Physics 308: Statistical, Molecular, and Solid State Physics

Final Exam

Wednesday, April 23, 2008, 09:00–12:00

Student's Name: _____

Constants

$k_B = 8.617 \times 10^{-5}\,\mathrm{eV/K}$

$k_B T_{\mathrm{room}} = 0.025\,\mathrm{eV}$

$\hbar = 1.055 \times 10^{-34}\,\mathrm{J\,s}$

$hc = 1.24 \times 10^{-6}\,\mathrm{eV\,m} = 1240\,\mathrm{eV\,nm}$

$m_e c^2 = 5.11 \times 10^5\,\mathrm{eV} = 0.511\,\mathrm{MeV}$

$\dfrac{\hbar^2}{2m_e} = \dfrac{(hc)^2}{8\pi^2(m_e c^2)} = 0.0381\,\mathrm{eV\,nm^{-2}}$

$1\,\mathrm{u} = 1.661 \times 10^{-27}\,\mathrm{kg} = 931.4\,\mathrm{MeV/c^2}$

$c = 299792458\,\mathrm{m/s}$

$a_0 = \dfrac{4\pi\epsilon_0\hbar^2}{m_e e^2} = 5.292 \times 10^{-2}\,\mathrm{nm}$

$e = 1.602 \times 10^{-19}\,\mathrm{C}$

$\mu_B = \dfrac{e\hbar}{2m_e} = 5.7883 \times 10^{-5}\,\mathrm{eV\,T^{-1}}$

Formulas

volume and surface area of a sphere:

$$V = \frac{4}{3}\pi R^3 \qquad A = 4\pi R^2$$

particle in a box:

$$\psi \sim \sin\frac{n\pi x}{L} \qquad E_n = \frac{h^2 n^2}{8mL^2}$$

quantum of light:

$$E = \hbar\omega = hf \qquad c = \lambda f$$

dipole moment:

$$p = (\Delta q) r_0$$

binding energy in an ionic solid:

$$U = -\frac{\alpha}{4\pi\epsilon_0 a}$$

Leonard-Jones potential:

$$U(r) = U_0\left[\left(\frac{a}{r}\right)^{12} - 2\left(\frac{a}{r}\right)^{6}\right]$$

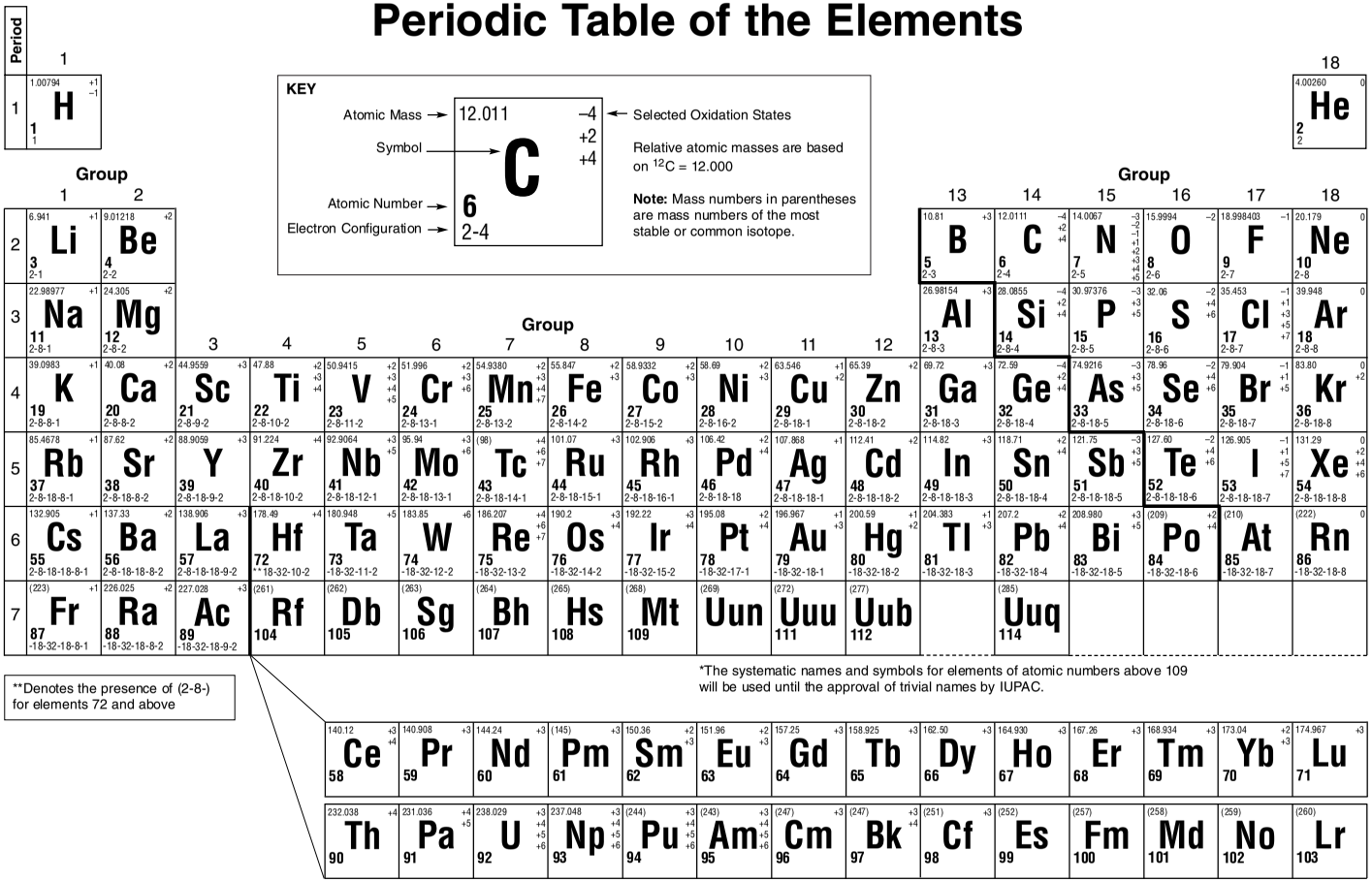
reduced mass:

$$\frac{1}{\mu} = \frac{1}{m_1} + \frac{1}{m_2}$$

current density and resistivity:

$$j = nqv \qquad \rho = \frac{mv}{nq^2 l}$$

Periodic Table of the Elements



Multiple Choice Questions (30 points)

Answer by circling one of (a), (b), (c), etc. Please be clear about which one you have selected.

1. Free electrons with wavefunction $\psi \sim e^{i\mathbf{k}\cdot\mathbf{r}}$ are characterized by a quantum number $\mathbf{k} = (k_x, k_y, k_z)$ and have energy $E(\mathbf{k}) = \hbar^2|\mathbf{k}|^2/2m$. Which of the following correctly describes the ground state of N electrons in a volume V with Fermi energy $E_F = (\hbar^2/2m)(3\pi^2 N/V)^{2/3}$?

   (a) all states with $(k_x^2 + k_y^2 + k_z^2)^{1/2} < (3\pi^2 N/V)^{1/3}$ are filled

   (b) all states with $(k_x^2 + k_y^2 + k_z^2)^{1/2} > (3\pi^2 N/V)^{1/3}$ are filled

   (c) all states with $-E_F/2 < E(\mathbf{k}) < E_F/2$ are filled

2. Blackbody radiation consists of photons populated according to

   (a) $\dfrac{1}{e^{\alpha}e^{\beta E} - 1}$ $(\alpha \neq 0)$

   (b) $\dfrac{1}{e^{\alpha}e^{\beta E} - 1}$ $(\alpha = 0)$

   (c) $\dfrac{1}{e^{\alpha}e^{\beta E} + 1}$

   (d) $e^{-\beta E}$

3. A Bose-Einstein condensate of ^{4}He atoms with N_0 atoms in the ground state is populated according to

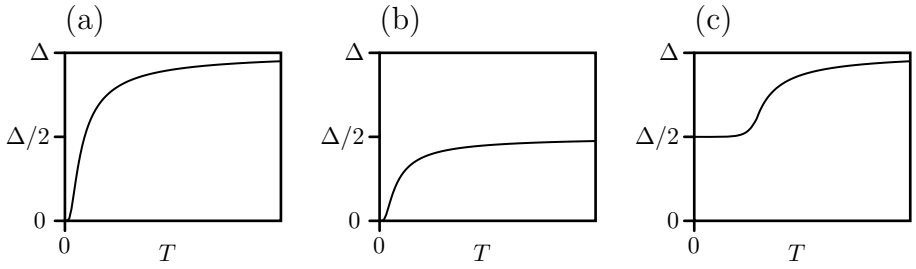
   (a) $\dfrac{1}{e^{\alpha}e^{\beta E} - 1}$ $(\alpha \sim 1/N_0)$

   (b) $\dfrac{1}{e^{\alpha}e^{\beta E} - 1}$ $(\alpha = 0)$

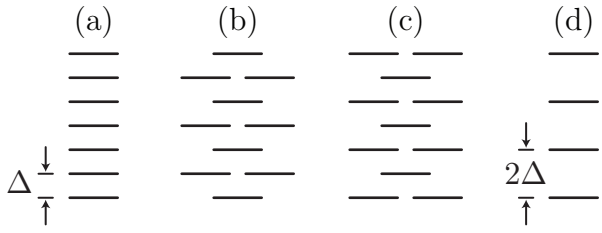
   (c) $\dfrac{1}{e^{\alpha}e^{\beta E} + 1}$

   (d) $e^{-\beta E}$

4. A system with two energy levels $\epsilon_1 = 0$ and $\epsilon_2 = \Delta$ is in thermal equilibrium. Which picture correctly illustrates the total energy as a function of T?



