

## Physics 234: Exercise 1

1. An  $N$ -bit binary register can hold  $2^N$  unique bit patterns. To represent an unsigned integer, the bit patterns are interpreted as the base-2 numbers 0 to  $2^N - 1$ . There are several possible ways to represent signed integers. Nonetheless, on all modern computers this is accomplished using the *two's complement* scheme (which we'll denote by base  $\bar{2}$ ), in which the the numbers  $-2^{N-1}$  to  $+2^{N-1} - 1$  are represented by the same bit patterns reinterpreted as 0 to  $2^{N-1} - 1$  and then  $-2^{N-1}$  to  $-1$  in lexical order.

The table below illustrates the unsigned (left) and signed (right) values for  $N = 8$ :

127	0	1	1	1	1	1	1	1	127
126	0	1	1	1	1	1	1	0	126
3	0	0	0	0	0	0	1	1	3
2	0	0	0	0	0	0	1	0	2
1	0	0	0	0	0	0	0	1	1
0	0	0	0	0	0	0	0	0	0
255	1	1	1	1	1	1	1	1	-1
254	1	1	1	1	1	1	1	0	-2
253	1	1	1	1	1	1	0	1	-3
129	1	0	0	0	0	0	0	1	-127
128	1	0	0	0	0	0	0	0	-128

The advantage of two's complement numbers is that they can be added in exactly the same way as conventional binary numbers:

·	·		1	1			1		
	0	1	0	1	1	0	0	1	
+	1	0	0	1	1	1	0	1	
	1	1	1	1	0	1	1	0	

Here, red ones represent the carry bits. The computation above can be interpreted as either  $89 + 157 = 246$  or  $89 - 97 = -8$ .

- (a) Determine the 8-bit unsigned binary representation of 116. Does the two's complement representation of that number have the same bit pattern?
- (b) In the  $N$ -bit two's complement scheme, negation (i.e., mapping a number  $x \rightarrow -x$ ) is accomplished by all flipping all  $N$  bits and then adding one (modulo  $2^N$ , of course). Negate the 8-bit two's complement number 116 from part (a).
- (c) One advantage of two's complement is that it encodes a unique zero (rather than both 0 and  $-0$ ). But there is one nonzero value that also behaves as  $x \rightarrow x$  under negation. What is it?
- (d) There are four possible values for the two leftmost carry bits (marked with dots): 00, 01, 10, 11. Which combinations denote an overflow for unsigned binary and which for signed (two's complement) binary?

2. Change the octal numbers  $60177756_8$ ,  $3006012_8$ ,  $3726755_8$ ,  $76545336_8$ ,  $5655_8$ , and  $67545336_8$  to hexadecimal notation using the digits  $0, 1, \dots, F$ . Hint: Doing this by hand is easiest if you use the base-2 bit patterns as an intermediate step.
3. We can generalize a positional number system to negative bases by writing

$$(\dots a_3 a_2 a_1 a_0 . a_{-1} a_{-2} \dots)_b = \dots + a_2 b^2 + a_1 b + a_0 + a_{-1} b^{-1} + \dots$$

in the usual way but requiring that the digits be in the range  $0 \leq a_k < |b|$ . For example,

$$1322_{-4} = -22$$

$$133.21_{-5} = 12.64$$

$$2050.23_{-6} = -462.25$$

$$6231.46_{-8} = -2967.40625$$

- (a) Negate each of the identities above.
- (b) Is there a unique negative base representation for each number?
- (c) Is it ever necessary to have a leading minus sign, e.g.,  $-54_{-3}$ , when the base is negative?
4. Consider a 6-digit *factoradic* system

$$(a_6 \dots a_3 a_2 a_1) = a_6 \cdot 6! + \dots + a_3 \cdot 3! + a_2 \cdot 2! + a_1, \quad \text{where } 0 \leq a_k \leq k$$

- (a) What are the largest and smallest numbers representable?
- (b) Is every number in between representable?