Distributed Model Predictive Control

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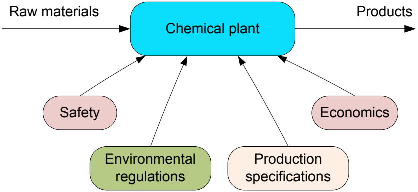
Introduction

- Incentives for chemical process control



  - Need for continuous monitoring and external intervention (control)

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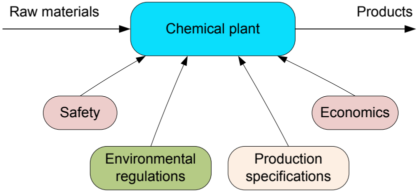
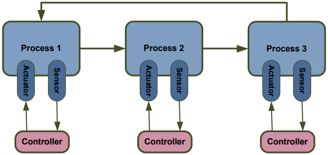


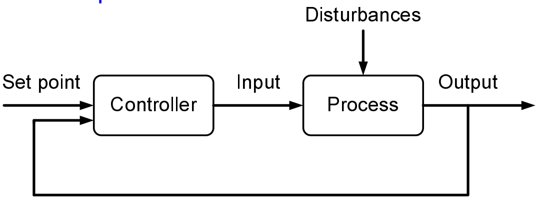
  - Need for continuous monitoring and external intervention (control)

- Objectives of a process control system
  - Ensuring stability of the process
  - Suppressing the influence of external disturbances
  - Optimizing process performance



Feedback Loop/Controller Design

- Feedback control loop



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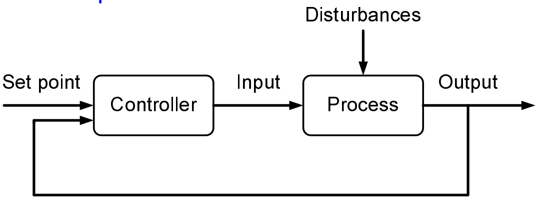

- Classical control (40s-60s): single-input/single-output (SISO) systems
  - Proportional-integral-derivative (PID) control
  - Simplicity of implementation

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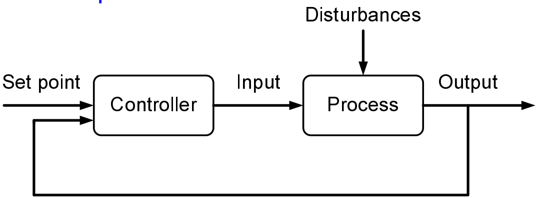

- Classical control (40s-60s): single-input/single-output (SISO) systems
  - Proportional-integral-derivative (PID) control
  - Simplicity of implementation
- Multi-input/multi-output systems
  - Many SISO PID loops/Decentralized approach
  - Does not account for interactions, constraints, nonlinear behavior

Model-Based Controller Design

- Controller design is based on a process dynamic model (60s-today)
  - A mathematical process model is constructed from first-principles or identified from input-output data to describe the process dynamics
  - Controllers are synthesized based on the process model

Model-Based Controller Design

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    - A mathematical process model is constructed from first-principles or identified from input-output data to describe the process dynamics
    - Controllers are synthesized based on the process model
- Advantages-disadvantages of model-based control
    - Possibility of improved closed-loop performance
        - Model accounts for inherent process characteristics (e.g., nonlinear behavior, spatial variations, multivariable interactions)
    - Characterization of limitations on achievable closed-loop stability, performance and robustness
    - It may be difficult to construct a model for a large-scale process

Model Predictive Control

(Carcia et al., Automatica, 1989; Mayne et al., Automatica, 2000)

- Model predictive control (MPC)

