ON CLASSIFYING SPACES OF SPIN GROUPS

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Abstract. For a split maximal torus T of a split spin group $G = \mathrm{Spin}(n)$ over an arbitrary field, we consider the restriction homomorphism $f\colon \mathrm{CH}(BG) \to \mathrm{CH}(BT)^W$ of the Chow rings of their classifying spaces with W the Weyl group of G. For $n \leq 6$, f is known to be surjective. For $n \geq 7$, an obstruction for an element of $\mathrm{CH}(BT)^W$ to be in the image of f is given by the Steenrod operations on $\mathrm{CH}(BT)/2\,\mathrm{CH}(BT)$. Using it, we show that several standard generators of $\mathrm{CH}(BT)^W$, including the defined for even n Euler class $e \in \mathrm{CH}^{n/2}(BT)^W$, are outside the image of f. This result differs from the analogues topological result.

Let F be a field and let G be a split reductive group over F with a split maximal torus $T \subset G$. We are interested in the restriction homomorphism $\mathrm{CH}(BG) \to \mathrm{CH}(BT)$ of the graded (by codimension of cycles) Chow rings of the classifying spaces (see [15]). This homomorphism relates the in general quite mysterious $\mathrm{CH}(BG)$ with the tame $\mathrm{CH}(BT)$ canonically isomorphic to the symmetric ring $S(\hat{T})$ on the character group $\hat{T}$ of T. Every element in the image of the restriction homomorphism is invariant under the action of the Weyl group $W = N_G(T)/T$ of G. By [4, Proposition 6], the homomorphism

$$f\colon \mathrm{CH}(BG) \to \mathrm{CH}(BT)^W$$

becomes bijective after tensoring with $\mathbb{Q}$. Therefore the kernel and the cokernel of f are torsion. More precisely, by [16, Theorem 1.3(1)], the kernel and the cokernel are killed by the torsion index $t(G)$ of G. In particular, f "computes" $\mathrm{CH}(BG)$ for any G with $t(G) = 1$. In general, since the group $\mathrm{CH}(BT)$ is torsion free, the kernel of f is actually precisely the ideal $\mathrm{Tors}\,\mathrm{CH}(BG)$ of torsion elements of the ring $\mathrm{CH}(BG)$ so that the image of f is identified with the quotient ring $\mathrm{CH}(BG)/\mathrm{Tors}\,\mathrm{CH}(BG)$.

Besides W-invariancy, the image of f satisfies another restriction: for every prime integer p it is stable under the total Steenrod operation

$$\mathrm{St}\colon \mathrm{Ch}(BT) \to \mathrm{Ch}(BT)$$

on the $\mathbb{F}_p$-coefficients version Ch of the Chow ring CH, where $\mathbb{F}_p := \mathbb{Z}/p\mathbb{Z}$. Indeed, the Steenrod operation for smooth varieties over an arbitrary field, constructed in [3] for characteristic $\neq p$ and in [13] for characteristic p, extends to $\mathrm{Ch}(BG)$ for any G including

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$G = T$. And the stability mentioned follows from commutativity of the square

$$\begin{array}{ccc} \mathrm{Ch}(BG) & \longrightarrow & \mathrm{Ch}(BT) \\ \downarrow{\scriptstyle \mathrm{St}} & & \downarrow{\scriptstyle \mathrm{St}} \\ \mathrm{Ch}(BG) & \longrightarrow & \mathrm{Ch}(BT) \end{array}$$

Therefore we have the following obstruction for an element $x \in \mathrm{CH}(BT)^W$ to be in the image of f:

Proposition 1. *If $x \in \operatorname{Im} f$, then $\mathrm{St}(g(x)) \in \operatorname{Im} g$, where g is the composition*

$$\mathrm{CH}(BT)^W \hookrightarrow \mathrm{CH}(BT) \to \mathrm{Ch}(BT)$$

of the embedding followed by the reduction modulo p. □

Note that the image of the homomorphism g is contained in (but, in general, not equal to) $\mathrm{Ch}(BT)^W$. An example of strict inclusion is given below.

We are going to apply the obstruction of Proposition 1 with $p = 2$ to investigate the image of f for G the standard split spin group $\mathrm{Spin}(n)$, $n \geq 2$, which is a split simply connected simple group of rank $l := [n/2]$. For $n = 2l + 1$, it has the Dynkin type B_l; for $n = 2l$ – the Dynkin type D_l. We take for T the standard split maximal torus $\mathbb{G}_{\mathrm{m}}^l \subset G$.

