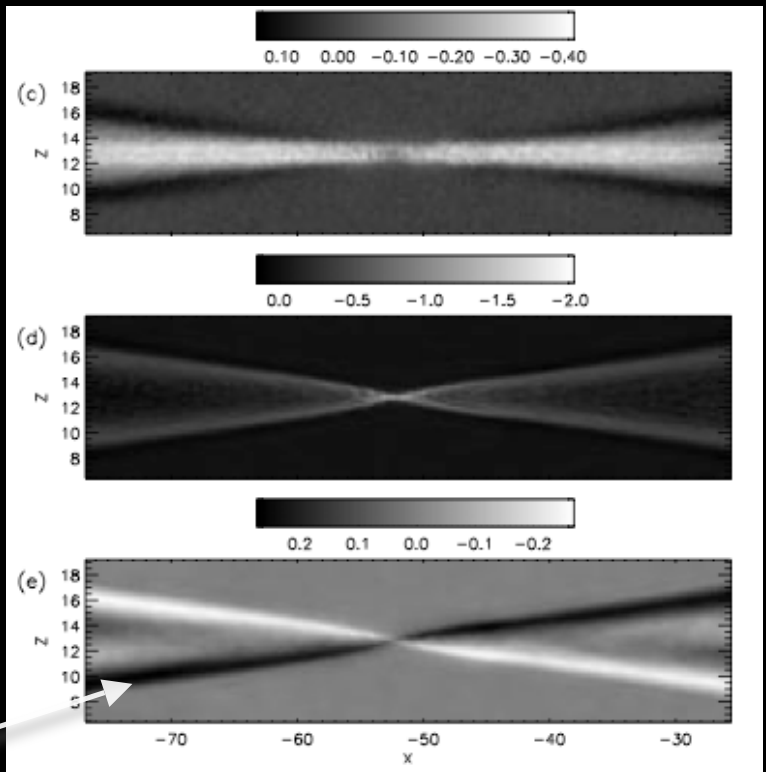
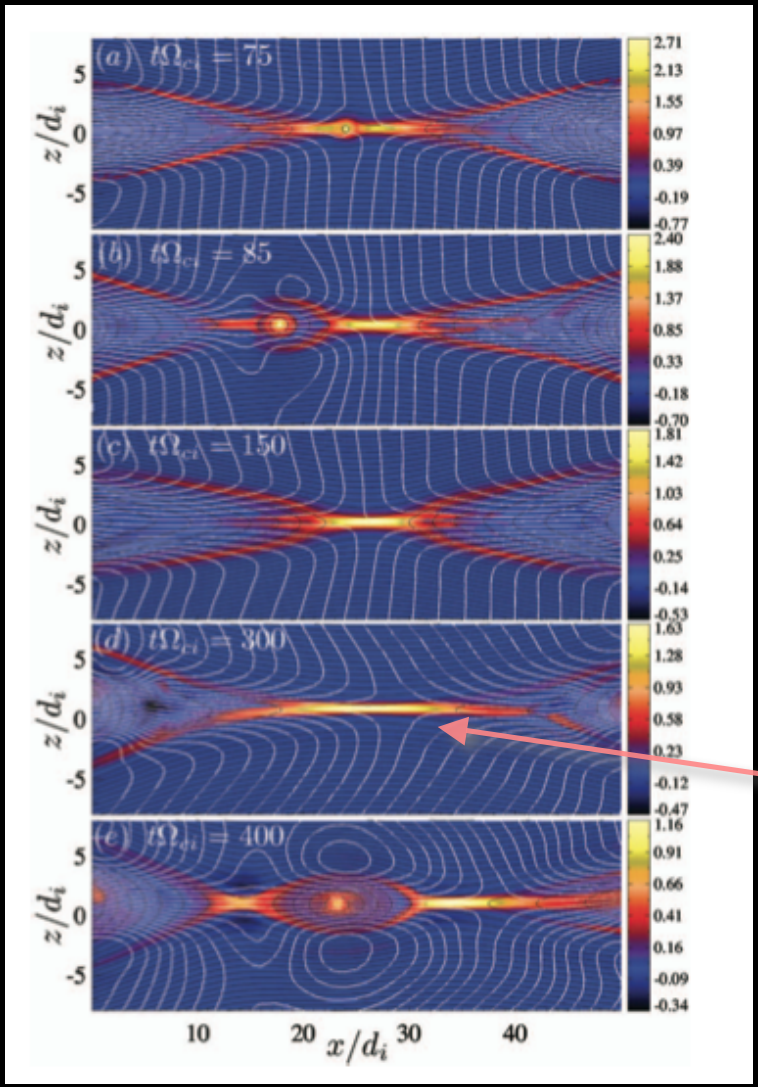


Figure 1. Structure of the x-line: (a) in-plane magnetic field, (b) in-plane ion velocity, (c) out-of-plane ion current, (d) out-of-plane electron current, (e) out-of-plane magnetic field.

"The reconnection rate is found to be a universal constant, corresponding to an inflow velocity…of around 0.1 V_A."
M. Shay

Shay, M. A., J. F. Drake and B. N. Rogers, The scaling of collisionless magnetic reconnection for large systems, *Geophys. Res. Lett.*, 26, 2163-2166, 1999.

Some "non-standard" ideas….



Reconnection is much slower in large systems (di/L <<1).

Wang and Bhattacharjee, PRL, 2001.
Dorelli and Birn, JGR, 2003.
Dorelli, Phys. Plasmas, 2003.
Fitzpatrick, Phys. Plasmas, 2004.
Knoll and Chacon, PRL, 2006.
Karimabadi et al., PRL, 2011.

"Open BC" PIC simulations show extended electron diffusion regions (limited only by secondary island formation?)