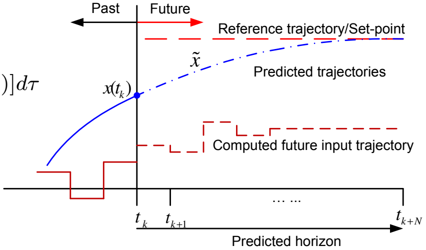
$$\min_{u \in S(\Delta)} \int_{t_k}^{t_{k+N}} [\tilde{x}(\tau)^T Q_c \tilde{x}(\tau) + u(\tau)^T R_c u(\tau)] d\tau$$

s.t. $\dot{\tilde{x}}(t) = f(\tilde{x}(t), u(t), 0)$

$\tilde{x}(t_k) = x(t_k)$

$u(t) \in U$

$\tilde{x}(t) \in X$



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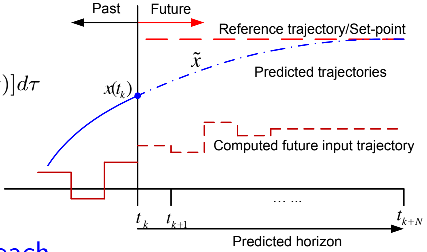

- On-line optimization-based approach
  - Incorporate optimization considerations
  - Explicitly address state and control input constraints

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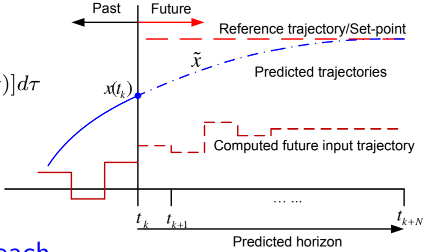
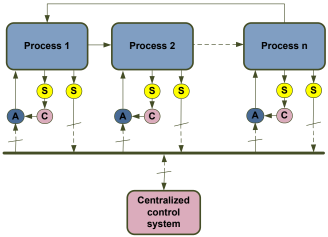
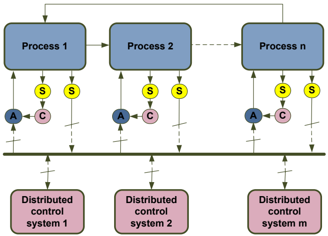


- On-line optimization-based approach
  - Incorporate optimization considerations
  - Explicitly address state and control input constraints
- Approaches to achieve closed-loop stability
  - Infinite prediction horizon
  - Terminal constraint or terminal cost
  - Constraint based on a Lypuanov function

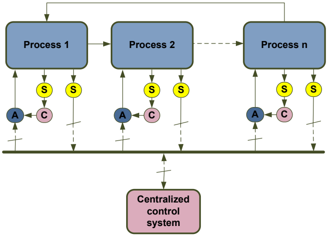
Centralized vs. Distributed Control

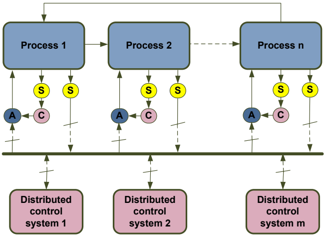




- Centralized process control architecture
  - Computational complexity, fault tolerance
- Move towards distributed process control architecture

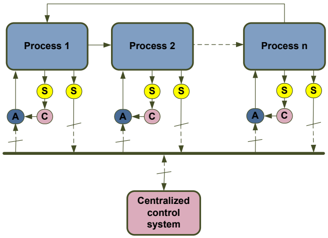
Centralized vs. Distributed Control

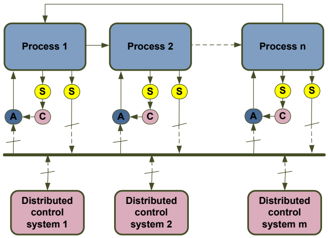




- Centralized process control architecture
  - Computational complexity, fault tolerance
- Move towards distributed process control architecture
- Issues need to be addressed when moving to distributed control
  - Coordination of controllers for stability and performance
  - Communication strategy between distributed controllers

Centralized vs. Distributed Control





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- Move towards distributed process control architecture
- Issues need to be addressed when moving to distributed control
  - Coordination of controllers for stability and performance
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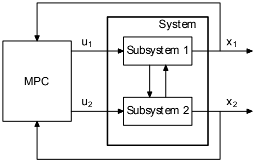
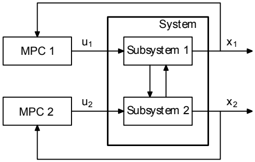
- MPC is a natural framework for distributed control system

