Physics 234: Practice Lab Test

1. How many of the numbers $1, 2, \ldots, 1000$ are perfect squares?

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   How many are perfect cubes?

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   (Hint: you can solve this question without using either `sqrt` or `pow`.)

2. Consider an unbounded square grid of points spaced by $\Delta x = 0.1$ and $\Delta y = 0.1$. How many points lie inside the diamond $|x| + |y| = 2$?

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   How many points lie inside the circle $x^2 + y^2 = 4$?

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   (Hints: (i) don't count the points on the boundary; (ii) you may be better off reformulating the problem so that you can use integer rather than floating-point types in your code.)

3. Here's a variant of *Buffon's needle problem.* Imagine placing a randomly-directed straight line segment of unit length in the plane so that one end lies on the x-axis somewhere between -1 and 1. What is the probability P that the line segment intersects the y-axis?

   Evaluate the probability as follows:

   (a) Set integers counter N and C to zero.

   (b) Choose three random numbers $\xi, \xi', \xi'' \in [-1, 1]$.

   (c) If the point (ξ', ξ'') lies in the annulus defined by $0.001 < (\xi')^2 + (\xi'')^2 \le 1$ then construct a unit vector
   $$\hat{\mathbf{n}} = \frac{(\xi', \xi'')}{\sqrt{(\xi')^2 + (\xi'')^2}},$$
   position the end-points at $\mathbf{r}_1 = (\xi, 0)$ and $\mathbf{r}_2 = \mathbf{r}_1 + \hat{\mathbf{n}}$, and increment N by one.

   (d) If the line segment connecting $\mathbf{r}_1$ and $\mathbf{r}_2$ crosses the y-axis then increment C by one.

   (e) Repeat from (b) until N numbers in the tens of millions.

   (f) Report the result $P \doteq C/N$.

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4. Consider the sequence $(a_n) = (a_0, a_1, a_2, \ldots)$ whose elements are the partial sums
   $$a_n = \sum_{k=0}^{n} \bigl(z^k + 2z^{k+1} + 3z^{2k}\bigr).$$
   This sequence converges linearly and attains the limit
   $$L = \lim_{n\to\infty} a_n = \frac{1}{1-z} + \frac{2z}{1-z} + \frac{3}{1-z^2} = \frac{4 + 3z + 2z^2}{1-z^2}.$$

In the case of $z = 0.1$, the first three terms are $a_0 = 4.2$, $a_1 = 4.35$, and $a_2 = 4.3623$. What is the smallest value p for which a_p agrees with L to within 1 part in 10^6.

$p = \square$ $\quad a_p = \square.\square\square\square\square\square\square\square\square\square\square$

Generate a new sequence (A_n) from (a_n) using the Aitken transformation:

$$A_n = a_{n+1} + \left(\frac{1}{a_{n+2} - a_{n+1}} - \frac{1}{a_{n+1} - a_n} \right)^{-1}$$

What is the smallest value q for which A_q agrees with L to within 1 part in 10^6.

$q = \square$ $\quad A_q = \square.\square\square\square\square\square\square\square\square\square\square\square\square\square\square$