Economic Model Predictive Control

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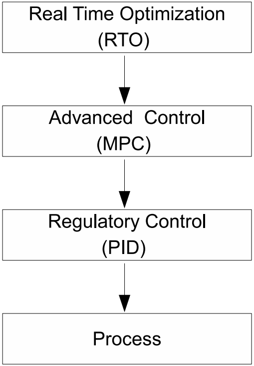
Introduction

- Current paradigm for achieving overall economic objectives



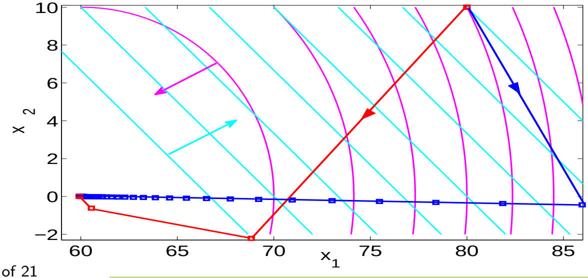
- Hierarchical partitioning of objectives and information
  - RTO layer: overall economic optimization
  - Advanced control layer: set-point tracking
- Issues that need to be addressed
  - Advanced control has different objectives
    - e.g., fast asymptotic tracking
  - Economic performance loss in the transient periods (Forbes and Marlin, CCE, 1996; Zhang and Forbes, CCE, 2000)

Motivating Example

- A numerical example with two states (Rawlings and Amrit, NMPC, 2008)

$$x(k+1) = \begin{bmatrix} 0.857 & 0.884 \\ -0.0147 & -0.0151 \end{bmatrix} x(k) + \begin{bmatrix} 8.57 \\ 0.884 \end{bmatrix}$$

  - Economic profit function $l(x,u) = -3x_1 - 2x_2 - 2u$
  - Cost function in MPC: $s(x,u) = |x - x_s|^2 + |u - u_s|^2$
  - Input constraint: $-1 \leq u \leq 1$
  - Optimal steady state: $u_s = 1$, $x_s = (60, 0)$



Tracking MPC: Red

7557

Economic MPC: Blue

8034

6%↑

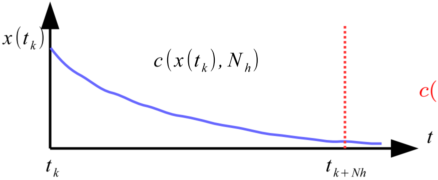
Introduction

- Economic MPC (EMPC): use an economic cost function in MPC
- Important topics in EMPC: stability, performance, robustness
- Existing results
  - Infinite horizon approach (Würth et al., ADCHEM, 2009; Huang et al., JPC, 2011; Chmielewski et al., CDC, 2012)
  - Terminal cost & terminal region constraint (Amrit et al., ARC, 2011; Rawlings et al., CDC, 2012)
  - Lyapunov-based approach (Heidarinejad et al., AIChE J., 2012; Ellis et al., JPC, 2014; Automatica, 2014)
- Drawbacks of existing results - Implementation difficulties
  - Reduced initial feasibility region, conservative terminal cost construction techniques, high computational complexity etc

Proposed EMPC design (Liu et al, ADCHEM 2015)

- Objectives: a computationally efficient EMPC with an easy-to-construct terminal cost and guaranteed stability & performance

- An auxiliary stabilizing controller $h(x)$ is in the design of the terminal cost



$$c(x(t_k), N_h) := \int_{t_k}^{t_{k+N_h}} l_e(\hat{x}(t|t_k), \hat{u}(t|t_k))\mathrm{d}t$$

  - $c(x(t_k), N_h)$: transient economic performance of $h(x)$ implemented in sample-and-hold of the first N_h steps with $\hat{x}(t_k|t_k) = x(t_k)$
  - If $N_h \geq N^*$, $c(x(t_k), N_h)$ covers the primary transient performance

Proposed EMPC design (Liu et al, ADCHEM 2015)