Control Architectures

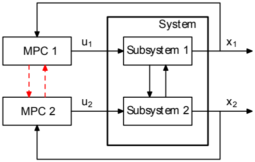
Different control architectures



Centralized control system



Decentralized control system



Distributed control system

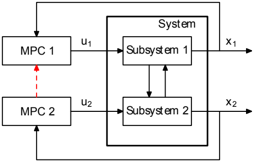
Classified by communication between controllers

- Decentralized control system
  - No communication between controllers
- Distributed control system
  - Controllers exchange information to coordinate their actions

Classification of DMPC

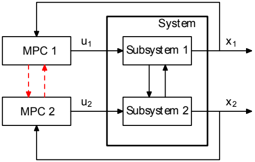
Non-Cooperative DMPC

- Sequential DMPC
  - One-directional communication
  - Controllers are evaluated in sequence



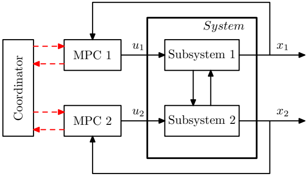
Sequential DMPC

- Non-iterative parallel DMPC
  - Controllers are evaluated once at a sampling time
- Iterative parallel DMPC
  - A local cost function is used in each controller

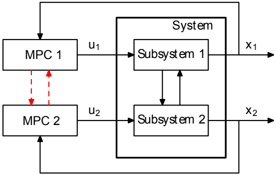


Parallel DMPC

Classification of DMPC



Coordinated DMPC

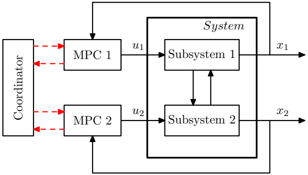


Cooperative DMPC

Coordinated DMPC

- There is a coordinator to coordinate the actions of distributed controllers

Classification of DMPC



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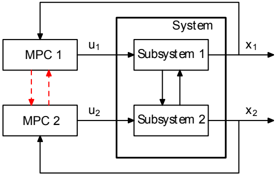

Cooperative DMPC

Coordinated DMPC

- There is a coordinator to coordinate the actions of distributed controllers

Cooperative DMPC

- In each controller, the same global cost function is optimized
- Achieve the performance of centralized MPC when iterate to convergence

Non-Cooperative DMPC

- DMPC for a class of decoupled systems with the distributed controllers are evaluated in sequence (Richards and How, International Journal of Control, 2007)
- DMPC for a class of discrete-time linear systems (Camponogara et al., IEEE Control Systems Magazine, 2002)
- DMPC for systems with dynamically decoupled subsystems (Keviczky et al., Automatica, 2006)
- DMPC scheme for linear systems coupled through the state (Jia and Krogh, ACC, 2001)

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Coordinated DMPC

- Coordinator-based DMPC (Cheng et al., Journal of Process Control, 2007; Marcos et al., ADCHEM 2009)

Cooperative DMPC

- Idea of cooperative DMPC was first introduced in 2005 (Venkat et al., CDC, 2005)
- Cooperative DMPC of linear systems (Rawlings and Stewart, Journal of Process Control, 2008; Stewart et al., Systems and Control Letters, 2010)
  - System-wide control objective functions
  - The closed-loop performance converges to the corresponding centralized control system as the iteration number increases
- Lyapunov-based iterative DMPC for nonlinear systems (Liu et al., AIChE Journal, 2009; 2010; Liu et al., Automatica, 2010; IEEE Transactions on Automatic Control, 2012)
  - Well-characterized regions of closed-loop stability
  - Accounting for asynchronous and delayed measurements
- Robust DMPC for linear systems accounting for model uncertainties explicitly (Al-Gherwi et al., Journal of Process Control, 2011)

