Physics 234: Exercise 3

1. Suppose we have an $N \times N$ matrix A whose elements $A_{i,j}$ are labelled by row i and column j, both counting from zero. The standard way to store such a matrix is to pack its N^2 elements into a one-dimensional array that can be accessed via a single, compressed index $I = j + Ni$ (row-major order) or $I = i + Nj$ (column-major order).

   If the matrix has additional structure (e.g., symmetries or large blocks of zero entries), we may be able to get away with storing fewer than N^2 terms, since the missing ones can always be reconstructed when needed. Consider the case where A is symmetric.

   (a) What's the minimum size of the array?

   (b) There are four obvious storage schemes (illustrated below) based on whether we keep the upper or lower triangle and whether we organize by row or column. Determine the indexing rule for each of them. Hint:

$$\sum_{k=0}^{i} k = \frac{i(i+1)}{2} \quad \text{and} \quad \sum_{k=0}^{i-1} (N-1-k) = \frac{i(2N-1-i)}{2}.$$

0	1	2	3	4	5	6	7
	8	9	10	11	12	13	14
		15	16	17	18	19	20
			21	22	23	24	25
				26	27	28	29
					30	31	32
						33	34
							35

0	1	3	6	10	15	21	28
	2	4	7	11	16	22	29
		5	8	12	17	23	30
			9	13	18	24	31
				14	19	25	32
					20	26	33
						27	34
							35

0							
1	2						
3	4	5					
6	7	8	9				
10	11	12	13	14			
15	16	17	18	19	20		
21	22	23	24	25	26	27	
28	29	30	31	32	33	34	35

0							
1	8						
2	9	15					
3	10	16	21				
4	11	17	22	26			
5	12	18	23	27	30		
6	13	19	24	28	31	33	
7	14	20	25	29	32	34	35

2. Derive the analogous results for the case of an *antisymmetric* matrix.

3. Compute a^{19} from a in the most efficient way.

4. Evaluate $a^4 - 6a^3 + 11a^2 - 4a + 1$, performing as few arithmetic operations as possible.