- EMPC design

$$\begin{aligned}
\min_{u(\tau)\in S(\Delta)} \quad & \int_{t_k}^{t_{k+N}} l_e(\tilde{x}(\tau), u(\tau))\mathrm{d}\tau + c(\tilde{x}(t_{k+N}), N_h) \\
\text{s.t.} \quad & \dot{\tilde{x}}(t) = f(\tilde{x}(t)) + g(\tilde{x}(t))u(t) \\
& \tilde{x}(t_k) = x(t_k) \\
& u(t) \in \mathbb{U} \\
& \tilde{x}(t) \in \mathbb{X} \\
& \tilde{x}(t_{k+N}) \in \mathbb{D}
\end{aligned}$$

- Achieving improved transient performance from t_k to t_{k+N+N_h}
- Recursive feasibility is ensured - $h(x)$ is a feasible solution
- Closed-loop stability is ensured via state constraints

Economic performance (Liu et al, ADCHEM 2015)

Theorem: If the initial state $x(t_0)$ is feasible, and if $N_h \geq N^*$, then the asymptotical average economic performance of the system under the EMPC:

$$\bar{J}_{asy}^{EMPC} := \lim_{F \to \infty} \frac{1}{F\Delta} \int_{t_0}^{t_F} l_e(x(t), u(t))\mathrm{d}t$$

is bounded as follows:

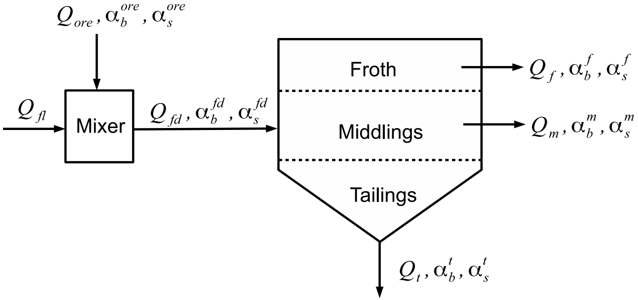
$$\bar{J}_{asy}^{EMPC} \leq \bar{J}_{\Delta}^{h}$$

with $\bar{J}_{\Delta}^{h} := \max \left\{ \frac{1}{\Delta} \int_{0}^{\Delta} l(x(t), h(x(0)))\mathrm{d}t : \; x(0) \in \Omega_{\rho^*} \right\}$.

- $\bar{J}_{\Delta}^{h}$ denotes the tailing part that $c(x, N_h)$ does not cover
- If $\bar{J}_{\Delta}^{h}$ is negligible, $\bar{J}_{asy}^{EMPC} \leq \bar{J}_{asy}^{ss} = \bar{J}_{asy}^{h} = \bar{J}_{asy}^{MPC}$
- No requirement on the length of N

Example 1 - Oilsand Separation (Liu et al, ADCHEM 2015)

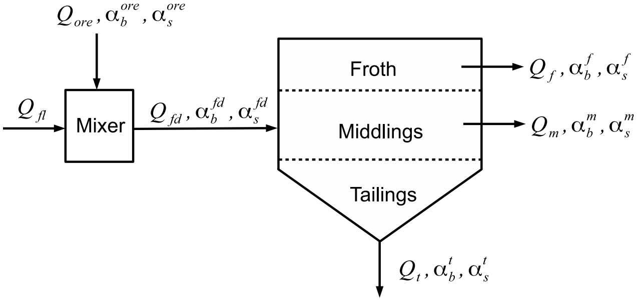
- Primary separation vessel



  - Three typical bitumen particles and three solid particles
  - 25 ODEs based on mass balance (Gilbert, 2004)
  - Dynamically modeled froth/middlings interface level
  - Mixing tank modeled as a continuous stirred tank

Example 1 - Oilsand Separation (Liu et al, ADCHEM 2015)

- Primary separation vessel



  - Three manipulated inputs: $u = [u_1, u_2, u_3]^T = [Q_{fl}, Q_m, Q_t]^T$
  - Economic objective: maximize bitumen recovery rate
  - A typical control configuration: maintain the froth/middlings interface at a constant level

Example 1 - Oilsand Separation (Liu et al, ADCHEM 2015)