Cooperative Nonlinear DMPC

System description

$$\dot{x}(t) = f(x(t)) + \sum_{i=1}^{m} g_i(x(t))u_i(t) + k(x(t))w(t)$$

- Fully coupled nonlinear processes with m sets of control inputs

Cooperative Nonlinear DMPC

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- Fully coupled nonlinear processes with m sets of control inputs

Nonlinear feedback control law, $u = h(x) = [h_1(x) \ldots h_m(x)]^T$

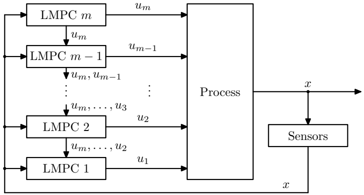
$$\dot{V}(x) = \frac{\partial V(x)}{\partial x}(f(x) + \sum_{i=1}^{m} g_i(x)h_i(x)) < 0$$

- Renders the origin of the nominal system asymptotically stable under the control: $u_i = h_i(x)$ $(i = 1, \ldots, m)$
- Satisfies the input constraints on u_i $(i = 1, \ldots, m)$
- Stability region: $\Omega \subset D$ is a compact set containing the origin

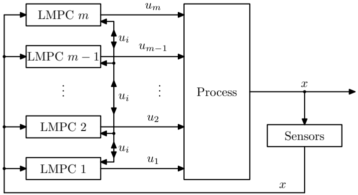
Sequential and Iterative DMPC

(Liu et al., AIChE J., 2009; AIChE J., 2010)

- m LMPCs will be designed to decide the m sets of control inputs



Sequential DMPC

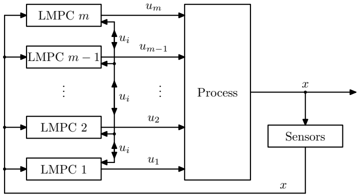


Iterative DMPC

- Sequential DMPC: One-directional communication, each controller is evaluated once at a sampling time
- Iterative DMPC: Bi-directional communication, controllers iterate to achieve convergence at a sampling time

Iterative DMPC

Implementation strategy



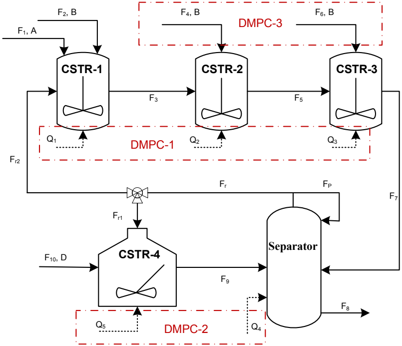
1. At t_k, controllers receive $x(t_k)$ and initialized with input guesses generated by $h(\cdot)$
2. At iteration c ($c \geq 1$):
   2.1. Each controller evaluates its own future input trajectory
   2.2. Controllers exchange information. Based on the latest information, each controller calculates and stores the value of the cost function
3. If a termination condition is satisfied, each controller sends the input trajectory corresponding to the smallest value of the cost function to its actuators; Else, go to Step 2 ($c = c + 1$)

Convergence of the Iterative DMPC

- The optimal cost of the iterative DMPC is upper bounded by the cost of the nonlinear controller $h(x)$
  - $h(x)$ is a feasible solution to the iterative DMPC ($x(0) \in \Omega$)
  - Implementation strategy of the iterative DMPC
- Guaranteed convergence for linear systems
  - The optimization problem of LMPC j is convex
  - Using a suitable input update rule, as $c \to \infty$, the cost of the iterative DMPC converges to the corresponding centralized MPC
- For general nonlinear systems, the convergence of the iterative DMPC cost to the centralized MPC is not guaranteed

Application to a Chemical Process

Alkylation of benzene with ethylene



- Three distributed LMPC controllers
  - MPC 1: Q_1, Q_2, Q_3 MPC 2: Q_4, Q_5 MPC 3: F_4, F_6
- Input constraints are considered