We first recall the situation with the similar homomorphism

$$f' \colon \mathrm{CH}(BG') \to \mathrm{CH}(BT')^W$$

for the standard split special orthogonal group $G' = \mathrm{SO}(n)$. Note that the inverse image of the standard split maximal torus $T' \subset G'$ under the cental isogeny $G \to G'$ is T and the Weyl group of G' coincides with W.

The ring $\mathrm{CH}(BT')$ is the polynomial ring over $\mathbb{Z}$ in l variables $y_1, \dots, y_l$. It is a graded ring with respect to the usual grading of the polynomial ring, where each variable has degree 1. Several special elements in this ring have traditional names and notation, c.f. [1, §2]. The elementary symmetric polynomials in $y_1, \dots, y_l$ are called the Chern classes and denoted $c_1, \dots, c_l$, where c_i is of degree i. The highest Chern class c_l is also called the Euler class and denoted e. The elementary symmetric polynomials in the squares $y_1^2, \dots, y_l^2$ are called the Pontrjagin classes and denoted $p_1, \dots, p_l$, where p_i is of degree $2i$.

For odd $n = 2l + 1$, the Weyl group W is a semidirect product by the symmetric group S_l, permuting the variables, of the direct product of l copies of $\mathbb{Z}/2\mathbb{Z}$, each of which acts by changing the sign of the respective variable. The ring of W-invariants $\mathrm{CH}(BT')^W = \mathbb{Z}[y_1, \dots, y_l]^W$ is therefore generated by the Pontrjagin classes $p_1, \dots, p_l$.

Note that the Weyl group W acts on the $\mathbb{F}_2$-version $\mathrm{Ch}(BT') = \mathbb{F}_2[y_1, \dots, y_l]$ of the Chow ring (only) by permutations of the variables $y_1, \dots, y_l$. Therefore the ring $\mathrm{Ch}(BT')^W$ is the polynomial ring $\mathbb{F}_2[c_1, \dots, c_l]$ which is strictly larger than the image of the integral invariants $\mathrm{CH}(BT')^W$ under the homomorphism g from Proposition 1.

For even $n = 2l$, the Weyl group is the subgroup in the Weyl group described above, generated by S_l and the even sign changes. In this case, the ring of W-invariants $\mathrm{CH}(BT')^W = \mathbb{Z}[y_1, \dots, y_l]^W$ is generated by the Pontrjagin classes $p_1, \dots, p_l$ and the Euler class e.

Let us consider the Chern classes in $\mathrm{CH}(BG')$ of the standard representation of G' given by the embedding $G' \hookrightarrow \mathrm{GL}(n)$. Their images in $\mathrm{CH}(BT')$ are the Chern classes of the representation of T' given by the embedding

$$T' \hookrightarrow \mathrm{GL}(2l), \;\; (a_1, \dots, a_l) \mapsto \mathrm{diag}(a_1, a_1^{-1}, \dots, a_l, a_l^{-1})$$

followed (if $n = 2l+1$) by the standard embedding $\mathrm{GL}(2l) \hookrightarrow \mathrm{GL}(n)$. This representation of T' is a direct sum of the 1-dimensional representations given by the characters $T' \to \mathbb{G}_\mathrm{m}$, $(a_1, \dots, a_l) \mapsto a_i$ and $T' \to \mathbb{G}_\mathrm{m}$, $(a_1, \dots, a_l) \mapsto a_i^{-1}$ $(i = 1, \dots, l)$ of T', having the first Chern classes y_i and $-y_i$. It follows that the images in $\mathrm{CH}(BT')$ of the Chern classes of the standard representation of G' are the elementary symmetric polynomials in $\pm y_1, \dots, \pm y_l$, i.e., the homogeneous components of the polynomial

$$(1 + y_1)(1 - y_1) \dots (1 + y_l)(1 - y_l) = (1 - y_1^2) \dots (1 - y_l^2).$$

The homogeneous components of odd degrees are trivial. The homogeneous components of even degrees are, up to signs, the Pontrjagin classes. In particular, the Pontrjagin classes are in the image of f'.

It follows that f' is surjective for odd n.