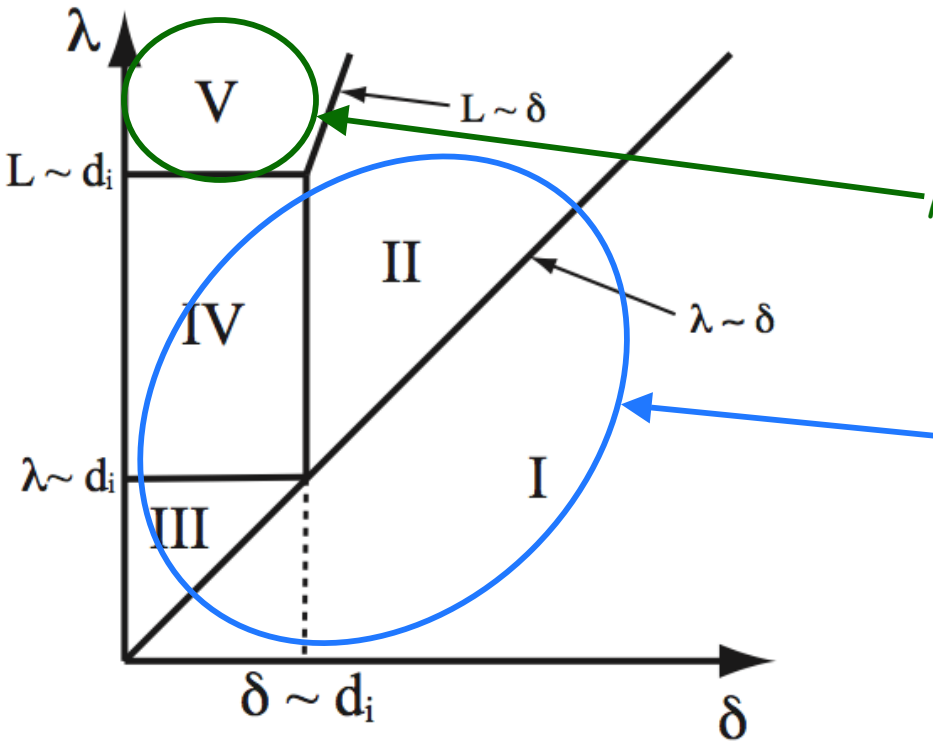
Daughton et al., Phys. Plasmas, 2006.

Though see the following interpretation (more in line with the Standard Model)

Hesse et al., Phys. Plasmas, 2008.

How does the Hall effect "scale up" to large system size?

