State Estimation and Economic MPC of Nonlinear Processes

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Outline

- State estimation of nonlinear systems
  - Observer-enhanced moving horizon estimation (MHE) - an output feedback perspective
  - Distributed implementation
    - Distributed observer-enhanced MHE
    - Forming distributed estimator networks from decentralized estimators
- Economic MPC
  - What is economic MPC?
  - Different approaches to economic MPC
  - Our approach - economic MPC with extended horizon
  - Applications
- Conclusions

Part I: State Estimation of Nonlinear Systems

1. Observer-enhanced moving horizon estimation (MHE)
2. Distributed implementation

Introduction to state estimation

- State estimation reconstructs the state of a system
  - Sufficient measured variables & a system model
- For linear systems, standard solutions are available
  - Luenberger observers and Kalman filters
- State estimation for nonlinear systems is much more challenging
  - Extensions of linear solutions based on successive linearization
    - Extended Kalman filters - *ad hoc* solutions (Eykhoff, Wiley, 1974)
  - Designs that explicitly account for nonlinearities
    - Deterministic approaches: High-gain observers etc. (Gauthier et al., TAC, 1992)
    - Stochastic approaches: Moving horizon estimation etc. (Rao et al., Automatica, 2001; TAC, 2003; Michalska and Mayne, TAC, 1995)

Deterministic nonlinear observers

- System description

$$\begin{aligned}\dot{x}(t) &= f(x(t), w(t)) \\ y(t) &= h(x) + v(t)\end{aligned}$$

  - x, y: system state vector & measured output vector
  - w, v: process & measurement noise
- Deterministic nonlinear observer $\dot{z}(t) = F(z(t), y(t))$
  - Noise information is not used
  - A common form of $F(z, y)$ (Gauthier et al., TAC, 1992; Ciccarella et al., IJC, 1993)

$$F(z, y) = f(z, 0) + K(z, y)(h(z) - y)$$

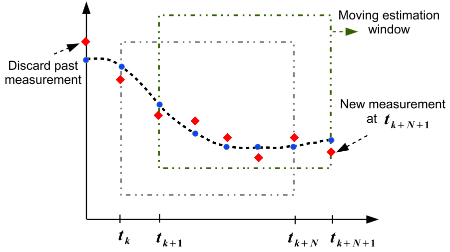
- Objective: z converges to x with tunable convergence rate
  - High-gain observers (Gauthier et al., TAC, 1992; Ahrens and Khalil, Automatica, 2009)
  - Separation principle is possible in output feedback control
  - Very sensitive to measurement noise (Ahrens and Khalil, Automatica, 2009)

Moving horizon estimation (Rao et al., TAC, 2003)

- Moving horizon estimation (MHE)

$$\min_{\tilde{X}(t_k)} \left\{ \sum_{i=k-N}^{k-1} |w(t_i)|^2_{Q^{-1}} + \sum_{i=k-N}^{k} |v(t_i)|^2_{R^{-1}} + V(\tilde{x}(t_{k-N})) \right\}$$

$$\text{s.t. } \dot{\tilde{x}}(t) = f(\tilde{x}(t), w(t_i)), t \in [t_i, t_{i+1}]$$
$$v(t_i) = y(t_i) - h(\tilde{x}(t_i))$$
$$w(t_i) \in W, \; v(t_i) \in V, \; \tilde{x}(t) \in X$$



- Online optimization based approach
  - Explicitly uses distribution/boundedness information of w, v, x
  - A moving estimation window with an arrival cost $V(\tilde{x}(t_{k-N}))$
- Objective: to obtain an estimate of x minimizing the cost function
  - Arrival cost approximation for constrained systems is difficult (Rao and Rawlings, AIChE, 2002; Ungarala, JPC, 2009; Lopez-Negrete et al., JPC, 2011)
  - Closed-loop stability in output feedback control cannot be established

Comparison of high-gain observers and MHE

- High-gain observers
  - Do use of noise information
  - Not optimal
  - Tunable convergence rate
  - Separation principle is possible
  - Sensitive to measurement noise
  - Use only current measurements
- Moving horizon estimation
  - Noise considered explicitly
  - Optimal
  - Unknown convergence rate
  - No available separation principle
  - Robust to measurement noise
  - Depends on arrival cost estimation

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Combine the advantages of high-gain observers and MHE

Comparison of high-gain observers and MHE

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Combine the advantages of high-gain observers and MHE

- Observer-enhanced MHE for nonlinear systems (Liu, CES, 2013)
  - ☐ Reduced sensitivity to noise
  - ☐ Reduced dependence on accuracy of the arrival cost
  - ☐ Has the potential to be used in output feedback control (Zhang and Liu, AIChE J., 2013; Ellis et al., SCL, 2013; Zhang et al., JPC, 2014)

Observer-enhanced MHE - Preliminaries (Liu, CES, 2013)

- System description

$$\begin{aligned} \dot{x}(t) &= f(x(t), w(t)) \\ y(t) &= h(x) + v(t) \end{aligned}$$