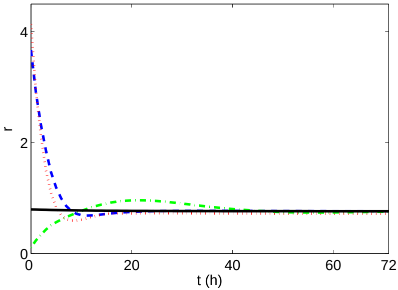
- EMPC design - representation of the control objective
  - Bitumen recovery rate

$$r(x(t), u(t)) = \frac{\sum_{j=1}^{3} \alpha_{bj}^{f}(t) Q_f(t)}{\sum_{j=1}^{3} \alpha_{bj}^{ore} Q_{ore}}$$

- EMPC design - stability of the process
  - State constraint on V_f and input constraints
- Four different control methods
  - Proposed EMPC with terminal cost
  - EMPC without terminal cost
  - Tracking MPC
  - Proportional control

Example 1 - Oilsand Separation (Liu et al, ADCHEM 2015)

- Simulation results - Bitumen recovery rates

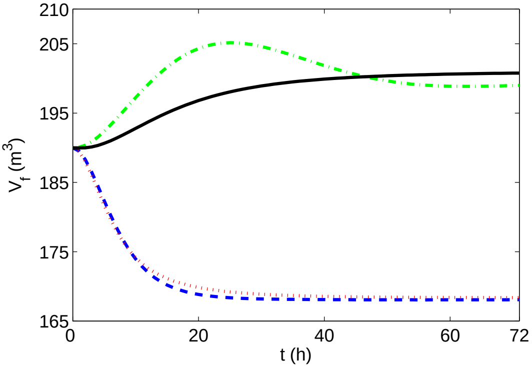


Proposed: Blue EMPC w/o TC: Red Tracking MPC: Green P: Black

- Average recovery rates: P=0.7690, MPC=0.7754, EMPC w/o TC=0.8267, Proposed EMPC= 0.8845
- 12%, 11%, 6% increases compared with P, MPC and EMPC w/o TC

Example 1 - Oilsand Separation (Liu et al, ADCHEM 2015)

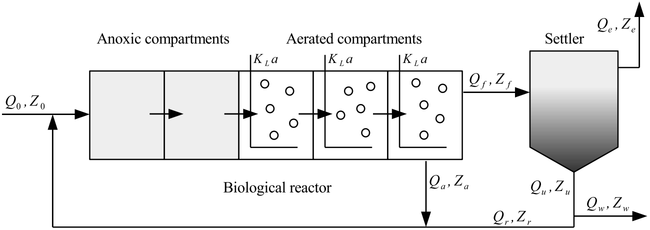
- Simulation results - Froth volume V_f



Proposed: Blue EMPC w/o TC: Red Tracking MPC: Green P: Black

Example 2 - Wastewater Treatment (Zeng and Liu, IECR 2015)

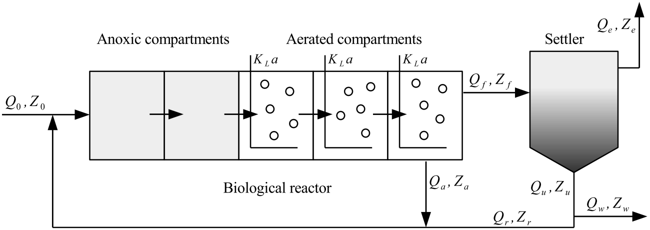
- Wastewater treatment plant



  - Model is developed by the International Water Association
  - Eight biological processes with a total of 145 states are considered
  - Two manipulated inputs: Q_a and K_La_5
  - Periodic operation subject to high uncertainties

Example 2 - Wastewater Treatment (Zeng and Liu, IECR 2015)

- Wastewater treatment plant



  - Economic objective: maximize the effluent quality
  - A typical control configuration: maintain $S_{NO,2}$ and $S_{O,5}$ at pre-determined set-points by manipulating the two control inputs

Example 2 - Wastewater Treatment (Zeng and Liu, IECR 2015)