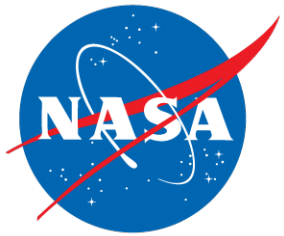
A Magnetic Reconnection Phase Diagram



A few of us are exploring region V

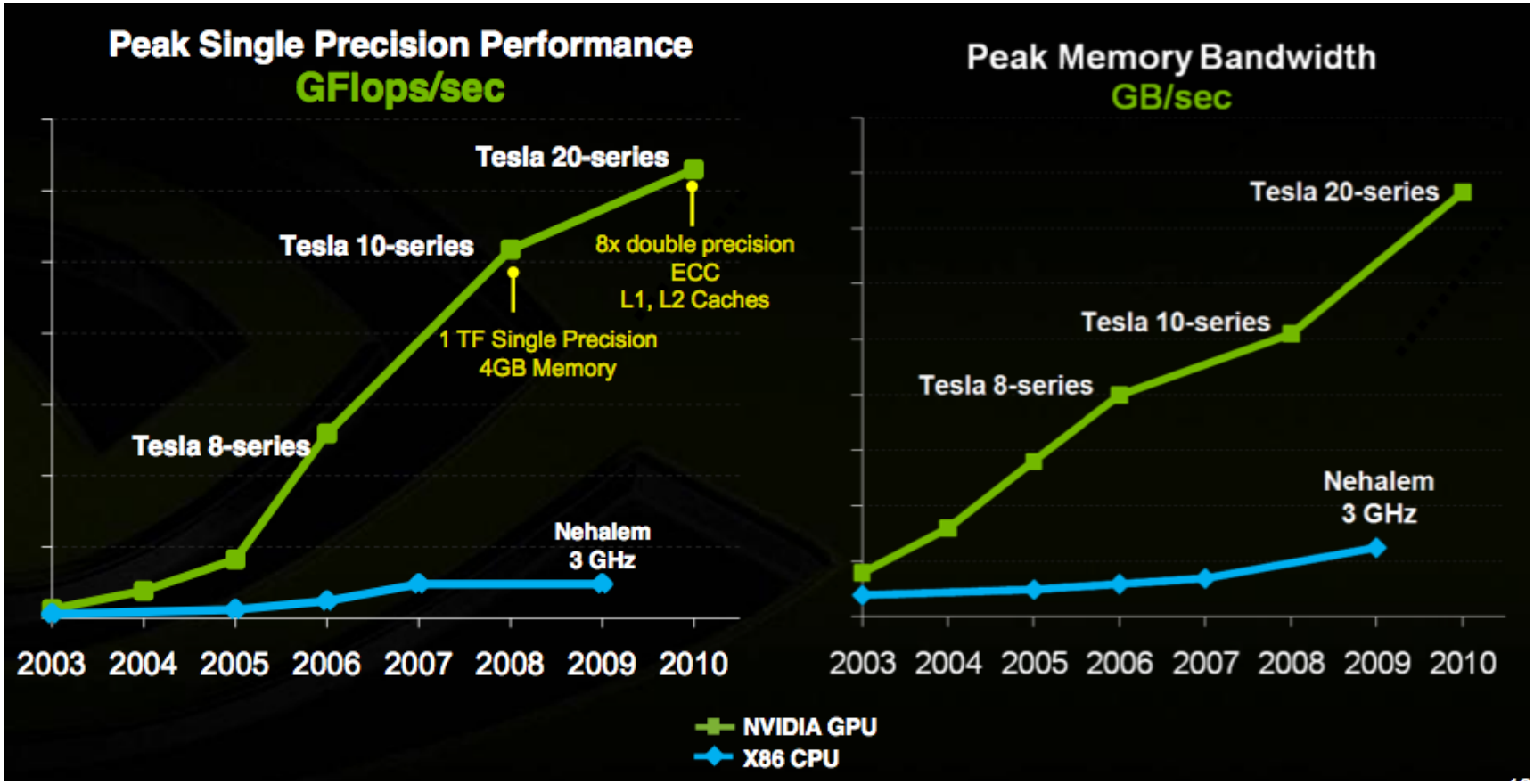
Most of us are stuck here

Does the reconnection rate drop with increasing system size?



Why GPUs?

from www.nvidia.com



GPU-CPU performance gap has been steadily increasing for the last 5 years: The latest Tesla 20-series cards ("Fermi" architecture, with up to 6GB GDDR5) are designed with large scale HPC apps in mind.

A Desktop Supercomputer

GeForce GTX 480

1.5 GB GDDR5
Fermi architecture
15 multiprocessors
(480 cores)
~$400

The GTX 480 provides the same Fermi architecture (albeit with 0.25-0.5 the memory and 0.5 the double precision performance) but at a tenth of the cost of the Tesla C2070 -- Total cost of our workstation: ~$3K!

Hall MHD Equations

$$\mathbf{V}_H = -\delta_i \frac{\mathbf{J}}{\rho}$$

$$\mathbf{J} = \nabla \times \mathbf{B}$$

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{V}) = 0$$

$$\frac{\partial}{\partial t}(\rho \mathbf{V}) + \nabla \cdot \left[\rho \mathbf{V}\mathbf{V} + \left(p + \frac{B^2}{2}\right)\mathbf{I} - \mathbf{B}\mathbf{B}\right] = 0$$

$$\frac{\partial}{\partial t}\left(\frac{1}{2}\rho V^2 + \frac{3}{2}p + \frac{B^2}{2}\right) + \nabla \cdot \left[\frac{1}{2}\rho V^2 \mathbf{V} + \frac{5}{2}p\mathbf{V} + (\mathbf{V} + \mathbf{V}_H)B^2 - (\mathbf{V} \cdot \mathbf{B} + \mathbf{V}_H \cdot \mathbf{B})\mathbf{B}\right] = 0$$

$$\frac{\partial \mathbf{B}}{\partial t} + \nabla \cdot [(\mathbf{V} + \mathbf{V_H})\mathbf{B} - \mathbf{B}(\mathbf{V} + \mathbf{V}_H)] = 0$$

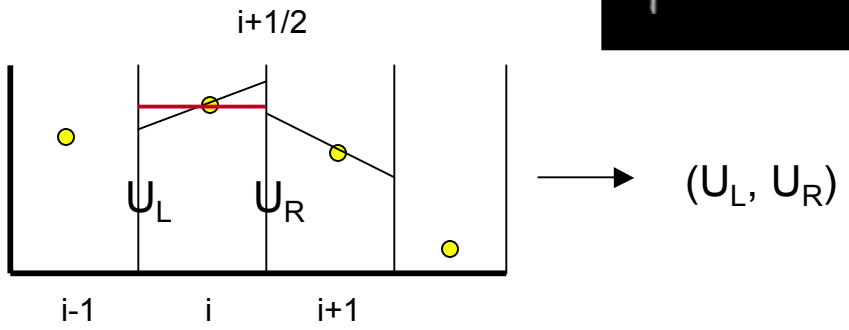
Godunov Solver

- 2nd order MUSCL-Hancock scheme on a uniform grid -- van Leer, 1985.
- Strang dimensional splitting for 2nd order time stepping (currently 2D) -- Strang, 1968.
- HLL approximate Riemann solver -- Toro, 1999
- Hyperbolic divergence cleaning to control magnetic field divergence errors -- Dedner et al., 2002.
- Second order differencing of current density for Hall term + MC slope limiter to reconstruct all conservative variables (acts like hyper-resistivity) -- Toth et al., J. Comp. Phys., 2008.

This scheme robustly captures MHD discontinuities -- the addition of the Hall effect tends to smear these out due to the physical dispersion introduced by whistler waves.

MUSCL-Hancock Scheme

Linearly extrapolate cell centered variables to cell interfaces



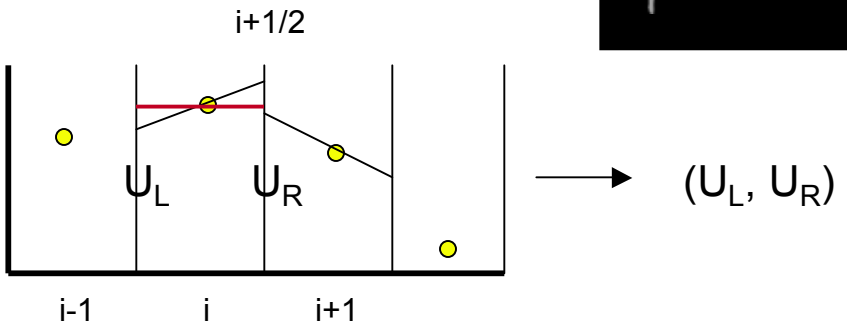
Slope limiters control the appearance of new local extrema (but they introduce numerical dissipation that can be hard to quantify)

