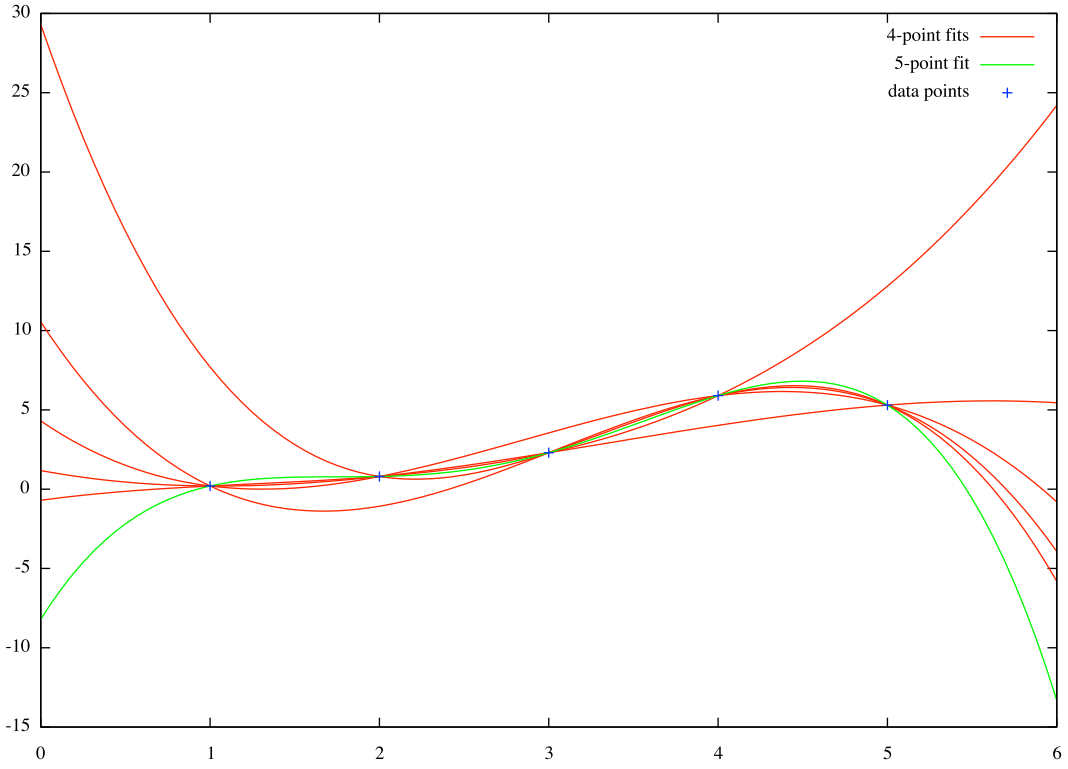
Physics 234: Quiz 7

Friday, March 25, 2011

Student's Name: _____



1. A 4th-order polynomial $p(x)$ passing through all the points in $S = \{(x_0, y_0), \ldots, (x_4, y_4)\}$ is a Lagrange interpolant for the data set S. $p(x)$ is plotted in the figure above. But superimposed on that curve are five additional fits using 3rd-order polynomials through all possible four-point subsets of S.

   What name do we give to this kind of analysis?

   (a) Bootstrap
   (**B**) Jackknife
   (c) Bowie knife
   (d) David Bowie's knife
   (e) Someone else's cutlery
   (f) Chebyshev
   (g) Quadrature
   (h) Rhomberg

2. Suppose that S was generated by evaluating a function $f(x)$ at points $(x_i, y_i) = (x_i, f(x_i))$. We understand $p(x)$ to be at most a plausible reconstruction of $f(x)$.

   Which of the following statements is false?

   (a) Since $f(1.0) = 0.2$, it must be that $p(1.0) = 0.2$.

   (b) $p(x)$ is unlikely to provide a good interpolation if $f(x)$ has a singularity at any point in the interval $[1, 5]$.

   (**C**) Uniformly spacing the x_i values (as in the figure) is the optimal choice of mesh.

   (d) It's very unlikely that $f(0) \approx -8$.

3. I now want to fit the leftmost four points in S to the function $a + be^{cx}$. I'll do this by making a good guess for the parameters (a, b, c) and then evolving them in the direction $\hat{\mathbf{g}} = (g_a, g_b, g_c)$ that is "downhill" with respect to the sum of the squares of the residuals.

$$g_a \sim -\sum_{i=0}^{4}(a + be^{cx_i} - y_i)$$

$$g_b \sim -\sum_{i=0}^{4}(a + be^{cx_i} - y_i)e^{cx_i}$$

   Write down the corresponding expression for g_c.

$$g_c \sim -\sum_{i=0}^{4}(a + be^{cx_i} - y_i)be^{cx_i}x_i$$