For even $n = 2l$, the computation of $\mathrm{CH}(BG')$, made in [12] over any field of characteristic not 2 (see also [5]), tells us that the image of f' is generated by the Pontrjagin classes and the multiple $2^{l-1}e$ of the Euler class. In particular, for $n \geq 4$, the Euler class itself is outside the image of f'. This can also be shown directly (and in any characteristic) as follows.

Let $\mathrm{Ch}(BT') = \mathbb{F}_2[y_1, \dots, y_l]$ be the Chow ring with coefficients $\mathbb{F}_2 = \mathbb{Z}/2\mathbb{Z}$ and consider the total Steenrod operation $\mathrm{St}\colon \mathrm{Ch}(BT') \to \mathrm{Ch}(BT')$ – the (non-homogeneous) ring endomorphism mapping each y_i to $y_i(1 + y_i)$. Its degree 1 homogeneous component St^1 (raising degrees by 1) applied to e yields ec_1. If e would be in $\mathrm{Im}\, f$, then ec_1 viewed as an element of $\mathbb{F}_2[y_1, \dots, y_l]$, would be inside the subring of $\mathbb{F}_2[y_1, \dots, y_l]$, generated by $p_1 = c_1^2, \dots, p_{l-1} = c_{l-1}^2$ and $e = c_l$, which is false.

We are going to study the group $G = \mathrm{Spin}(n)$ using the similar approach. For $n \leq 6$, the torsion index of G is 1 so that f is an isomorphism. Therefore we need to deal with $n \geq 7$ only. The torsion index of G (computed in [16]) is then a power of 2 (with a positive exponent). Besides results on $n = 7$ ([6]) and on $n = 8$ ([14]), the ring $\mathrm{CH}(BG)$ is far from being understood. One can say that the special orthogonal group G', whose torsion index is a power of 2 as well, constitutes a rare example of a split reductive group with a nontrivial torsion index, for which the Chow ring of its classifying space is computed.

We start with a description of $\mathrm{CH}(BT)$.

Note that $\mathrm{CH}(BT')$ is a subring in $\mathrm{CH}(BT)$. The $\mathrm{CH}(BT')$-algebra $\mathrm{CH}(BT)$ is generated by a single element a satisfying the relation $2a = y_1 + \dots + y_l$. The action of W on $\mathrm{CH}(BT')$ extends uniquely to $\mathrm{CH}(BT)$: the symmetric group $S_l \subset W$ acts on a trivially and the action on a of the change of sign of y_i yields $a - y_i$.

The maps f and f' are related by the commutative square

$$\begin{array}{ccc} \mathrm{CH}(BG) & \xrightarrow{f} & \mathrm{CH}(BT)^W \\ \uparrow & & \uparrow \\ \mathrm{CH}(BG') & \xrightarrow{f'} & \mathrm{CH}(BT')^W \end{array}$$

In particular, $\operatorname{Im} f \supset \operatorname{Im} f'$.

The ring of W-invariants $\mathbb{Z}[a, y_1, \dots, y_l]^W$ is computed (in a topological context) by D. Benson and J. Wood in [1, Theorem 7.1]. To formulate the result, they first inductively construct in [1, Proposition 3.3] for every $i \geq 1$ certain homogeneous element $q_i \in \mathbb{Z}[a, y_1, \dots, y_l]^W$ of degree 2^i. Besides, they define one more homogeneous element $\alpha \in \mathbb{Z}[a, y_1, \dots, y_l]^W$. If n is 0 modulo 4, the degree of α is 2^{l-2}. If n is not 0 modulo 4 (i.e., n is ± 1 or 2 modulo 4), the degree of α is 2^{l-1}. If n is 2 modulo 4, then α is the orbit product α' of a, i.e., α coincides with the product α' of the elements in the W-orbit of a. If n is not 2 modulo 4 (i.e., n is 0 or ± 1 modulo 4), then $\alpha' = \alpha^2$, i.e., α is a square root of the orbit product α'.