MUSCL-Hancock Scheme

Linearly extrapolate cell centered variables to cell interfaces

↓

Advance cell interfaces by 1/2 time step (conservative differencing)



$$\tilde{U}_{L,R} = U_{L,R} - \frac{\Delta t}{2\Delta x}[F(U_R) - F(U_L)]$$

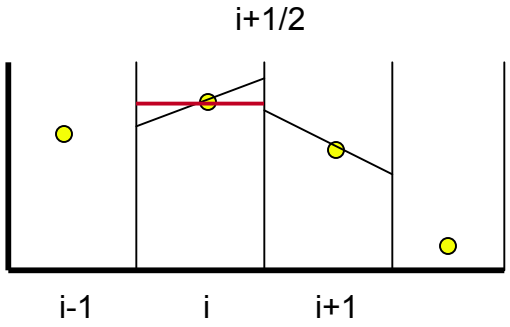
"Predictor step" used to achieve higher (second order) temporal accuracy for numerical flux.

MUSCL-Hancock Scheme

Linearly extrapolate cell centered variables to cell interfaces

Advance cell interfaces by 1/2 time step (conservative differencing)

Apply Riemann solver (HLL) to compute numerical flux at interface



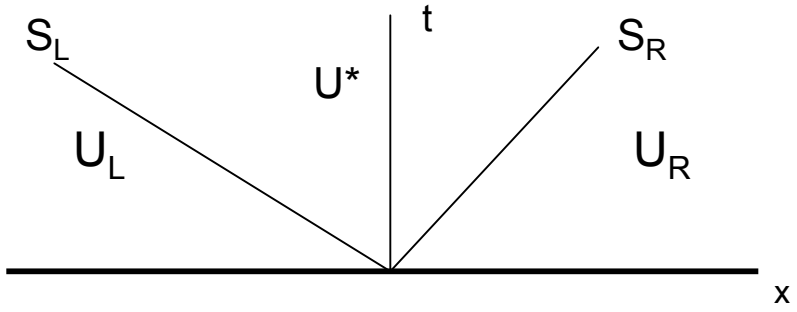
$\rightarrow (U_L, U_R)$

$$\tilde{U}_{L,R} = U_{L,R} - \frac{\Delta t}{2\Delta x}[F(U_R) - F(U_L)]$$

$$Riemann(\tilde{U}_{L,i+1}, \tilde{U}_{R,i}) \rightarrow \hat{F}_{i+1/2}$$

("Numerical Flux")

Riemann Problem: Given a 1D interface with constant left and right states, compute the time evolution of the interface state (U*).



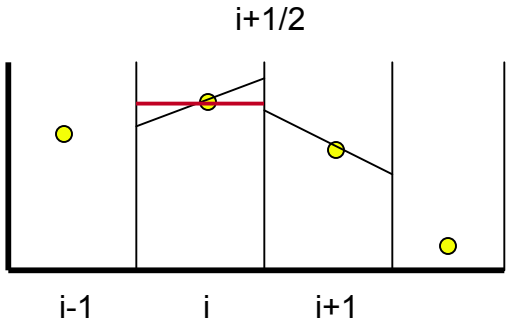
MUSCL-Hancock Scheme

Linearly extrapolate cell centered variables to cell interfaces

Advance cell interfaces by 1/2 time step (conservative differencing)

Apply Riemann solver (HLL) to compute numerical flux at interface

Advance cell centers full time step (conservative differencing)



$\longrightarrow (U_L, U_R)$

$$\tilde{U}_{L,R} = U_{L,R} - \frac{\Delta t}{2\Delta x}[F(U_R) - F(U_L)]$$

$$Riemann(\tilde{U}_{L,i+1}, \tilde{U}_{R,i}) \longrightarrow \hat{F}_{i+1/2}$$

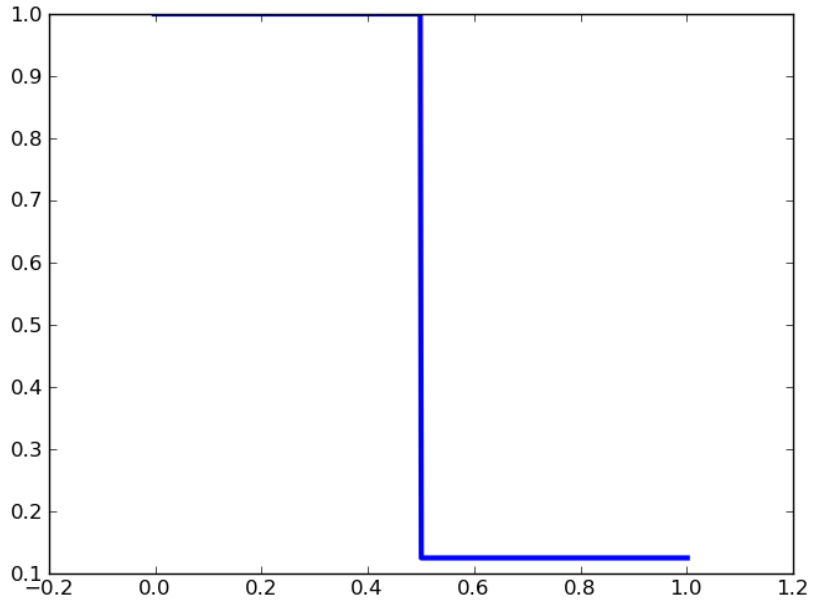
("Numerical Flux")