  - w and v are bounded and $x \in X$

- Existence of a nonlinear deterministic observer $\dot{z} = F(z, y)$
  - Estimation error decays asymptotically for the nominal system

$$|z(t) - x(t)| \leq \beta(|z(0) - x(0)|, t)$$

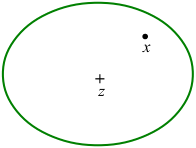
    - β is a KL function

- The estimation error is bounded when w and v are bounded

$$|z(t) - x(t)| \leq \beta(|z(t_k) - x(t_k)|, t - t_k) + \gamma(t - t_k)$$

  - $\gamma(t - t_k)$: an increasing function that characterizes the effects of w, v

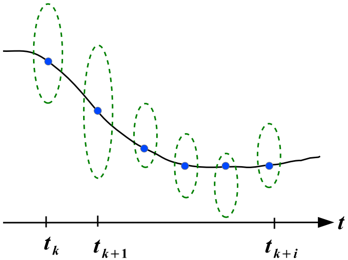
- The difference between $y(t_k)$ and $h(z(t_k))$ can be used to measure the accuracy of the estimate $z(t_k)$



Observer-enhanced MHE - Formulation (Liu, CES, 2013)

- Observer-enhanced MHE

$$
\begin{aligned}
\min_{\tilde{X}(t_k)} \quad & \left\{ \begin{array}{c} \sum_{i=k-N}^{k-1} |w(t_i)|^2_{Q^{-1}} + \sum_{i=k-N}^{k} |v(t_i)|^2_{R^{-1}} \\ +V(\tilde{x}(t_{k-N})) \end{array} \right\} \\
\text{s.t.} \quad & \dot{\tilde{x}}(t) = f(\tilde{x}(t), w(t_i)), t \in [t_i, t_{i+1}] \\
& v(t_i) = y(t_i) - h(\tilde{x}(t_i)) \\
& w(t_i) \in W,\ v(t_i) \in V,\ \tilde{x}(t) \in X \\
& \dot{z}(t) = F(z(t), y(t_{k-1})) \\
& z(t_{k-1}) = \hat{x}(t_{k-1}) \\
& |\tilde{x}(t_k) - z(t_k)| \leq \kappa |y(t_k) - h(z(t_k))|
\end{aligned}
$$



- The observer is used to calculate a confidence region every sampling time
- $\tilde{x}(t_k)$ is optimized within the region
- κ is a parameter that determines the size of the confidence region
  - When $\kappa = 0$, it reduces to the observer implemented in sampled and hold
  - When κ is too large, it reduces to the regular MHE

Application to a CSTR example - Simulation settings

- A non-isothermal continuous stirred tank reactor

$$\frac{dT}{dt} = \frac{F}{V_r}(T_{A0} - T) - \sum_{i=1}^{3} \frac{\Delta H_i}{\sigma c_p} k_{i0} e^{\frac{-E_i}{RT}} C_A + \frac{Q_c}{\sigma c_p V_r}$$
$$\frac{dC_A}{dt} = \frac{F}{V_r}(C_{A0} - C_A) + \sum_{i=1}^{3} k_{i0} e^{\frac{-E_i}{RT}} C_A$$

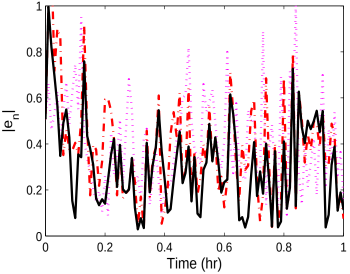
- The reactor temperature T is measured
- Bounded uncertainties: $-5 \leq v \leq 5$, $-10 \leq w_T,\ w_{C_A} \leq 10$
- A reduced-order deterministic observer (Soroush, CES, 1997)

$$\frac{d\hat{C}_A}{dt} = \frac{F}{V_r}(C_{A0} - \hat{C}_A) + \sum_{i=1}^{3} k_{i0} e^{\frac{-E_i}{RT}} \hat{C}_A$$

- Parameters: $\Delta = 0.01\ h$, $\kappa = 0.02$

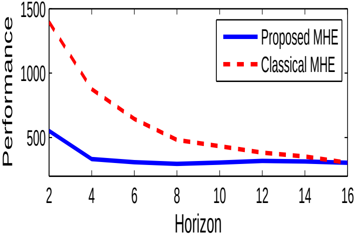
Application to a CSTR example - Results

- Simulation results



$N = 10$

BLACK: Proposed; RED: regular MHE; PINK: observer

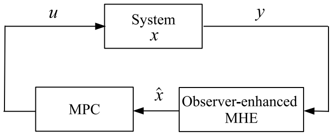


$$J = \sum_{k=0}^{k=f} |\hat{x}(t_k) - x(t_k)|_S^2 \text{ with } S = \begin{bmatrix} 1 & 0 \\ 0 & 50 \end{bmatrix}$$

- Observer-enhanced MHE gives better estimates in both T and C_A
- Averages of the normalized error: 0.3667, 0.3494, 0.2836
- Observer-enhanced MHE depends less on N or the arrival cost