5. A system with many energy levels (some degenerate) has a lowest level $\epsilon_1 = 0$ and additional levels spaced above it (in increments of Δ or 2Δ, as indicated in the diagrams). Which has average energy $\sim 2\Delta e^{-\Delta/k_B T}$ when the system is very cold?



6. The quantum numbers of an electron in an atomic orbital describe the radial character of the wavefunction (n), the angular momentum (l, m_l), and the spin (m_S). What is the total number of states with $n = 3$?

   (a) 32

   (b) 30

   (c) 24

   (d) 18

7. LiF has a dipole moment of $2.09 \times 10^{-29}\,\mathrm{C\,m}$. The atomic separation is 0.1539 nm. What is the percentage ionic character of the bond?

   (a) 10%

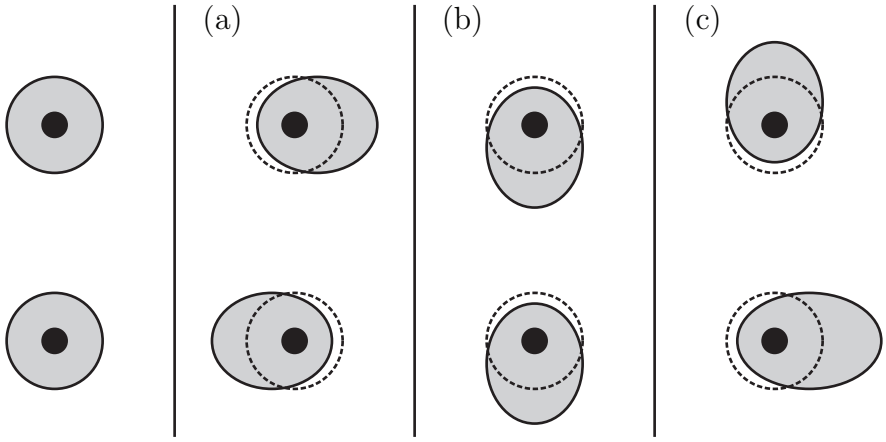
   (b) 15%

   (c) 85%

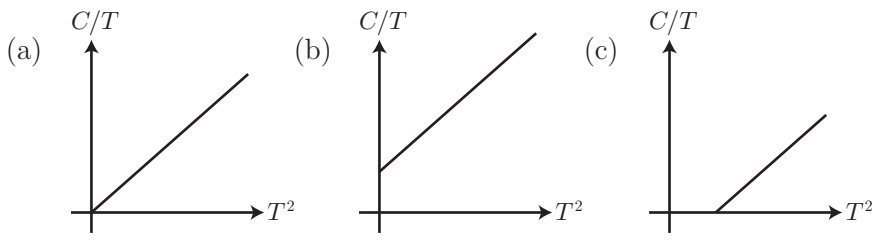
   (d) 90%

   (e) 115%

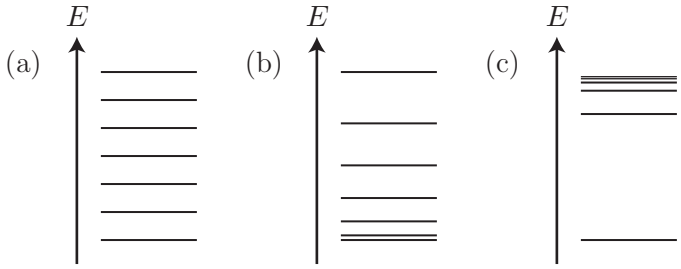
8. Van der Waals forces arise from the dipole-dipole interaction between the fluctuations of the electron cloud (shaded grey) around the nucleus (black circle). Which diagram shows the strongest dipole-dipole attraction?



9. The total heat capacity of a material is the sum $C = C_{\mathrm{el}} + C_{\mathrm{ph}}$ of its electron and phonon contributions. Which of the following graphs describes a metal?



10. (2 points) Which diagram is consistent with the energy levels described? Write the corresponding letter in the spaces provided.