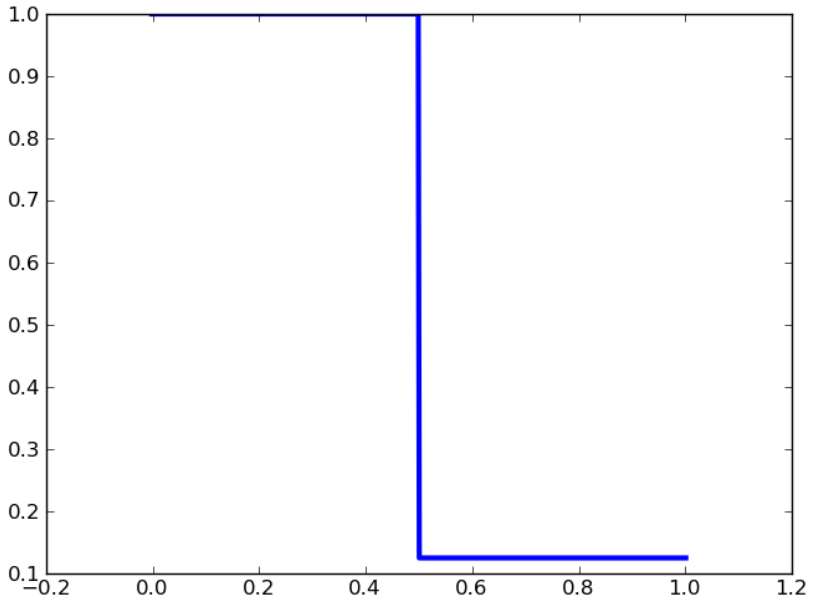
$$U_i^{n+1} = U_i^n - \frac{\Delta t}{\Delta x}(\hat{F}_{i+1/2} - \hat{F}_{i-1/2})$$

This scheme robustly captures MHD discontinuities -- the addition of the Hall effect tends to smear these out due to the physical dispersion introduced by whistler waves.

Hall MHD Riemann Problem

(Brio-Wu Shock Tube)



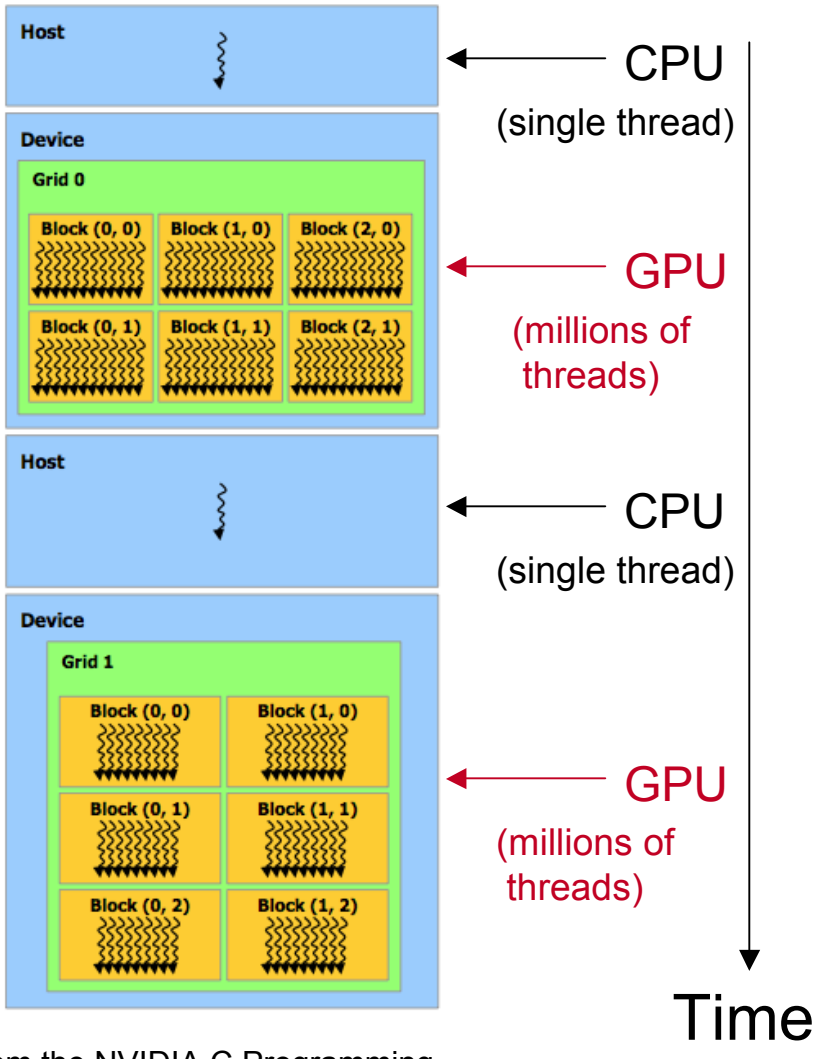


M. Brio and C. C. Wu, An upwind differencing scheme for the equations of ideal magnetohydrodynamics, J. Comp. Phys., 75, 400-422, 1988.

