Physics 234: Lab Test

Tuesday, March 31, 2009 / Thursday, April 2, 2009

Student's Name: ________________________ UserID: `p234u`______

1. Consider the finite sequence of numbers

$$S = (n^2 + 3n^5)_{n=1}^{65} = (4, 100, 738, \ldots, 3480876100).$$

How many of the numbers in S are divisible by 12? (Hint: beware of integer overflow.)

$\boxed{15}$

2. Implement Newton's method for the function $f(x) = x^4 - 20x^2 + 3x$. Starting from an initial guess $x_0 = -20$, iterate according to $x_{n+1} := x_n - f(x_n)/f'(x_n)$ to construct a sequence

$$(x_n) = (x_0, x_1, x_2, \ldots) = (-20, -15.1297, -11.5226, \ldots)$$

that converges toward one of the roots of $f(x)$. What are the first six digits of x_7? (Watch out: x_7 is the *eighth* term in the sequence.)

$\boxed{-4.61171}$

3. What is the globally minimum value of $f(x) = x^4 - 20x^2 + 3x$? I suggest you plot the function first (in `gnuplot`, say) to visually identify a good starting point and then proceed by steepest-descent. (Watch out: I'm looking for $f(x_{\text{min}})$ rather than x_{min}.)

$\boxed{-109.542}$

4. Two 10×10 matrices A and B are defined element-wise by the formulas

$$A_{ij} = \frac{1}{2i + j} \quad \text{and} \quad B_{ij} = \frac{1}{1 + (i - j)^4},$$

where the row and column indices range over $i, j = 1, 2, \ldots, 10$ (as opposed to the zero-based indexing of C arrays). The product matrix $C = AB$ is partially revealed below:

$$C = \begin{pmatrix} 0.473048 & \cdots & 0.137063 \\ \vdots & & \vdots \\ 0.073686 & \cdots & 0.0534094 \end{pmatrix}$$

What is the trace of C? That is, what is the *sum of its diagonal elements*?

$\boxed{1.82561}$

5. Consider a disc D of unit radius centred on $(0, 0)$. We want to compute its moment of inertia for rotation about an arbitray point $\mathbf{r}_0 = (x_0, y_0)$. If the disc has uniform mass density and total mass $M = 1$, then the moment of intertia is given by

$$I(\mathbf{r}_0) = \int_D d^2r \, (\mathbf{r} - \mathbf{r}_0)^2 = \int_{-1}^{1} dx \int_{-\sqrt{1-x^2}}^{+\sqrt{1-x^2}} dy \, [(x - x_0)^2 + (y - y_0)^2].$$

Evaluate the integral as follows:

(a) Set an integer counter N and a floating-point accumulator S to zero.
(b) Choose two random numbers $x, y \in [-1, 1]$.
(c) If $x^2+y^2 \le 1$ (the point is in the disc) then increment N by one and increment S by $(x-x_0)^2+(y-y_0)^2$.
(d) Repeat from (b) until N numbers in the tens of millions.
(e) Report the result $I \doteq S/N$.

You should be able to verify the numerical estimates $I(0, 0) \doteq 0.500$ and $I(\frac{1}{2}, 0) \doteq 0.750$. What is the value for $I(\frac{1}{4}, \frac{1}{4})$?

$\boxed{0.625}$