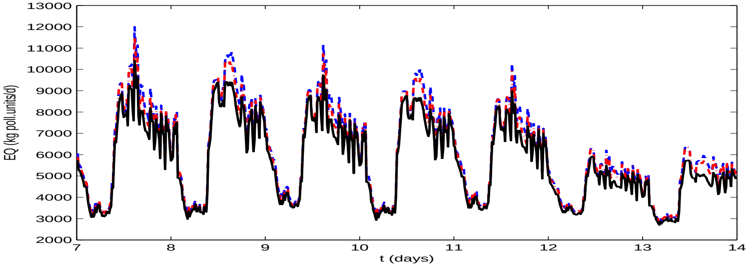
- EMPC design - representation of the control objective
  - Effluent quality: daily average of a weighted summation of the concentrations of different compounds in the effluent

$$EQ = \frac{1}{T}\int_{t_0}^{t_f}\left(2TSS_e(t) + COD_e(t) + 30S_{NKj,e}(t) + 10S_{NO,e}(t) + 2BOD_e(t)\right)Q_e(t)dt$$

- EMPC design - stability of the process
  - State constraints on $S_{NO,2}$ and $S_{O,5}$ as well as input constraints
- Control configurations
  - Proportional-integral control
  - Tracking MPC
  - Proposed EMPC with terminal cost

Example 2 - Wastewater Treatment (Zeng and Liu, IECR 2015)

- Simulation results

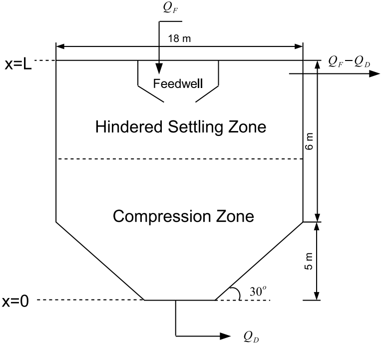


Proposed: Black   Tracking MPC: Red   PI: $h(x)$: Blue

- PI control: EQ $= 6123.53\ kg/d$
- Tracking MPC: EQ $= 6022.64\ kg/d$
- EMPC: EQ $= 5671.86\ kg/d$
  - 7.4% and 5.8% decreases compared with PI and MPC

Example 3 - Thickener in Coal Beneficiation

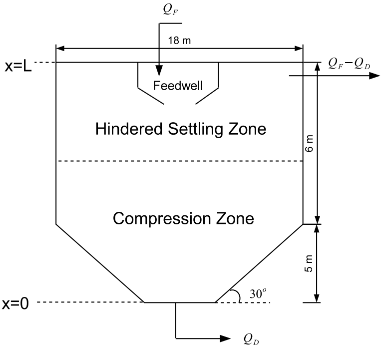
- Deep cone thickener



  - An important operating unit in coal handling and preparation plant
  - Approximation of the distributed parameter process with 34 ODEs
  - One manipulated input: bottom discharge flow rate Q_D

Example 3 - Thickener in Coal Beneficiation

- Deep cone thickener



  - Economic objective: maximize water recovery rate
  - A typical control configuration: maintain the bottom discharge flow rate at a pre-determined set-point

Example 3 - Thickener in Coal Beneficiation

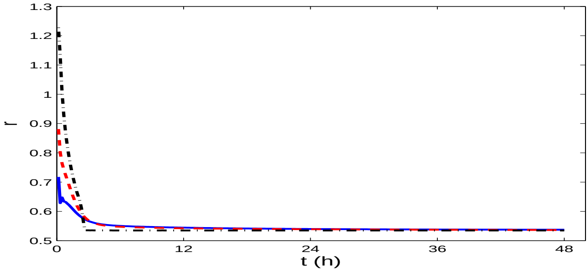
- EMPC design - representation of the control objective
  - Water recovery rate:

$$l(x,u) = -\frac{Q_F - Q_D}{Q_F(1-\phi_F)}$$

- EMPC design - stability of the process
  - State constraints on volumetric solid concentrations as well as input constraint
- Control configurations
  - Proportional control
  - Tracking MPC
  - Proposed EMPC with terminal cost

Example 3 - Thickener in Coal Beneficiation

- Simulation results



EMPC: Black Tracking MPC: Red P: Blue

- P control: average recovery rate = 0.57
- Tracking MPC: average recovery rate = 0.59
- EMPC: average recovery rate = 0.62
  - 5% and 3% improvements compared with P and MPC

Conclusions

- EMPC with ensured economic performance and stability
  - An auxiliary asymptotically stabilizing nonlinear controller is used
- Demonstrated the effectiveness via simulation examples
  - An oilsand primary separation vessel
  - A wastewater treatment plant
  - A thickener in coal handling and preparation

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