Solutions to assignment 7

1. (a) The current density is $j = I/A = I/\pi(d/2)^2$ and has the value

$$j = \frac{4(10^{-3}\,\mathrm{A})}{\pi(1.63\times10^{-3}\,\mathrm{m})^2} = 479\,\mathrm{A\,m^{-2}}.$$

(b) The drift velocity is $v_\mathrm{d} = I/Ane = j/ne$. Using the j value from part (a), we find

$$\begin{aligned} v_\mathrm{d} &= \frac{479\,\mathrm{A\,m^{-2}}}{(8.47\times10^{28}\,\mathrm{m^{-3}})(1.602\times10^{-10}\,\mathrm{C})} \\ &= 3.53\times10^{-8}\,\mathrm{m/s} \\ &= 35.3\,\mathrm{nm/s}. \end{aligned}$$

2. The Fermi speed is found by inverting $E_F = \frac{1}{2}mu_F^2$. That is,

$$u_F = \left(\frac{2E_F}{m_e}\right)^{1/2} = c\left(\frac{2E_F}{m_ec^2}\right)^{1/2},$$

where c is the speed of light.

(a) For Na:

$$\begin{aligned} u_F &= (3.00\times10^8\,\mathrm{m/s})\sqrt{\frac{2(3.26\,\mathrm{eV})}{5.11\times10^5\,\mathrm{eV}}} \\ &= 1.07\times10^6\,\mathrm{m/s}. \end{aligned}$$

(b) For Au:

$$\begin{aligned} u_F &= (3.00\times10^8\,\mathrm{m/s})\sqrt{\frac{2(5.55\,\mathrm{eV})}{5.11\times10^5\,\mathrm{eV}}} \\ &= 1.40\times10^6\,\mathrm{m/s}. \end{aligned}$$

(c) For Sn:

$$\begin{aligned} u_F &= (3.00\times10^8\,\mathrm{m/s})\sqrt{\frac{2(10.3\,\mathrm{eV})}{5.11\times10^5\,\mathrm{eV}}} \\ &= 1.90\times10^6\,\mathrm{m/s}. \end{aligned}$$

3. The resistivity is given by $\rho = m_eu_F/ne^2\lambda$, where n is the electronic density and λ is the mean free path. The electronic densities of Na, Au, and Sb are listed in Table 10-3 on page 458 of the text. The corresponding Fermi velocities are known from question 2. The corresponding resistivities are $4.2\,\mu\Omega\,\mathrm{cm}$, $2.04\,\mu\Omega\,\mathrm{cm}$, and $10.6\,\mu\Omega\,\mathrm{cm}$. Hence,

$$\begin{aligned} \lambda_\mathrm{Na} &= \frac{m_eu_F}{ne^2\rho} \\ &= \frac{(9.109\times10^{-31}\,\mathrm{kg})(1.07\times10^6\,\mathrm{m/s})}{(2.65\times10^{28}\,\mathrm{m^{-3}})(1.602\times10^{-19}\,C)^2(4.2\times10^{-8}\,\Omega\,\mathrm{m})} \\ &= 3.4\times10^{-8}\,\mathrm{m} \\ &= 34\,\mathrm{nm} \end{aligned}$$

$$\begin{aligned} \lambda_\mathrm{Au} &= \frac{(9.109\times10^{-31}\,\mathrm{kg})(1.40\times10^6\,\mathrm{m/s})}{(5.9\times10^{28}\,\mathrm{m^{-3}})(1.602\times10^{-19}\,C)^2(2.04\times10^{-8}\,\Omega\,\mathrm{m})} \\ &= 4.1\times10^{-8}\,\mathrm{m} \\ &= 41\,\mathrm{nm} \end{aligned}$$

$$\begin{aligned} \lambda_\mathrm{Sn} &= \frac{(9.109\times10^{-31}\,\mathrm{kg})(1.90\times10^6\,\mathrm{m/s})}{(14.8\times10^{28}\,\mathrm{m^{-3}})(1.602\times10^{-19}\,C)^2(10.6\times10^{-8}\,\Omega\,\mathrm{m})} \\ &= 4.3\times10^{-9}\,\mathrm{m} \\ &= 4.3\,\mathrm{nm} \end{aligned}$$

4. (a) The photon energy is related to its wavelength by $E = hc/\lambda$. Hence,

$$E = \frac{1240\,\mathrm{eV\,nm}}{3.35\times10^3\,\mathrm{nm}} = 0.37\,\mathrm{eV}.$$

(b) The corresponding temperature scale is

$$T = \frac{E}{k_B} = \frac{0.37\,\mathrm{eV}}{8.617\times10^{-5}\,\mathrm{eV/K}} = 4300\,\mathrm{K}.$$

5. (a) The energy as a function of the radial quantum number n can be organized as follows:

$$E_n = \underbrace{-\frac{m_e}{2}\left(\frac{e^2}{4\pi\epsilon_0\hbar}\right)^2}_{-13.6\,\mathrm{eV}} \frac{m^*}{m_e}\frac{1}{\kappa^2}\frac{1}{n^2}.$$

Therefore, the ionization energy is

$$E_\infty - E_1 = \frac{0.2(13.6\,\mathrm{eV})}{(11.8)^2} = 0.0195\,\mathrm{eV}.$$

(b) The average orbital size is

$$\langle r_n\rangle = a_0n^2\frac{m_e}{m^*}\kappa,$$

where $a_0 = 0.0529\,\mathrm{nm}$ is the Bohr radius. Hence,

$$\langle r_1\rangle = (0.0529\,\mathrm{nm})(1)^2(1/0.2)(11.8) = 3.12\,\mathrm{nm}.$$

(c) For silicon, $E_g = 1.11\,\mathrm{eV}$ at room temperature. Thus, $E_1/E_g = 0.0195/1.11 = 0.0176$ or about 2%.