(____) electronic donor levels in a doped semiconductor

(____) rotational energy levels in a diatomic molecule

(____) vibrational energy levels in a diatomic molecule

11. A diatomic gas is held at room temperature. Which of the following excitation modes is NOT significantly occupied?

    (a) rotational

    (b) vibrational

    (c) electronic

    (d) vibrational and electronic

    (e) rotational, vibrational, and electronic

12. The number of different Bravais lattices is

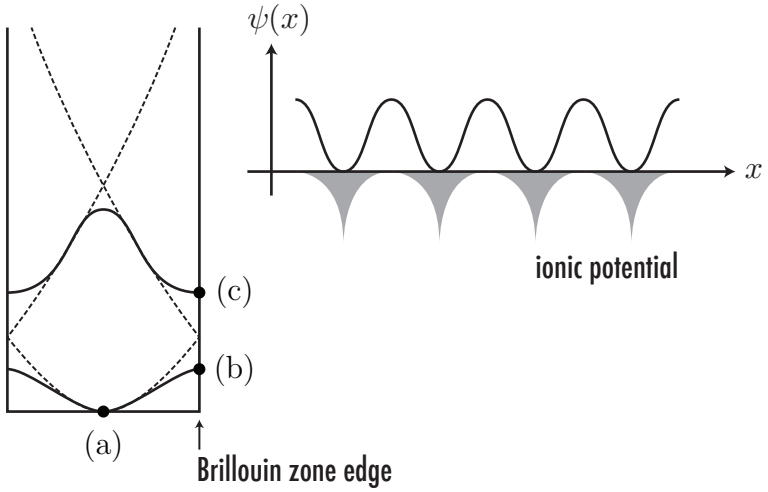
    (a) 3

    (b) 14

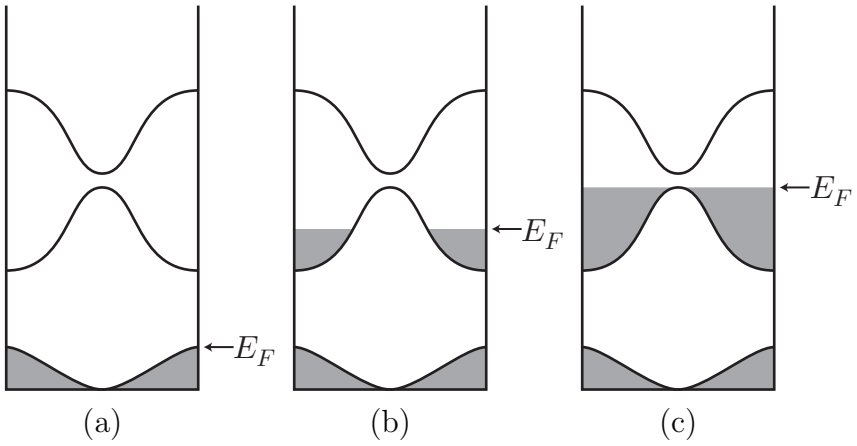
    (c) 167

    (c) unlimited

13. The band structure diagram on the left shows the dispersion (energy versus **k** vector) for electrons in a crystal. The dotted line is the result for noninteracting electrons; the solid line includes the effect of the electrons' Coloumb attraction to the positive ions. Which point corresponds to the electronic wavefunction shown on the right?

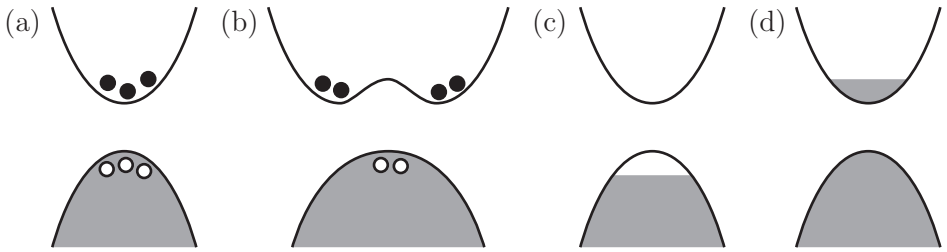


14. (2 points) Match the band structure with the electronic behaviour. Write the appropriate letter in the spaces provided.