Output feedback control & some remarks

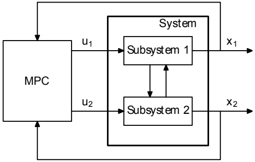
- Observer-enhanced MHE in output feedback control
  - Output feedback MPC and its triggered implementation (Zhang and Liu, AIChE J., 2013)
  - Output feedback economic MPC (Ellis et al., SCL, 2013)

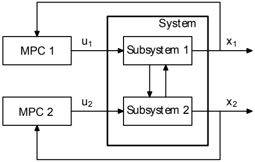


- ▷ Provable closed-loop stability
- ▷ Improved control performance

- Remarks on observer-enhanced MHE
  - Theoretical advancement for output feedback nonlinear control
  - If a nonlinear observer can be designed, it is appealing
  - If regular MHE requires a large N, it may be used to address the computational issue

Spectrum of Plant-wide Control Schemes

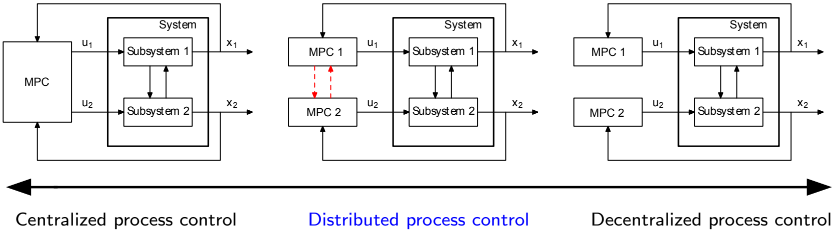




Centralized process control

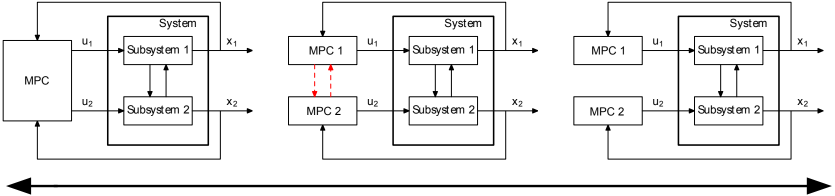
Decentralized process control

Spectrum of Plant-wide Control Schemes



Distributed process control is between
centralized and decentralized process control

Spectrum of Plant-wide Control Schemes



Centralized process control | Distributed process control | Decentralized process control

Distributed process control is between
centralized and decentralized process control

- Motivation of distributed process control/estimation
  - Reduced computational complexity and increased fault tolerance
  - Increased estimation performance to decentralized state estimation
  - Distributed output feedback control
    - Distributed MPC based on state feedback (Christofides et al., Springer, 2011; CCE, 2013; Cai et al., JPC, 2014; Li and Shi, SCL, 2013; Li and Zheng, Wiley, 2016)

Distributed MHE - System description

- System description

$$\dot{x}_i(t) = f_i(x_i(t), w_i(t)) + \tilde{f}_i(X_i(t))$$

$$y_i(t) = h_i(x_i(t)) + v_i(t)$$

  - f_i , $\tilde{f}_i$ and h_i are Lipschitz functions
  - w_i and v_i are bounded and $x_i \in X_i$
  - y_i is sampled every Δ at time instants t_k

- Observability assumption - Auxiliary observers

$$\dot{z}_i(t) = F_i(z_i(t), h_i(x_i(t)))$$

  - Estimation error decays asymptotically for the nominal system when $\tilde{f}_i(X_i(t)) = 0$

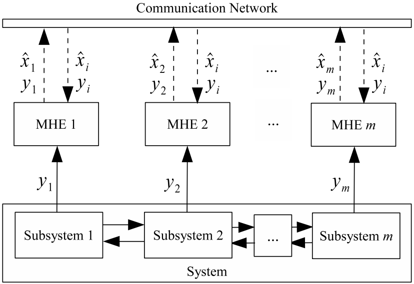
$$|z_i(t) - x_i(t)| \leq \beta_i(|z_i(0) - x_i(0)|, t)$$

    - β_i is a KL function and F_i is a Lipschitz function

  - Different techniques to design the auxiliary observer (Cicccarella et al., IJC, 1993; Kazantzis and Kravaris, SCL, 1998; Soroush, CCE, 1998; Kravaris et al., CCE, 2013)

Distributed MHE - Algorithm (Zhang and Liu, JPC, 2013)

1. At t_0, all MHEs are initialized with initial subsystem guess $\hat{x}_i(0)$ and the actual subsystem output measurements $y_i(0)$
2. At $t_k > 0$, carry out the following:



   2.1. MHE i receives its local measurement $y_i(t_k)$
   2.2. MHE i requests and receives the output measurements $y_j(t_{k-1})$ and state estimate $\hat{x}_j(t_{k-1})$ from other subsystems that directly affect its dynamics
   2.3. Based on the received information, MHE i calculates its current state estimate $\hat{x}_i(t_k)$
3. Go to Step 2 at t_{k+1}

