Physics 234: Lab Test

Thursday, April 7, 2011

Student's Name: ______________________________ UserID: `p234u`_______

1. How many of the numbers $1000, 1001, \ldots, 7999, 8000$ are perfect squares?

   $\boxed{58}$

2. The number of ways to group $2n$ objects into n pairs is

   $$T_{2n} = \frac{(2n)!}{n! \cdot 2^n}.$$

   (Remember that $n! = 1 \cdot 2 \cdot 3 \cdots n$ is the *factorial* of n.) You should find that $T_2 = 1$, $T_4 = 3$, $T_6 = 15$, and $T_8 = 105$. Report the value of T_{18}. You'll have to be very careful about avoiding integer overflow.

   $\boxed{34459425}$

3. Apply Newton's method to find the local extrema of the function $f(x) = \frac{3}{4}x^4 - \frac{21}{4}x^3 + \frac{3}{2}x^2 + 30x$. Starting from an initial guess $x_0 = 1$, iterate according to $x_{n+1} := x_n - f'(x_n)/f''(x_n)$ to construct a sequence

   $$(x_n) = (x_0, x_1, x_2, \ldots) = (1, 2.03846, 1.87387, \ldots).$$

   This converges to the local maximum at $x = 1.8762$. Now build the sequence starting from $x_0 = -8$. What is the value of $\lim_{n\to\infty} f(x_n)$?

   $\boxed{-23.2329}$

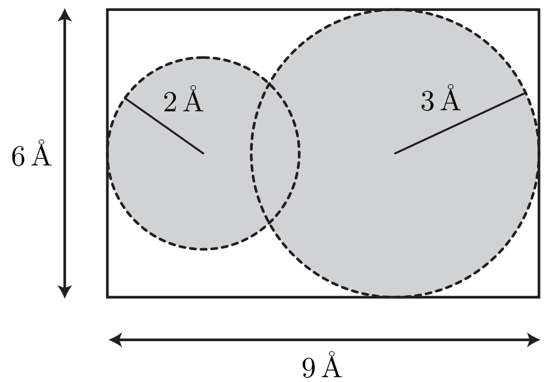
4. The 8×2 matrix

   $$A = \begin{pmatrix} \frac{3}{4} & -\frac{1}{2} \\ \vdots & \vdots \\ -\frac{17}{67} & \frac{33}{73} \end{pmatrix} \quad \text{is defined element-wise by} \quad A_{ij} = (-1)^{i+j} \frac{1 + 2ij}{1 + i^2 + 2j^2}.$$

   Here, the row and column indices range over $i = 1, 2, \ldots, 8$ and $j = 1, 2$ (in typical math style, as opposed to the zero-based indexing of C arrays). Compute the product $B = A^{\mathrm{T}}A$ (with the superscript T denoting the *transpose*) and check that $B_{1,2} = B_{2,1} = -2.299670537$. Report the determinant $\det B = \det(A^{\mathrm{T}}A)$.

   $\boxed{0.5710928332}$

5. A diatomic molecule—modelled as two overlapping spheres located at $(2, 0)$ and $(6, 0)$—is shown in profile below.



Compute its cross-sectional area as follows:

(a) Set two integer counters C and N to zero.

(b) Choose two random numbers $x \in [0, 9]$ and $y \in [-3, 3]$.

(c) If the point (x, y) lies within either of the atomic circumferences then increment C by one.

(d) Increment N by one.

(e) Repeat from (b) until N numbers in the tens of millions.

(f) Report the result $54 \times C \div N$, determined as a double-precision floating-point calculation.

You should be able to verify that the answer is 35.46 when the larger atom is relocated to $(5, 0)$; and 31.52 at $(4, 0)$. What's the value when the atom is in its original position, $(6, 0)$?

38.85