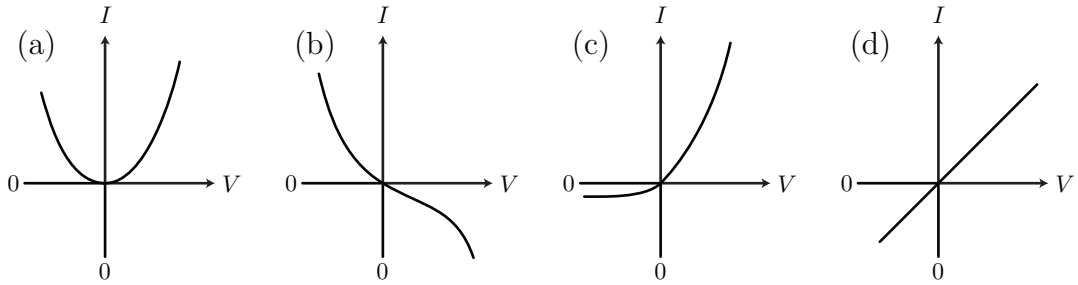


(____) metal (____) semiconductor (____) insulator

15. Which diagram best represents the band structure of a light-emitting diode (LED)?



16. Which plot of current versus bias voltage is that of a diode rectifier?



17. An electron orbiting a proton in the H atom is analogous to an electron orbiting its donor impurity in a doped semiconductor. The main difference is that inside the semiconductor the relative dielectric constant is enhanced: $\kappa = \epsilon/\epsilon_0 \gg 1$. How do the sizes of these orbits compare?

    (a) the orbital radius in the semiconductor is much larger than in the H atom

    (b) the orbital radius in the semiconductor is much smaller than in the H atom

    (c) the two orbits are roughly equal in size

18. The superconducting current is carried by so-called Cooper pairs. Cooper pairs are what?

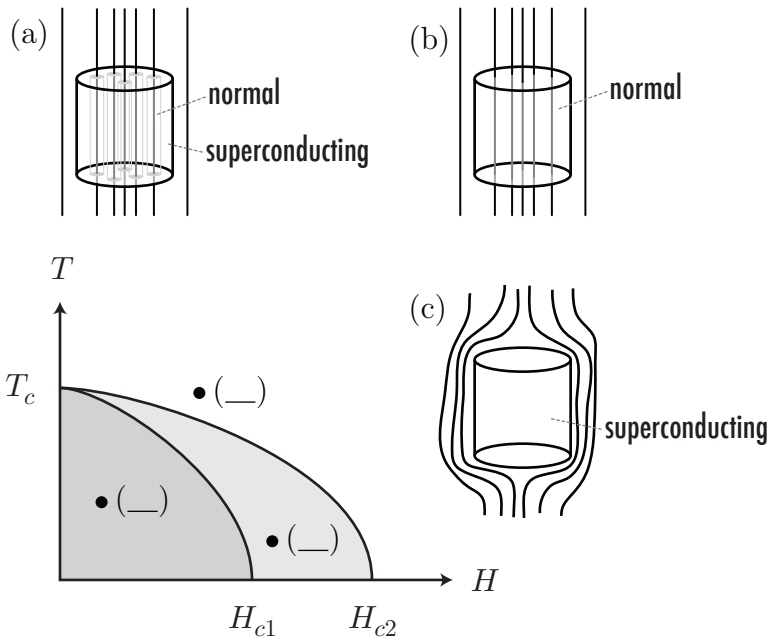
    (a) two strongly bound phonons of opposite polarization

    (b) two weakly bound phonons of equal polarization

    (c) two strongly bound electrons of equal spin and momenta

    (d) two weakly bound electrons of opposite spin and momenta

19. (2 points) *Meissner effect:* A metallic cylinder is held at temperature T in a magnetic field H. Match the three illustrations with the three points in the phase diagram.

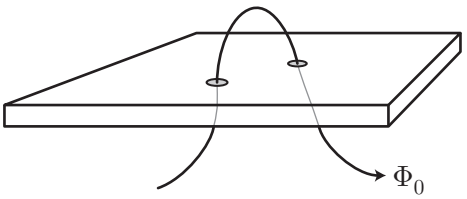


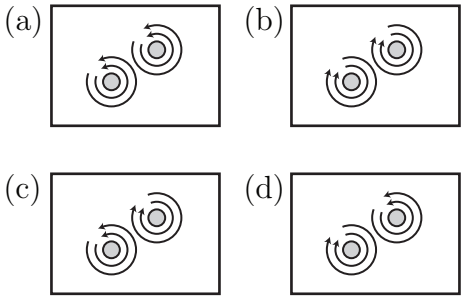
20. Is the material in the previous question type-I or type-II?

    (a) type-I

    (b) type-II

21. A thin slab of superconductor is threaded by a single flux quantum, as shown. Where the magnetic field line penetrates, the system is nonsuperconducting (i.e., in the normal state). The superconductor responds by setting up a vortex of circulating supercurrent. Which diagram—representing a view from above the slab—gives a correct account of those currents.