Whistler waves introduce physical dispersion: "signal speed" increases with decreasing wavelength (CFL challenge for explicit solvers)

GPU Implementation

Heterogeneous CPU-GPU programming model



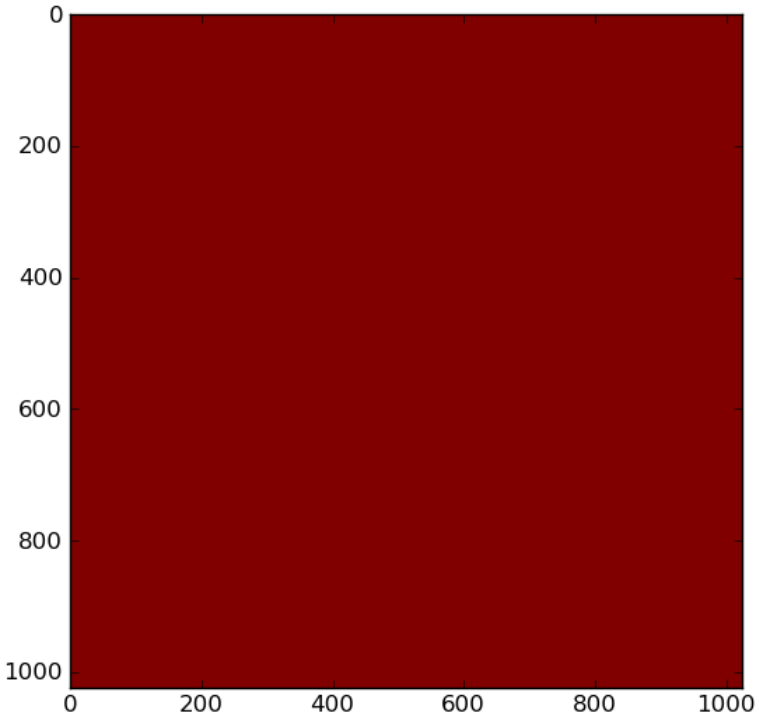
from the NVIDIA C Programming Guide v3.1.1

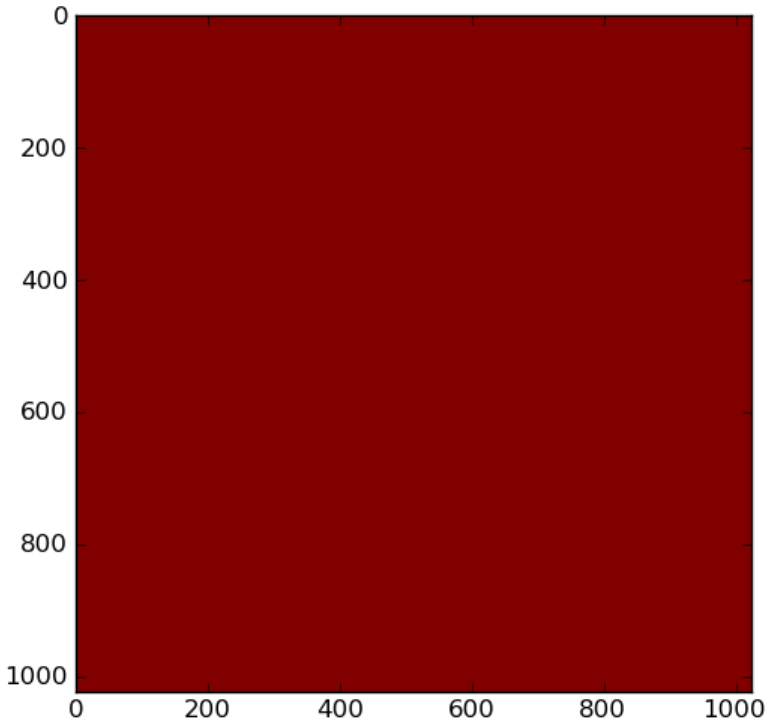
```
void evolvex(double* p_cons, double dt){

   dim3 dimGrid(NX/TILE_WIDTH, NZ/TILE_WIDTH);
   dim3 dimBlock(TILE_WIDTH, TILE_WIDTH);
   size3 = NX*NZ*NVAR*sizeof(double);

   cudaMalloc((void**)&p_xslope, size3);

   xslope_ker<<<dimGrid, dimBlock>>>(p_cons,p_xslope);

   cudaMalloc((void**)&p_dright, size3);
   cudaMalloc((void**)&p_dleft, size3);

   d_from_slope_ker<<<dimGrid, dimBlock>>>(p_dright, \
      p_dleft, p_cons, p_xslope);

   cudaMalloc((void**)&p_fleft);
   cudaMalloc((void**)&p_fright);

   physical_flux_f_ker<<<dimGrid, dimBlock>>>(p_dright, \
      p_fright);
   physical_flux_f_ker<<<dimGrid, dimBlock>>>(p_dleft, \
      p_dleft);

                                *
                                *
                                *
```

CPU sequentially launches GPU “kernels”, each of which consists of millions of threads; each thread is responsible for a single computational cell.