Distributed MHE - Augmenting auxiliary observers (Zhang and Liu, JPC, 2013)

- Augmented auxiliary observers

$$\dot{z}_i(t) = F_i(z_i(t), y_i(t_{k-1})) \quad \text{– auxiliary observer}$$
$$+\tilde{f}_i(\hat{X}_i(t_{k-1})) \quad \text{– interaction model}$$
$$+\sum_{l\in I_i} K_{i,l}(\hat{x}_l)(y_l(t_{k-1}) - h_l(\hat{x}_l(t_{k-1}))) \quad \text{– correction term}$$

  - $\tilde{f}_i(\hat{X}_i(t_{k-1})) \neq \tilde{f}_i(X_i(t_{k-1}))$

- The gain $K_{i,l}$ is time-varying

$$K_{i,l} = \frac{\partial \tilde{f}_i}{\partial x_l}\left(\frac{\partial h_l}{\partial x_l}\right)^+\Bigg|_{x_l=\hat{x}_l(t_{k-1})}$$

  - Linear dynamics in the error dynamics caused by the interaction is compensated for by the correction term

Distributed MHE - Local MHE (Zhang and Liu, JPC, 2013)

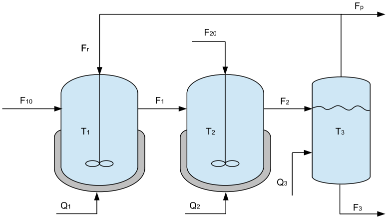
$$\min_{\tilde{x}_i(t_{k-N}),\ldots,\tilde{x}_i(t_k)} \quad \sum_{q=k-N}^{k-1} |w_i(t_q)|^2_{Q_i^{-1}} + \sum_{q=k-N}^{k} |v_i(t_q)|^2_{R_i^{-1}} + V_i(\tilde{x}_i(t_{k-N}))$$

$$\begin{aligned}
\text{s.t.} \quad & \dot{\tilde{x}}_i(t) = f_i(\tilde{x_i}(t), w_i(t_i)) + \tilde{f}_i(\hat{X}_i(t_q)), t \in [t_q, t_{q+1}] \\
& v_i(t_q) = y_i(t_q) - h_i(\tilde{x}_i(t_q)) \\
& w_i(t_q) \in W,\ v_i(t_q) \in V,\ \tilde{x}_i(t) \in X_i \\
& \dot{z}_i(t) = F_i(z_i(t), y_i(t_{k-1})) + \tilde{f}_i(\hat{X}_i(t_{k-1})) \\
& \qquad + \textstyle\sum_{l \in I_i} K_{i,l}(\hat{x}_l)(y_l(t_{k-1}) - h_l(\hat{x}_l(t_{k-1}))) \\
& z_i(t_{k-1}) = \hat{x}_i(t_{k-1}) \\
& |\tilde{x}_i(t_k) - z_i(t_k)| \leq \kappa_i |y_i(t_k) - h_i(z_i(t_k))|
\end{aligned}$$

- The local MHEs are formulated in terms of subsystems and subsystem interactions are considered
- A confidence region is created based on both the output and the reference state estimate calculated by the nonlinear observer
- The estimate of the current state is only allowed to be optimized within this region

Distributed MHE - A chemical process example

- Application to a reactor-separator process



States: $x_{A,i},\ x_{B,i},\ T_i$

Inputs: Q_i

Outputs: T_i

$i = 1,\ 2,\ 3$

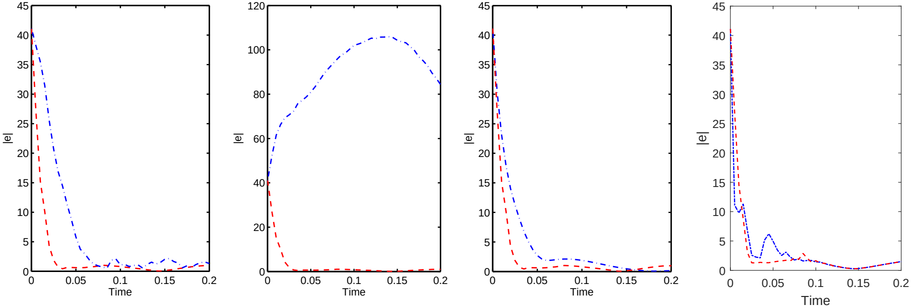
- Three subsystems according to the three tanks
- Auxiliary observers are designed as follows (Ciccarella et al., IJC, 1993)

$$\dot{\hat{x}}_i(t) = f_i(\hat{x}_i(t), 0) + G_i(\hat{x}_i(t))^{-1} K_{o,i}(y_i(t) - \hat{y}_i(t))$$

