

Physics 234: Lab Test

Tuesday, April 5, 2011

Student's Name: _____ UserID: p234u_____

1. How many of the numbers 1000, 1001, ..., 7999, 8000 are perfect cubes?

11

2. The number of ways to group $2n$ objects into n pairs is

$$T_{2n} = \frac{(2n)!}{n! \cdot 2^n}.$$

(Remember that $n! = 1 \cdot 2 \cdot 3 \cdots n$ is the *factorial* of n .) You should find that $T_2 = 1$, $T_4 = 3$, $T_6 = 15$, and $T_8 = 105$. Report the value of T_{16} . You'll have to be very careful about avoiding integer overflow.

2027025

3. Apply Newton's method to find the local extrema of the function $f(x) = \frac{1}{2}x^4 + 2x^3 - 15x^2 + 5$. Starting from an initial guess $x_0 = 1$, iterate according to $x_{n+1} := x_n - f'(x_n)/f''(x_n)$ to construct a sequence

$$(x_n) = (x_0, x_1, x_2, \dots) = (1, -0.833333, -0.0516796, \dots).$$

This converges to the local maximum at $x = 0$. Now build the sequence starting from $x_0 = -4$. What is the value of $\lim_{n \rightarrow \infty} f(x_n)$?

-325.039

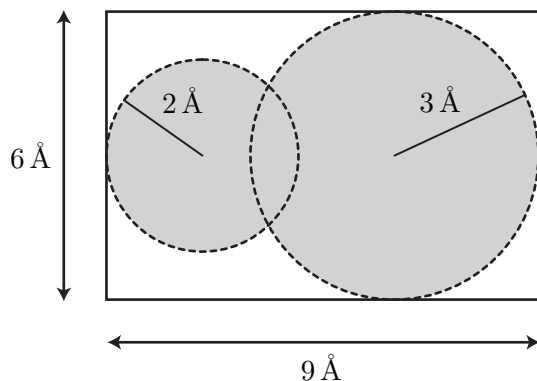
4. The 7×2 matrix

$$A = \begin{pmatrix} \frac{2}{3} & -\frac{1}{2} \\ \vdots & \vdots \\ \frac{8}{51} & -\frac{5}{18} \end{pmatrix} \text{ is defined element-wise by } A_{ij} = (-1)^{i+j} \frac{1+ij}{1+i^2+j^2}.$$

Here, the row and column indices range over $i = 1, 2, \dots, 7$ and $j = 1, 2$ (in typical math style, as opposed to the zero-based indexing of C arrays). Compute the product $B = A^T A$ (with the superscript T denoting the *transpose*) and check that $B_{1,2} = B_{2,1} = -1.095439594$. Report the determinant $\det B = \det(A^T A)$.

0.1198063155

5. A diatomic molecule—modelled as two overlapping spheres located at $(2, 0)$ and $(6, 0)$ —is shown in profile below.



Compute its cross-sectional area as follows:

- Set two integer counters C and N to zero.
- Choose two random numbers $x \in [0, 9]$ and $y \in [-3, 3]$.
- If the point (x, y) lies within either of the atomic circumferences then increment C by one.
- Increment N by one.
- Repeat from (b) until N numbers in the tens of millions.
- Report the result $54 \times C \div N$, determined as a double-precision floating-point calculation.

You should be able to verify that the answer is 35.46 when the larger atom is relocated to $(5, 0)$; and 31.52 at $(4, 0)$. What's the value when the atom is in its original position, $(6, 0)$?

38.85