Theorem 2 (c.f. [1, Theorem 7.1])**.** *The $\mathbb{Z}[y_1, \dots, y_l]^W$-algebra $\mathbb{Z}[a, y_1, \dots, y_l]^W$ is generated by α together with all q_i of lower (than that of α) degree.*

Here is our main result:

Theorem 3. *For $G = \mathrm{Spin}(n)$, the generators q_i of Theorem 2 with $2^i + 1 < n/2$ are outside the image of the homomorphism $f\colon \mathrm{CH}(BG) \to \mathrm{CH}(BT)^W$. Moreover, for even $n \geq 7$, the Euler class $e \in \mathrm{CH}(BT')^W \subset \mathrm{CH}(BT)^W$ is also outside the image of f.*

Remark 4. Theorem 3 states, inter alia, that $q_1 \notin \operatorname{Im} f$ for $n \geq 7$. In particular, the map f is not surjective in degree 2 for such n. The latter statement is apparent already from [11]. See also [18, Theorem 3.3] together with [2, Théorème 12.1(b)].

Let us explain how to determine the image of f in degree 2 without using the Steenrod operation. The representation ring $R(G)$ of G is the subring $\mathbb{Z}[\hat{T}]^W$ of W-invariants in the group ring $\mathbb{Z}[\hat{T}]$ of the character group $\hat{T}$ of T. The second Chern class map $c_2\colon R(G) \to \mathrm{CH}^2(BG)$ is surjective and the square

$$\begin{array}{ccc} R(G) = \mathbb{Z}[\hat{T}]^W & \longrightarrow & R(T) = \mathbb{Z}[\hat{T}] \\ \downarrow{\scriptstyle c_2} & & \downarrow{\scriptstyle c_2} \\ \mathrm{CH}^2(BG) & \xrightarrow{f} & \mathrm{CH}^2(BT)^W = S^2(\hat{T})^W \end{array}$$

commutes. It follows that the image of f in degree 2 is the image of

$$c_2\colon \mathbb{Z}[\hat{T}]^W \to S^2(\hat{T})^W. \tag{5}$$

Remark 6. In topology, the similar to f map f_H, departing out of the integral cohomology $H(BG)$ and having the same destination as f (see Remark 8), is surjective if and only if n is not congruent to ± 3 or 4 modulo 8, see [1, Theorem 10.2]. Moreover, all the generators q_i (for any n) as well as the Euler class (for any even n) are always in the image.

Remark 7. Concerning the generator α, note that α', which appears in the above description of α, is a Chern class of the orbit sum of the element of $\mathbb{Z}[\hat{T}]$ given by a (c.f. [8, Proof of Proposition 3.4]). (This orbit sum is an element of the representation ring $R(G) = \mathbb{Z}[\hat{T}]^W$ of G. The Chern classes $R(G) \to \mathrm{CH}(BG)$ we are using are defined, e.g., in [10, §4]. They already appeared above for G' in place of G during the discussion of the Pontrjagin classes. The second Chern class also appeared in Remark 4.) Therefore $\alpha' \in \operatorname{Im} f$. It follows that $\alpha \in \operatorname{Im} f$ if n is 2 modulo 4 (or, equivalently, n is ± 2 modulo 8). If n is ± 3 or 4 modulo 8, then by [1, Theorem 10.2] α is not in the image of the topological analogue f_H of f (all the remaining generators of $\mathrm{CH}(BT)^W$ are in the image); therefore $\alpha \notin \operatorname{Im} f$ (at least in characteristic 0). Finally, when n is ± 1 or 0 modulo 8, α is in the image in topology, but we do not know whether $\alpha \in \operatorname{Im} f$ for our f.

Proof of Theorem 3. The generators of the modulo 2 reduction $\mathrm{Ch}(BT) = \mathbb{F}_2[a, y_1, \dots, y_l]$ of the ring $\mathrm{CH}(BT) = \mathbb{Z}[a, y_1, \dots, y_l]$ are subject to the only relation $y_1 + \cdots + y_l = 0$. For the images in $\mathrm{Ch}(BT)$ of the special elements of $\mathrm{CH}(BT') = \mathbb{Z}[y_1, \dots, y_l]$ we are still using the same notation. The first Chern class c_1 vanishes and the remaining Chern classes $c_2, \dots, c_l$ are algebraically independent. The Pontrjagin classes are simply the squares of the respective Chern classes: $p_i = c_i^2$ for every $i = 1, \dots, l$. The Steenrod operation

$$\mathrm{St}\colon \mathbb{F}_2[a, y_1, \dots, y_l] \to \mathbb{F}_2[a, y_1, \dots, y_l]$$

is the ring endomorphism mapping $y_i \mapsto y_i(1 + y_i)$ and $a \mapsto a(1 + a)$.