  - $G_i = \dfrac{d\Phi_i(\hat{x}_i)}{d\hat{x}_i}$, $\Phi_i(\hat{x}_i) = [h_i(\hat{x}_i),\ L_{f_i} h_i(\hat{x}_i),\ L_{f_i}^2 h_i(\hat{x}_i)]^T$
  - $K_{o,i}$ is a fixed gain matrix
- Sampling time: $\Delta = 18\ sec$, $N = 3$, $\kappa_i = 0.5$
- Correction gain: $K_{1,3} = [0\ 0\ 50.4]^T$ $K_{2,1} = [0\ 0\ 110.88]^T$ $K_{3,2} = [0\ 0\ 60.48]^T$

Distributed MHE - Simulation results (Zhang and Liu, JPC, 2013)

- Trajectories of normalized estimation error

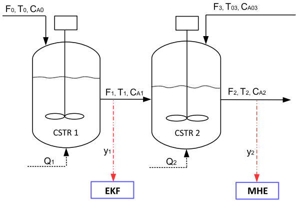


Proposed v.s. Observers | Proposed v.s. Decentralized | Proposed v.s. w/o correction | Proposed v.s. Regular

- In the observers, the correction terms are also implemented
- Observer-enhanced distributed MHE has a much faster convergence rate
- Information exchange can be used to significantly improve the performance
- Correction terms play an important role

Forming distributed estimators from decentralized estimators

- The concept can be extended to connect decentralized estimators
- An illustrative example



Decentralized estimation

Forming distributed estimators from decentralized estimators

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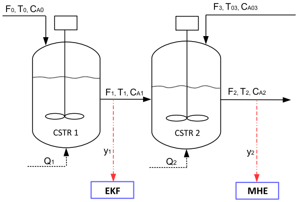
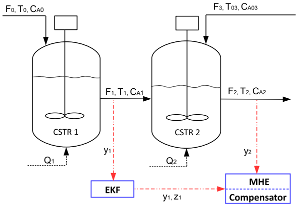


Decentralized estimation



Distributed estimation

Forming distributed estimators from decentralized estimators

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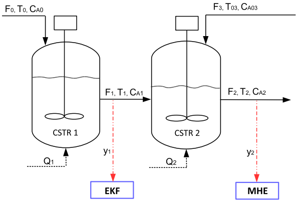
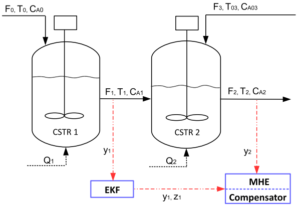
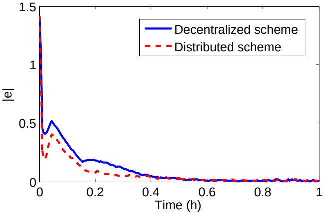


Decentralized estimation



Distributed estimation



Normalized estimation error

Forming distributed estimators from decentralized estimators

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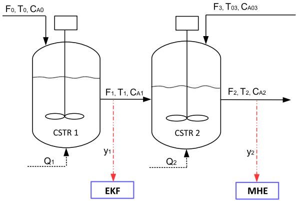
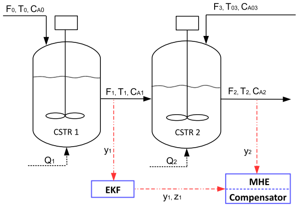
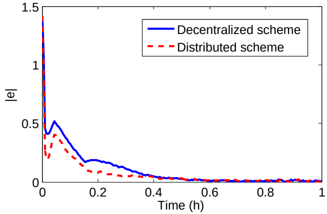
- An illustrative example



Decentralized estimation



Distributed estimation



Normalized estimation error

- Different types of estimators can be connected
- Improved estimation performance
- Weakly coupled subsystem error dynamics

Other related work

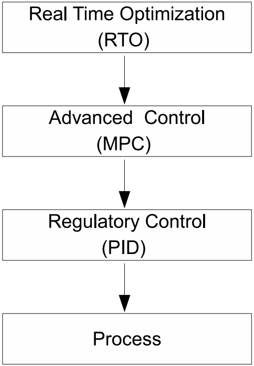
- Distributed adaptive high-gain extended Kalman filters (Rashedi et al., ADECHEM, 2015)
- Coordinated distributed moving horizon state estimation (An et al., CDC, 2016)
- Communication delays and losses in distributed state estimation (Rashedi et al., AIChE Journal, 2016; Zhang and Liu, JPC, 2014; Zeng and Liu, SCL, 2015)
- Triggered communication in distributed state estimation (Zhang and Liu, SCL, 2014; Rashedi et al., submitted)
- Subsystem decomposition in distributed state estimation (Yin et al., AIChE Journal, 2016; Yin and Liu, submitted)

Part II: Economic Model Predictive Control

1. Economic MPC with extended horizon
2. Applications

Introduction to economic MPC

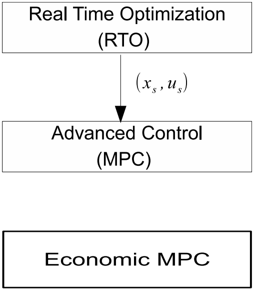
- Current paradigm for achieving overall economic objectives