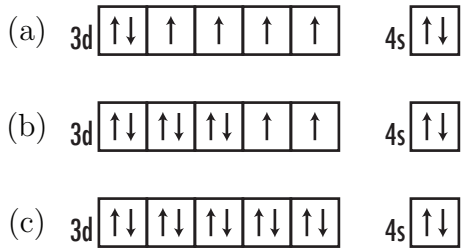
22. Do the two vortices attract or repel one another?

    (a) attract

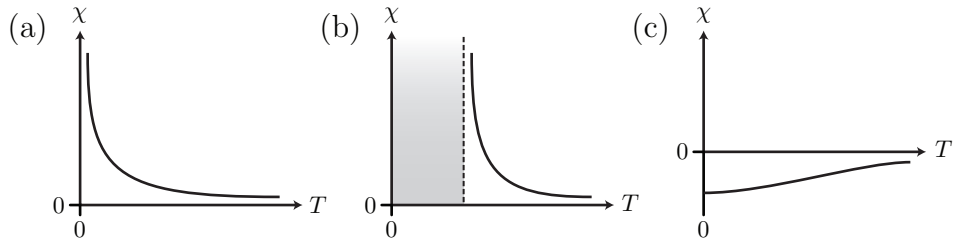
    (b) repel

    (c) neither; they don't interact

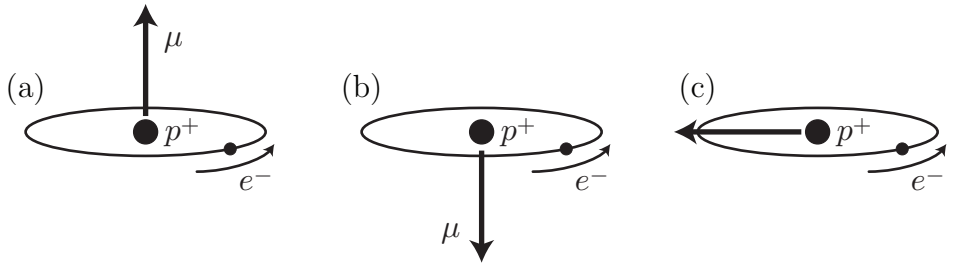
23. Which of the following outer-shell electronic configurations corresponds to an atom with no magnetic moment?



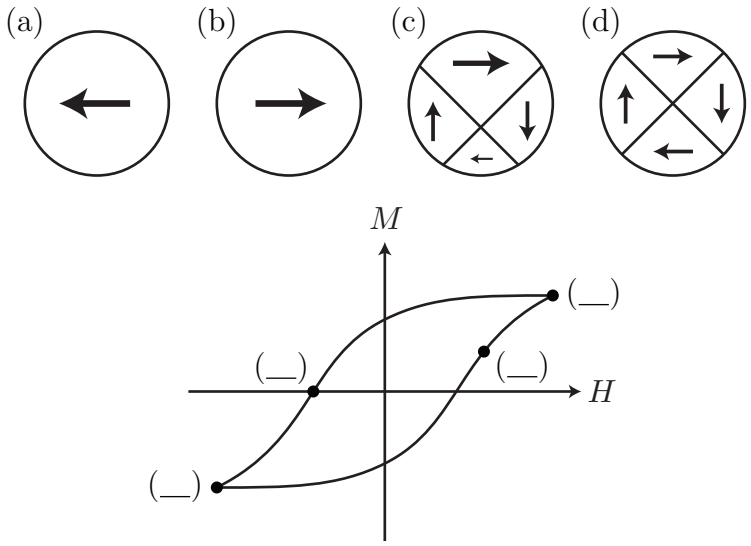
24. Identify the magnetic susceptibility of a paramagnet.



25. Circulation of current produces a magnetic moment. Accordingly, there is an orbital magnetic moment μ associated with the motion of an electron in an atomic orbital. How is that moment directed?

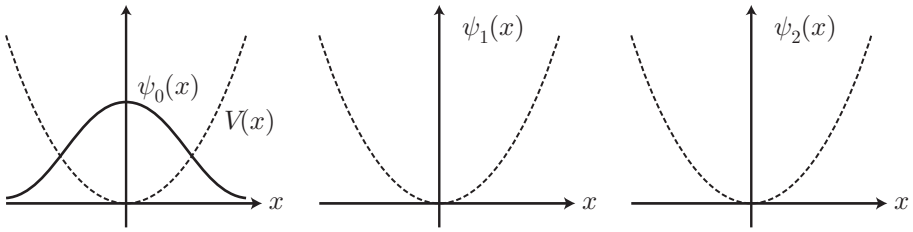


26. (2 points) *Hysteresis and ferromagnetic domains:* The arrows represent the magnetic moments of the ferromagnetic domains of an iron disc. The graph below shows the total magnetization as a function of the applied field H. Positive M and H are directed to the right. Match the domain configurations with the points on the graph.