Hall MHD Orszag-Tang

Pressure





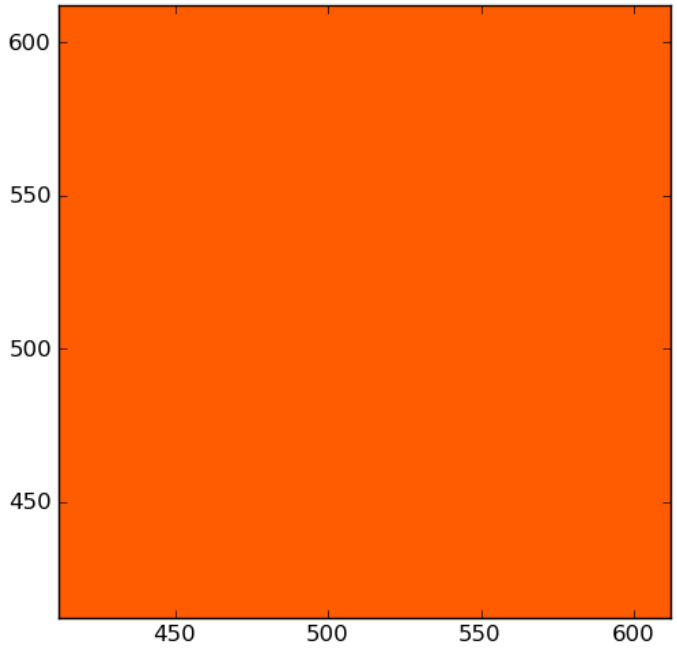
1024x1024 MHD

1024x1024 Hall MHD

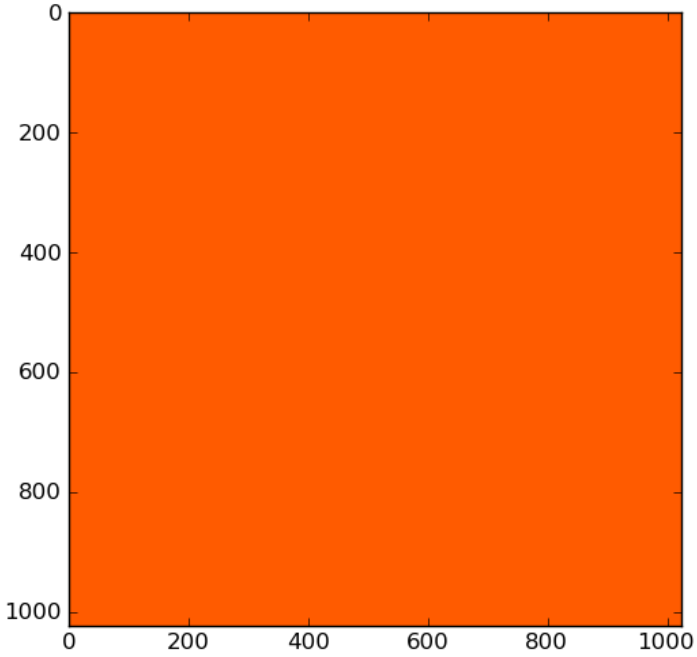
Hall MHD simulation of Orszag-Tang [1979] Vortex: Dispersive waves smear out sharp boundaries and add additional structure at short scales. Stretched grids or AMR will have trouble capturing this.

Secondary Island Formation in Hall MHD

Toroidal (out-of-plane) Magnetic Field

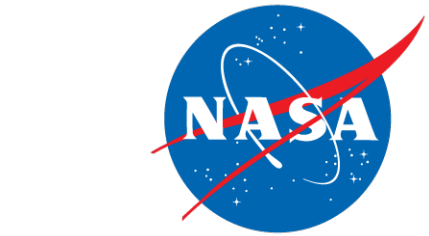


1024x1024 Hall MHD

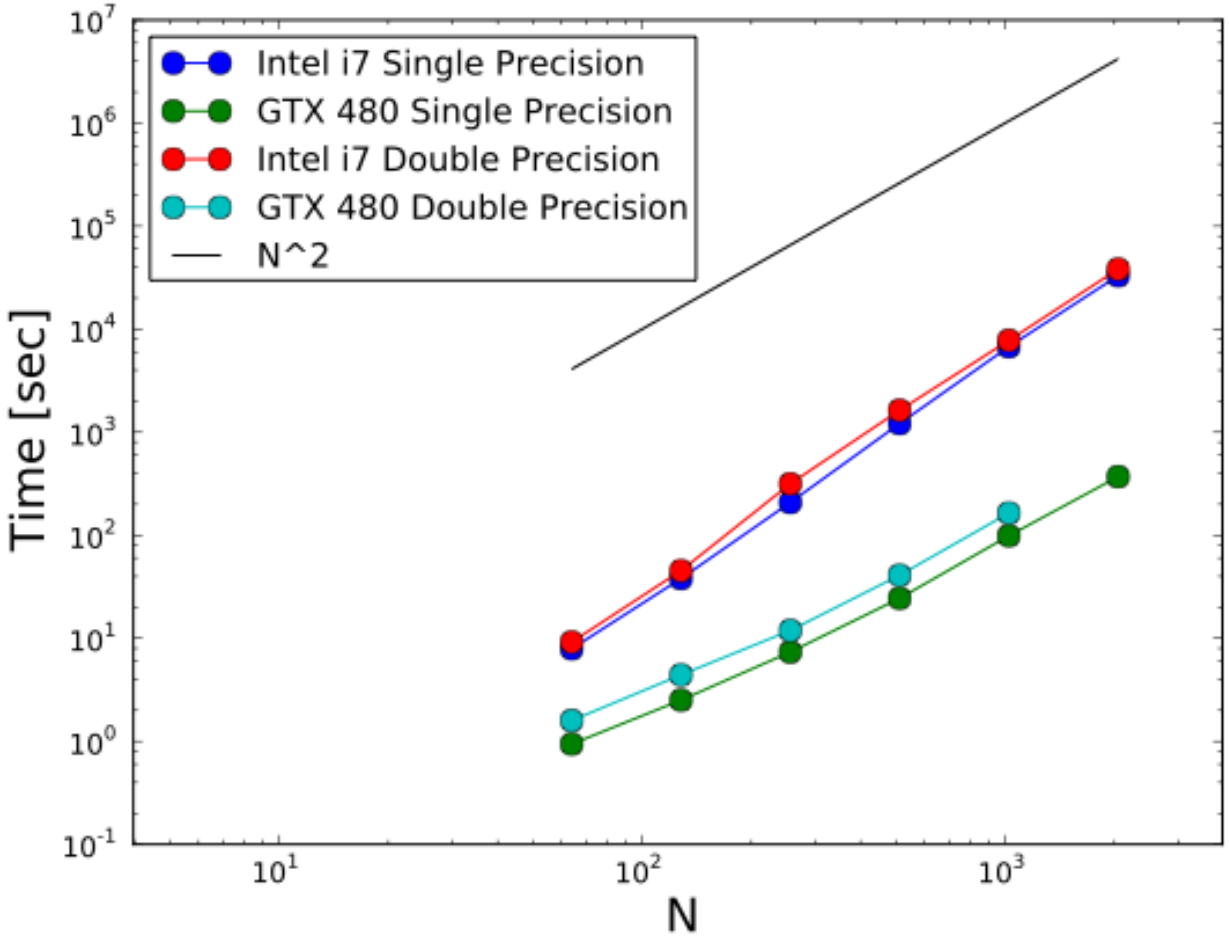


1024x1024 Hall MHD

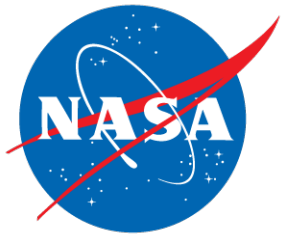
Hall MHD simulation of Orszag-Tang [1979] Vortex: Whistler turbulence may be playing an important role in secondary island formation.