- Hierarchical partitioning of objectives and information
  - RTO layer: overall economic optimization
  - Advanced control layer: set-point tracking
- Issues that need to be addressed
  - Advanced control has different objectives
    - e.g., fast asymptotic tracking
  - Economic performance loss in the transient periods (Forbes and Marlin, CCE, 1996; Zhang and Forbes, CCE, 2000)
    - More important for slow processes

Introduction to economic MPC

- Different approaches to address these issues
  - Dynamic RTO (Marquardt et al, FOCAPO, 2003; LNCIS, 2007)
  - MPC with an economic terminal cost (Zanin et al., CEP, 2002)
  - Economic model predictive control (EMPC) (Rawlings et al., NMPC, 2009)



$$\begin{aligned}(x_s, u_s) = \quad & \arg\min \; l_e(x,u) \\ \text{s.t.} \quad & f(x,u) = 0\end{aligned}$$

$$\downarrow (x_s, u_s)$$

$$\begin{aligned}\min_u \quad & \int (|x - x_s|_Q^2 + |u - u_s|_R^2)\mathrm{d}t \\ \text{s.t.} \quad & \dot{x} = f(x,u)\end{aligned}$$

$$\Downarrow$$

$$\begin{aligned}\min_u \quad & \int l_e(x,u)\mathrm{d}t \\ \text{s.t.} \quad & \dot{x} = f(x,u)\end{aligned}$$

Different approaches to economic MPC

- Important topics in EMPC: stability, performance, robustness
- Different approaches
  - Terminal cost and constraints
    - Point-wise terminal constraint (Diehl et al., TAC, 2011)
    - Terminal cost and terminal region constraints (Amrit et al., ARC, 2011; Müller et al., JPC, 2014)
    - Lyapunov-based constraints (Heidarinejad et al., AIChE J, 2012; Ellis et al., JPC, 2014; Automatica, 2014)
  - Extension of control horizon
    - Extension of control horizon (Grüne Automatica, 2013), (Grüne, JPC, 2014)
      - Finite horizon can provide near optimal performance
  - Our approach: extension of prediction horizon (Liu et al., CES 2015; ADCHEM, 2015; Automatica, in press)
    - Separation between prediction and control horizon
    - Significantly improved computational efficiency

Preliminaries

- System description

$$x(k+1) = f(x(k), u(k))$$

  - f is continuous
  - x and u are bounded in compact set $x \in \mathbb{X}$, $u \in \mathbb{U}$

- Optimal steady state

$$\begin{aligned} (x_s, u_s) = \quad & \arg\min_{x,u} \; l(x,u) \\ \text{s.t.} \quad & x = f(x,u) \\ & x \in \mathbb{X} \\ & u \in \mathbb{U} \end{aligned}$$

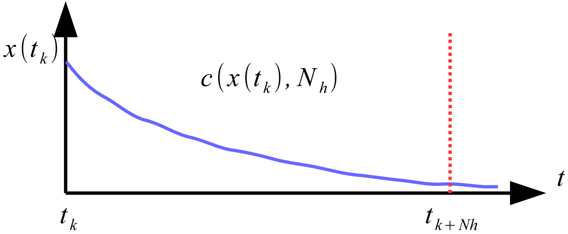
  - l: continuous economic cost function

- Auxiliary controller $h(x)$
  - $h(x)$ is Lipschitz continuous
  - x_s is asymptotically stable in $\mathbb{D} \subset \mathbb{X}$
  - $h(x) \in \mathbb{U}, \quad \forall x \in \mathbb{D}$
  - $\mathbb{D}$ is forward-invariant: $x \in \mathbb{D}$, $f(x, h(x)) \in \mathbb{D}$.

Proposed EMPC - Implicit terminal cost (Liu and Liu, Automatica, in press)

- Objectives: a computationally efficient EMPC with an easy-to-construct terminal cost and guaranteed stability & performance

- Implicit terminal cost based on the auxiliary controller



$$c(x, N_h) := \sum_{k=0}^{N_h-1} l(x_h(k,x), h(x_h(k,x)))$$

  - $x_h(k,x)$: state trajectory under controller $h(x)$ with initial state x
  - $c(x, N_h)$: accumulated economic stage cost under $h(x)$ for N_h steps

Proposed EMPC - Formulation (Liu and Liu, Automatica, in press)

- EMPC formulation

$$
\begin{aligned}
\min_{u(0),u(1),\ldots,u(N-1)} \quad & \textstyle\sum_{k=0}^{N-1} l(\tilde{x}(k),u(k)) + c(\tilde{x}(N),N_h) \\
\text{s.t.} \quad & \tilde{x}(k+1) = f(\tilde{x}(k),u(k)), \quad k=0,\ldots,N-1 \\
& \tilde{x}(0) = x(n) \\
& \tilde{x}(k) \in \mathbb{X}, \quad k=0,\ldots,N-1 \\
& u(k) \in \mathbb{U}, \quad k=0,\ldots,N-1 \\
& \tilde{x}(N) \in \mathbb{D}
\end{aligned}
$$