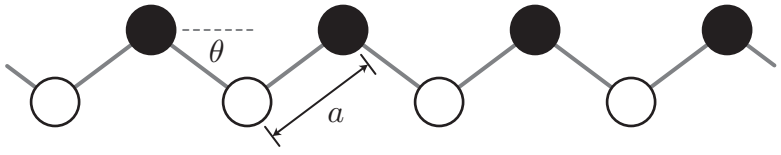
Long Answer Questions (45 points)

27. (3 points) An electron in a parabolic potential $V(x) = \frac{1}{2}m\omega^2 x^2$ has evenly-spaced energy levels $E_n = \hbar\omega(n + \frac{1}{2})$ with $n = 0, 1, 2, 3, \ldots$ The ground state wavefunction $\psi_0(x)$ is drawn below on the left. In the remaining two panels, sketch the first and second excited-state wavefunctions.



28. (5 points) Draw the molecular diagrams for (a) H_2S, (b) H_2Te, (c) H_3P, (d) H_3Sb, and (e) CH_4. As usual, use a line to denote a covalent bond and a dot to indicate any unpaired electrons.

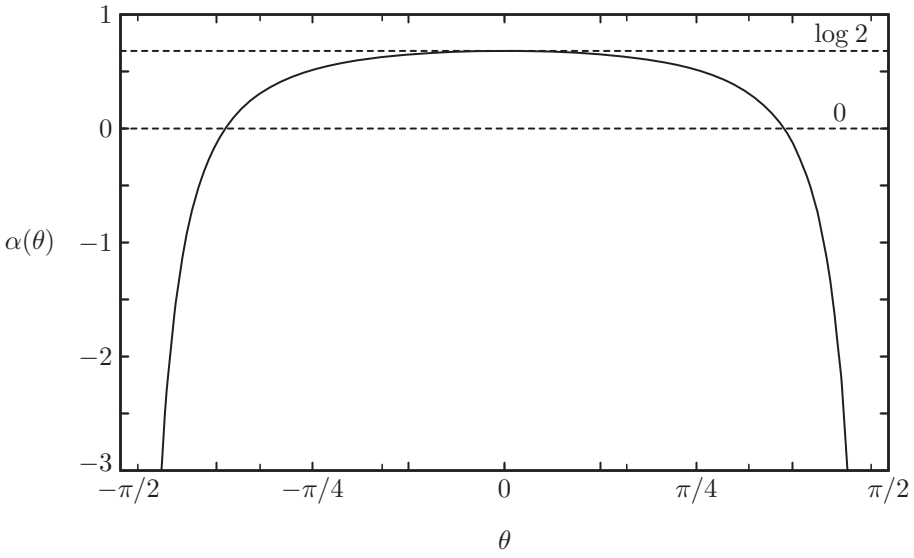
29. (7 points) (a) A zig-zag chain of positive and negative ions has bond-length a and bending angle θ.



Show that its Madelung constant is

$$\alpha = \sum_{n=1}^{\infty} \left[\frac{1}{\sqrt{4n(n-1)\cos^2\theta + 1}} - \frac{1}{2n\cos\theta} \right].$$

(Remember that (i) a right-angled triangle of hypotenuse a has an edge of length $a\cos\theta$ adjacent to a vertex of angle θ and an edge of length $a\sin\theta$ across from the vertex; and (ii) $\sin^2\theta + \cos^2\theta = 1$.)



(b) Over the range $-\pi/2 < \theta < \pi/2$, the Madelung constant has its maximum value at $\theta = 0$. What is the significance of that?

(c) What does it mean that α is negative when the bending exceeds about 65°?

30. (6 points) (a) What is the Fermi speed u_F for gold (Au)? Remember that u_F is the speed of a conduction electron whose energy is equal to the Fermi energy E_F. For Au, $E_F = 5.55\,\text{eV}$.