By [1, Proposition 3.3(i) and Proof of Proposition 3.3 (1st Displayed Formula)], we have $g(q_1) = c_2$. By [1, Proof of Proposition 3.3 (3d Displayed Formula and Definition of q_i)], one sees that $g(q_i) \in \mathbb{F}_2[y_1, \dots, y_l]$ for any $i \geq 1$ and that $g(q_{i+1})$ is the sum of all pairwise products of distinct monomials of $g(q_i)$. It follows that for i with $2^i + 1 < n/2$, $g(q_i)$ is equal to c_{2^i} plus a polynomial in the Chern classes of smaller degree. Since $\mathrm{St}^1(c_{2^i})$ is equal to c_{2^i+1} plus a polynomial in the Chern classes of smaller degree, $\mathrm{St}^1(g(q_i))$ is also equal to c_{2^i+1} plus a polynomial in the Chern classes of smaller degree.

If n is odd or divisible by 4, the subring $g(\mathrm{CH}(BT)^W) \subset \mathrm{Ch}(BT)$ is generated by homogeneous elements of even degrees. In the remaining case, it is generated by homogeneous elements of even degrees and the Euler class $e = c_l = c_{n/2}$. It follows that $\mathrm{St}^1(g(q_i)) \notin \operatorname{Im} g$ so that $q_i \notin \operatorname{Im} f$ by Proposition 1. This proves the first part of Theorem 3. Note that we used the first Steenrod square only. Therefore, in characteristic 2, instead of the newer [13], we may refer to the older [7].

Regarding the Euler class, we have $\mathrm{St}^i(e) = ec_i$ for $i = 1, \dots, l$. In particular,

$$\mathrm{St}^3(e) = ec_3 \notin \operatorname{Im} g.$$

Therefore $e \notin \operatorname{Im} f$. □

Remark 8. To explain relations and differences with topology, let us recall that for any affine algebraic group G over the complex numbers, the cycle class map

$$\mathrm{cl}\colon\ \mathrm{CH}(BG) \to H(BG)$$

is a functorial in G homogeneous ring homomorphism, where $H(BG)$ is the integral cohomology ring of the classifying space of G studied in topology. In general, the map cl is neither surjective nor injective; it is an isomorphism provided that G is a torus. By [17, Theorem 2.14], for arbitrary G, tensoring with $\mathbb{Q}$ makes cl an isomorphism.

For G as in Theorem 3 (still over $\mathbb{C}$), the homomorphism f fits into the commutative square

$$\begin{array}{ccc} \mathrm{CH}(BG) & \xrightarrow{f} & \mathrm{CH}(BT)^W \\ \downarrow{\scriptstyle \mathrm{cl}} & & \downarrow{\scriptstyle \simeq} \\ H(BG) & \xrightarrow{f_H} & H(BT)^W \end{array}$$

It follows that the image of f is contained in the image of f_H. In the cases of strict inclusion (e.g., provided by Theorem 3), the map cl is not surjective.

Let us now explain why the proof of Theorem 3 does not work in topology. As a replacement of the Steenrod operation, used in the proof, one can try to use the Steenrod operation on the cohomology $H(BG, \mathbb{F}_2)$ with coefficients $\mathbb{F}_2$. However, unlike the homomorphism $\mathrm{CH}(BG) \to \mathrm{Ch}(BG)$, the homomorphism

$$H(BG) = H(BG, \mathbb{Z}) \to H(BG, \mathbb{F}_2)$$

is not surjective. Because of that, we do not get the analogue of Proposition 1.

The positive answer to the following question would allow one to determine the indexes of generic grassmannians for even spin groups. Lower bounds on these indexes, which are within 1 from the exact values, are recently obtained in [9]. For the odd spin groups, a procedure for determination of the exact values is described in [8].

Question 9. *For even $n \geq 12$, is the image of $f\colon \mathrm{CH}(B\,\mathrm{Spin}(n)) \to \mathrm{CH}(BT)^W$ contained in the subring of $\mathrm{CH}(BT)^W$ generated by $2e$ together with the remaining (without e) generators (including the Pontrjagin classes) of $\mathrm{CH}(BT)^W$, listed in Theorem 2?*

One can show that for each $n \leq 10$ the answer to Question 9 is negative. The Euler class part of Theorem 3 can be viewed as a first step towards resolution of Question 9.

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