  - $c(\tilde{x}(N),N_h)$ extends the prediction horizon
  - Achieving improved transient performance from t_k to t_{k+N+N_h}
  - Recursively feasible
  - Computationally efficient

Proposed EMPC - Performance & stability (Liu and Liu, Automatica, in press)

- Asymptotic average performance

$$\bar{J}_{asy} := \lim_{F\to\infty} \sup \frac{1}{F} \sum_{k=0}^{F-1} l(x(k), u(k))$$

- Properties of the proposed EMPC
  - $\bar{J}_{asy}^{EMPC} \leq l(x_s, u_s) + \beta_l(d_{\max}, N_h)$
  - State will be driven into an open ball $\mathcal{B}_r(x_s)$ where r depends on N_h
    - Achieve practical stability
    - Sufficient conditions: strict dissipativity and finite supply under $h(x)$
  - Transient performance is upper bounded by the auxiliary controller
    - An optimally designed auxiliary controller may contribute to improved computationally efficiency and economic performance - back to the basis
- No requirement on the length of N

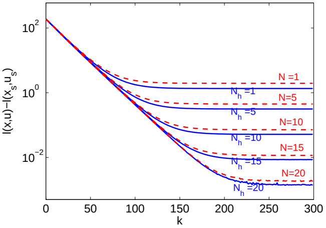
A numerical example

- Linearized continuous stirred-tank (Diehl, et al., TAC, 2011; Grüne, Automatica, 2013)

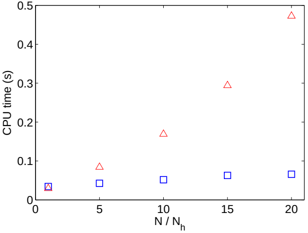
$$x(k+1) = \begin{pmatrix} 0.8353 & 0 \\ 0.1065 & 0.9418 \end{pmatrix} x(k) + \begin{pmatrix} 0.00457 \\ -0.00457 \end{pmatrix} u(k) + \begin{pmatrix} 0.5559 \\ 0.5033 \end{pmatrix}$$

  - Stage cost $l(x,u) = |x|^2 + 0.05u^2$, $\mathbb{X} = [-100, 100]^2$, $\mathbb{U} = [-10, 10]$.
  - Optimal steady state $x_s \approx [3.5463, 14.6531]^T$, $u_s \approx 6.1637$
  - Auxiliary controller $h = u_s$, $\mathbb{D} = \{x : |x - x_s| \leq 85\} \subset \mathbb{X}$

- Proposed with $N = 1$ v.s. EMPC without terminal cost (Grüne, Automatica, 2013)



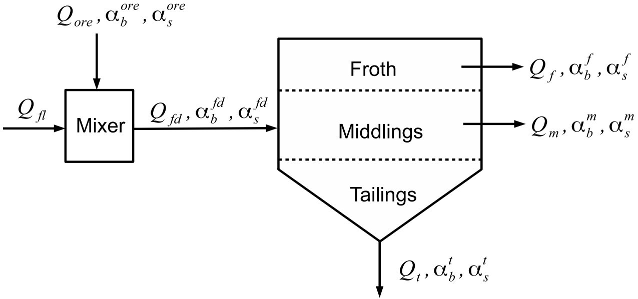
Performance trajectories



Computational times

Oilsand separation example (Liu et al, ADCHEM 2015; CES, 2015)

- Primary separation vessel



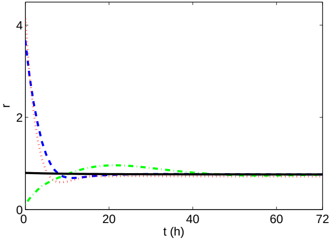
- Three manipulated inputs: $u = [u_1, u_2, u_3]^T = [Q_{fl}, Q_m, Q_t]^T$
- Economic objective: maximize bitumen recovery rate
- A typical control configuration: maintain the froth/middlings interface at a constant level

Oilsand separation example (Liu et al, ADCHEM 2015; CES, 2015)

- EMPC design
  - Control objective - maximize bitumen recovery rate

$$r(x(t), u(t)) = \frac{\sum_{j=1}^{3} \alpha_{bj}^{f}(t) Q_f(t)}{\sum_{j=1}^{3} \alpha_{bj}^{ore} Q_{ore}}$$

  - Auxiliary proportional controllers
- Simulation results: $N = 5$, $N_h = 30$, $\Delta = 1hr$



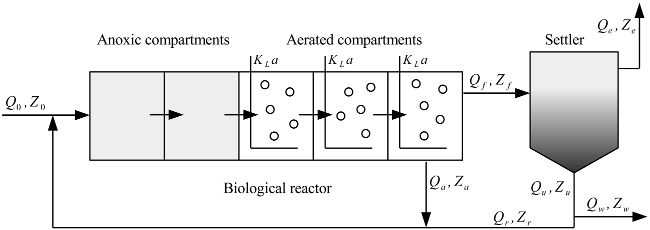
Proposed: Blue EMPC w/o TC: Red

Tracking MPC: Green P: Black

- Average recovery rates
  - P=0.7690, MPC=0.7754
  - EMPC w/o TC=0.8267
  - Proposed EMPC= 0.8845
- 12%, 11%, 6% increases compared with P, MPC and EMPC w/o TC