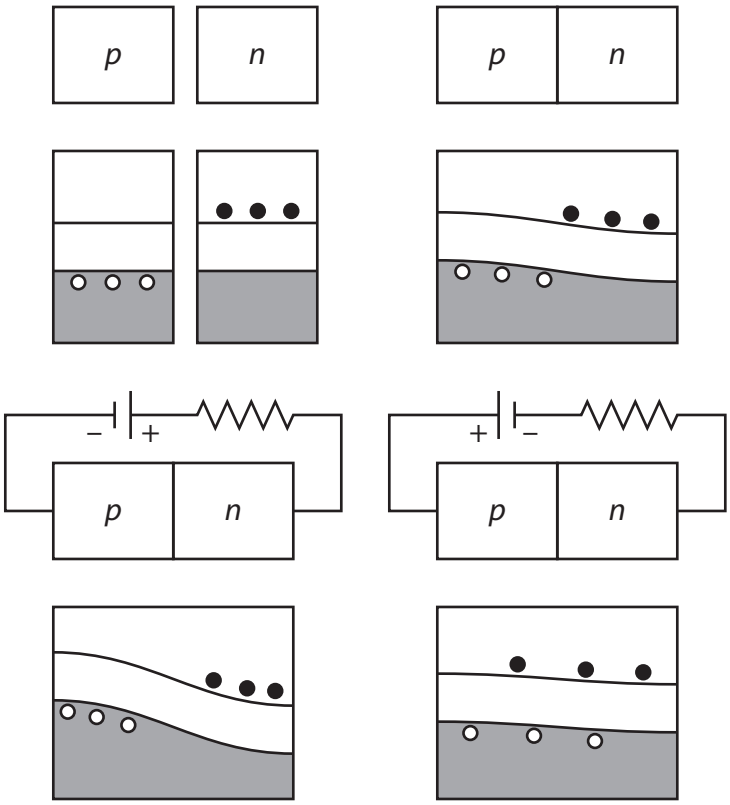
(b) The conduction electron density of Au is $5.9 \times 10^{28}\,\text{m}^{-3}$. Its resistivity at room temperature is $2.04\,\mu\Omega\,\text{cm}$. Compute the mean free path of the conduction electrons.

31. (6 points) The four panels below show the energy level diagrams for p- and n-type doped semiconductors in various junction configurations.

(a) The top-left panel represents two disjoint semiconductor samples. Indicate the forbidden region that constitutes the semiconductor gap.

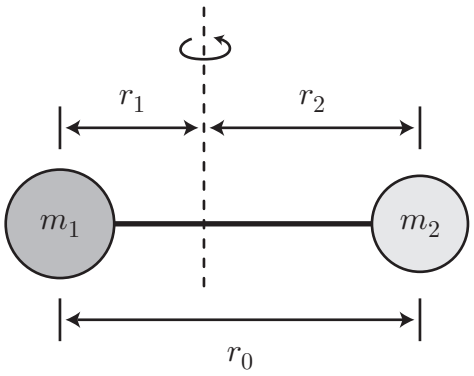
(b) The top-right panel shows a pn junction in equilibrium. Draw an arrow to show the direction of the *diffusive forces* felt by the donor *electrons*. Draw another arrow for the direction of the *electrostatic forces* felt by the acceptor *holes*.

(c) The bottom two panels show pn junctions in a circuit. Indicate which one is forward biased and which is reverse biased. In the forward-biased case, draw arrows to denote the drift velocities of the electrons and holes.



32. (5 points) A doped n-type silicon sample with 10^{16} electrons per cubic centimeter in the conduction band has a resistivity of $5 \times 10^{-3}\,\Omega\,\mathrm{m}$ at $300\,\mathrm{K}$. Find the mean free path of the electrons. Use $0.2\,m_e$ for the effective mass of the electron.

33. (7 points) The rotational energy of a diatomic molecule is quantized according to $E_{\text{rot}}(l) = \hbar^2 l(l+1)/2I$, where $l = 0, 1, 2, 3, \ldots$ is the angular momentum quantum number and I is the molecule's moment of intertia.



(a) The moment of inertia is $I = m_1 r_1^2 + m_2 r_2^2 = \mu r_0^2$, where μ is the reduced mass. Carbon monoxide (CO) has a bond length $r_0 = 0.1128\,\text{nm}$. Compute its moment of inertia.

(b) The rotational modes have degeneracy $g(l) = 2l + 1$. If the system is in thermal equilibrium at room temperature, what is the ratio n_5/n_1 of the occupation of the $l = 5$ and $l = 1$ modes?

(c) Draw all the transitions between rotational states $l = 0, 1, \ldots, 5$ that result in the emission of a photon. Compute the longest possible photon wavelength.

$l = 5$ _____

$l = 4$ _____

$l = 3$ _____

$l = 2$ _____

$l = 1$ _____

$l = 0$ _____

34. (3 points) Brass is an alloy of copper (Cu) and zinc (Zn). For low Zn concentrations, brass maintains the face-centred-cubic crystal structure of Cu; the Zn atoms simply substitute for Cu atoms in the lattice. When the temperature is lowered from 300 K to 4 K, the resistivity of pure copper drops by a much greater factor than that of brass. Why?

35. (3 points) Explain why the mean free path of electrons in a pure metal decreases as the temperature increases from zero.