Application to a Chemical Process

Mean Evaluation Times

- Mean evaluation times for 100 evaluations

		$N = 1\ (s)$	$N = 3\ (s)$	$N = 6\ (s)$
Centralized MPC		2.192	8.694	27.890
Sequential	MPC 1	0.472	2.358	6.515
	MPC 2	0.497	1.700	4.493
	MPC 3	0.365	1.453	3.991
Iterative (1 iteration)	MPC 1	0.484	2.371	6.280
	MPC 2	0.426	1.716	4.413
	MPC 3	0.185	0.854	2.355

- Sequential DMPC evaluation time is reduced by 36% - 46%
- Iterative DMPC evaluation time (1 iteration) is reduced by more than 70%; 3 - 4 iterations are possible in 1 evaluation of the Centralized MPC

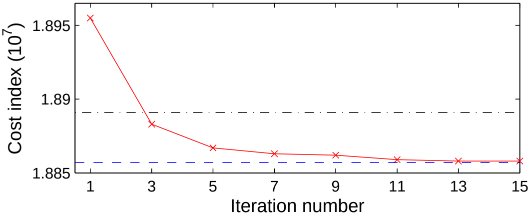
Application to a Chemical Process

Optimality

- Performance index

$$J = \sum_{i=0}^{M} \left[x(t_i)^T Q_c x(t_i) + \sum_{j=1}^{3} u_j(t_i)^T R_{cj} u_j(t_i) \right]$$

- Simulation time: $t_M = 1000\ s$, $N = 1$

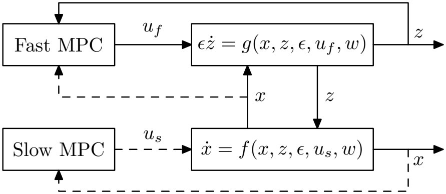


Blue: Centralized   Red: Iterative   Black: Sequential

- The cost of the iterative DMPC converges to the centralized MPC

DMPC for Two-Time-Scale Processes

(Chen et al., Journal of Process Control, 2011; AIChE Journal, 2012)



- Slow dynamics is regulated by slow MPC
- Fast dynamics is regulated by fast MPC (or explicit controller)
- No communication between the two MPCs is necessary
- Near optimality of fast-slow MPC system
  - $J \to J_s^* + J_f^*$ as $\epsilon \to 0$
  - ϵ is a parameter that indicates the level of separation between the fast and slow dynamics

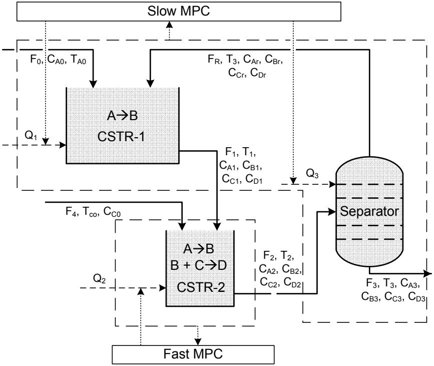
Reactor-Separator with Large Recycle

An example of two-time-scale process

- Two reactions:

$$A \xrightarrow{r_1, exothermic} B$$
$$B + C \xrightarrow{r_1, exothermic} D$$

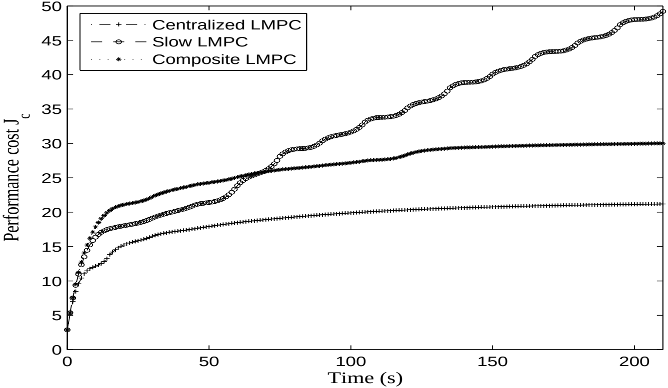
- Fast dynamics: CSTR-2
  - Residence time: $\frac{F_1}{V_2} = 0.11 \ sec$