Wastewater treatment plant (Zeng and Liu, IECR 2015)

- Wastewater treatment plant



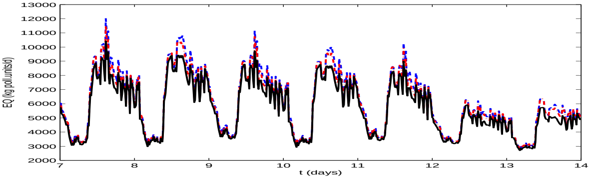
- Model is developed by the International Water Association
- Periodic operation subject to high uncertainties
- Two manipulated inputs: Q_a and $K_L a_5$
- Economic objective: maximize the effluent quality
- A typical control configuration: maintain $S_{NO,2}$ and $S_{O,5}$ at pre-determined set-points by manipulating the two control inputs

Wastewater treatment plant (Zeng and Liu, IECR 2015)

- EMPC design - representation of the control objective
  - Effluent quality: daily average of a weighted summation of the concentrations of different compounds in the effluent

$$EQ = \frac{1}{T}\int_{t_0}^{t_f} \Big(2TSS_e(t) + COD_e(t) + 30S_{NKj,e}(t) + 10S_{NO,e}(t) + 2BOD_e(t)\Big)Q_e(t)dt$$

- Simulation results: MPC with $N_p = 2,\ N_u = 1$, EMPC with $N = 8$, $N_h = 60$

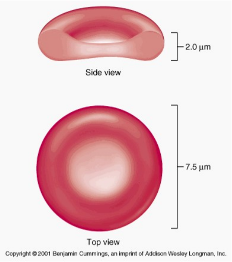


Proposed: Black Tracking MPC: Red PI: Blue

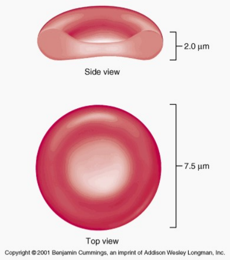
  - PI control $= 6123.53\ kg/d$, Tracking MPC $= 6022.64\ kg/d$
  - Proposed EMPC $= 5671.86\ kg/d$
    - Improved 7.4% and 5.8% compared with PI and MPC

EMPC in anemia management

- Anemia is caused by compromised hemoglobin levels
- Patients with End Stage Renal Disease have a compromised ability to produce erythropoietin (EPO) by which the body creates red blood cells
- Recombinant human EPO (rHuEPO) is used to treat anemic patients



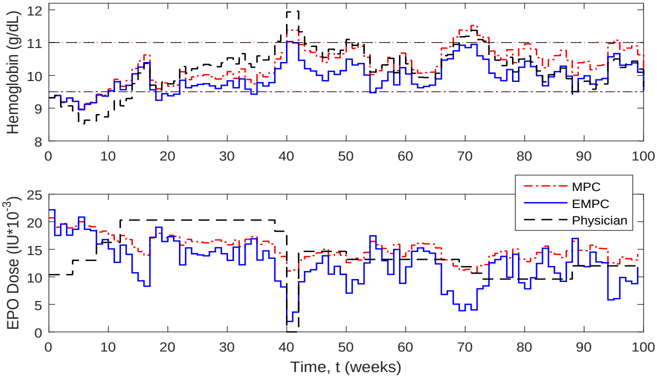
EMPC in anemia management



- Anemia is caused by compromised hemoglobin levels
- Patients with End Stage Renal Disease have a compromised ability to produce erythropoietin (EPO) by which the body creates red blood cells
- Recombinant human EPO (rHuEPO) is used to treat anemic patients
- Objectives: to develop control algorithms to maintain hemoglobin within target range and to save rHuEPO
- An ARX model is identified for each patient based on input-output data
- Economic zone MPC is used to minimize rHuEPO consumption
- Soft state constraints are used to ensure hemoglobin is within target

EMPC in anemia management

- Simulation results of economic zone MPC and zone MPC



- Percent of points in zone: MPC=86.8, EMPC=85.2, Physician=78.8
- rHuEPO consumptions ($\times 10^8$): MPC=1.53, EMPC=1.33, Physician=1.55
  - Reduced over 13%

Conclusions

- State estimation of nonlinear systems
  - Observer-enhanced MHE - an output feedback perspective
    - Less dependent on the horizon
    - Less sensitive to noise
    - May be used in output feedback control
  - Distributed MHE
    - Communication is important
    - Correction terms are important
    - May be extended to connect different types of estimators
- Economic MPC
  - Economic MPC with extended prediction horizon
    - Extended prediction horizon via an auxiliary stabilizing controller
    - Improved computational efficiency
    - Guaranteed stability and performance
  - Applications: oilsand separation, wastewater treatment, anemia management

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