Timing Results



NX = NZ = N: For N = 1024, we achieve a single (double) precision speedup of 87x (58x).



The Future (i.e., last year….)

Tianhe-1A: CPU-GPU Cluster at the National Supercomputing Center in Tianjin, China

7168 NVIDIA M2050 GPUs

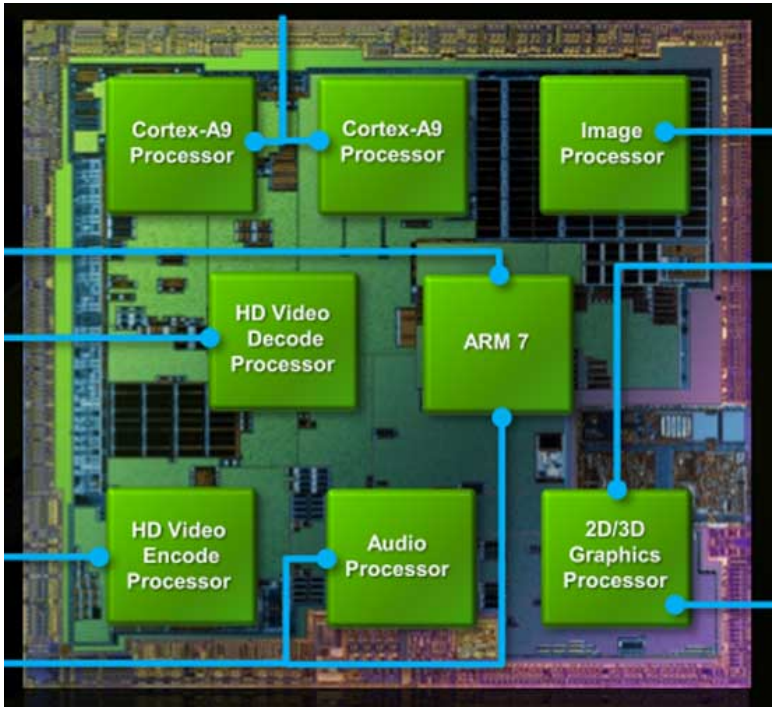
2.5 Pflops LINPACK

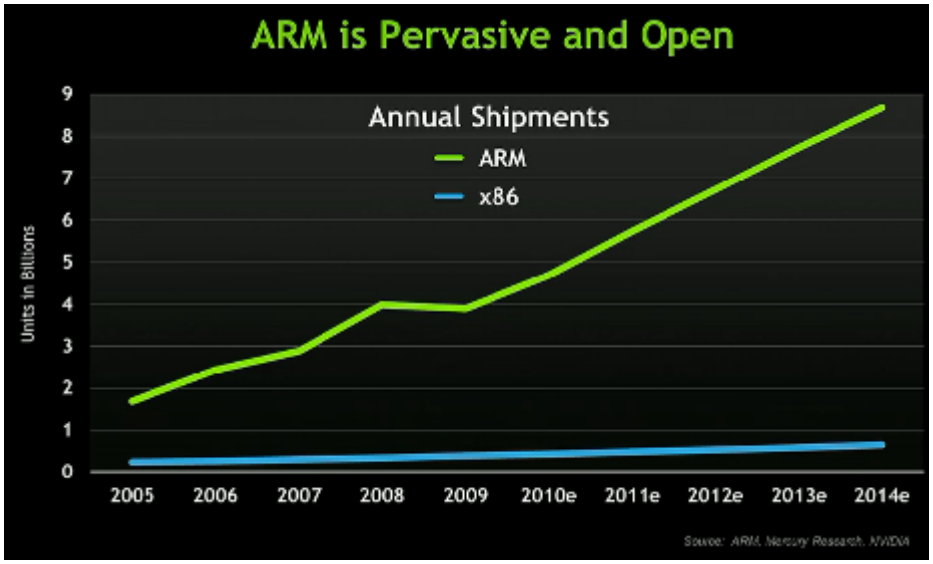
#1 in the Top500 (ORNL's Jaguar is #2)

"Heterogeneous Computing" -- Clusters of CPUs/GPUs will compete with larger scale multi-core systems (like Blue Waters).

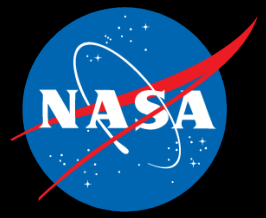
The Future (next several years….)

NVIDIA Embraces the ARM Processor for Heterogeneous HPC (Project Denver)





"Heterogeneous Computing" -- CPUs (ARM architecture) and GPUs on the same chip will make CPU-GPU clusters much more attractive in the coming years.



Summary

1) Large scale (several billion computational cells; large separation between dissipation scale and di; simulation box thousands of di long) Hall MHD simulations will become increasingly important in the next several years.

2) GPU computing will make such large scale simulations feasible on cheaper, physically smaller machines.