- Control inputs associated with slow dynamics: Q_1, Q_3
- Control inputs associated with fast dynamics: Q_2

Reactor-Separator with Large Recycle

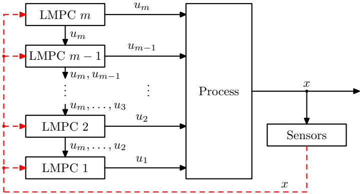
Simulation results: Performance trajectories



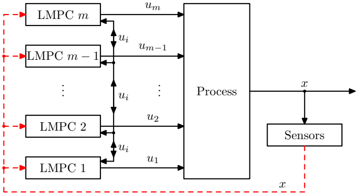
- Control methods
  - centralized MPC, fast-slow MPC, slow MPC with explicit controller

DMPC with Asynchronous/Delayed Feedback

(Liu et al., Automatica, 2010; IEEE Transactions on Automatic Control, 2012)



Sequential DMPC



Iterative DMPC

Proposed approaches

- Modify the implementation strategies to take into account that the control loop may be open
- Redesign the formulations of the LMPCs to take into account asynchronous and delayed feedback explicitly
- In the case of delayed measurements, iterative DMPC has to be used

DMPC for Switched Nonlinear Processes

(Heidarinejad et al., ACC, 2012)

System description

$$\dot{x} = f_{\sigma(t)}(x) + \sum_{i=1}^{m} g_{i_{\sigma(t)}}(x) u_{i_{\sigma(t)}}$$

- Switching signal $\sigma : [0, \infty) \to \mathcal{I} = \{1, 2, \ldots, p\}$
- Frequently arise in process operation (demand changes, phase changes, etc.)

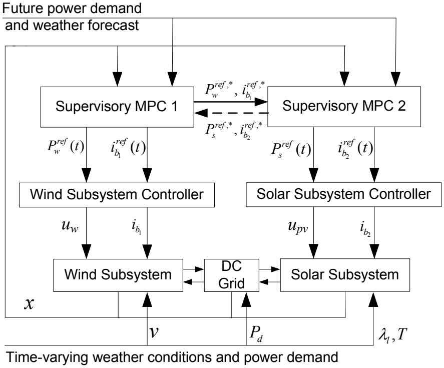
Proposed approach

- Focused on nonlinear processes with scheduled mode transitions
- Initial feasibility is assumed
- A stability constraint based on multiple Lyapunov function is checked at each iteration

Distributed Energy Generation Systems

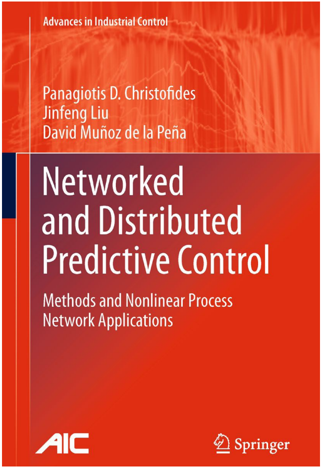
(Qi et al., IEEE Transactions on Control Systems and Technology, in press)

- System description
  - Wind subsystem
  - Solar subsystem
  - Loads of the system
  - DC bus
- Control system
  - One MPC for wind subsystem
  - One MPC for solar subsystem
  - Controllers communicate to meet total power demand



Conclusions

- Trends in process control
  - Control of large-scale complex processes
  - Distributed model predictive control is an appealing approach
- Our work on DMPC for nonlinear processes
  - Sequential and iterative DMPC
  - DMPC for two-time-scale processes
  - DMPC for with asynchronous/delayed measurements
  - DMPC for switched nonlinear processes
  - Distributed energy generation systems



Future Research Directions

- Distributed state estimation and integration with DMPC
- DMPC accounting for process topology
- DMPC with asynchronous evaluation
- Performance assessment of DMPC
- Loop partitioning and decomposition for DMPC
- Monitoring and reconfiguration of DMPC
- Applications