Solutions to assignment 8

1. Si, with electronic structure $1s^22s^22p^63s^23p^2$, has 4 electrons ($3s^23p^2$) outside its closed $n = 2$ shell.

   (a) Al has 3 valence electrons ($3s^23p^1$). Thus, it will act as an acceptor for one electron in the Si structure; a p-type semiconductor will result.

   (b) P has 5 valence electrons ($3s^23p^3$). It will act as a donor of one electron; an n-type semiconductor will result.

2. The net current flow I as a function of the bias voltage V_b is given by

$$I(V_b) = I_0\big(e^{eV_b/k_BT} - 1\big).$$

   At temperature $T = 300\,\mathrm{K}$, the fractional change in current is

$$\begin{aligned}&\frac{I(0.2\,\mathrm{V}) - I(0.1\,\mathrm{V})}{I(0.1\,\mathrm{V})}\\ &= \frac{e^{e(0.2\,\mathrm{V})/0.025\,\mathrm{eV}} - e^{e(0.1\,\mathrm{V})/0.025\,\mathrm{eV}}}{e^{e(0.1\,\mathrm{V})/0.025\,\mathrm{eV}}}\\ &= 47.6\end{aligned}$$

3. Beta decay of ^{137}Cs leads to the emission of photons of energy 660 keV. When a photon of energy E_{ph} is incident on a Ge semiconductor, it can be absorbed and re-emitted at energy $E_{\mathrm{ph}} - E_{\mathrm{g}}$, where $E_{\mathrm{g}} = 0.72\,\mathrm{eV}$ is the energy gap between the valence and conduction band. In this process, an electron is promoted from the valence band to the conduction band, producing a single electron-hole pair.

   (a) An incoming photon can undergo many of these scattering events, so that its energy is repeatedly reduced to $E_{\mathrm{ph}} - E_{\mathrm{g}}$, $E_{\mathrm{ph}} - 2E_{\mathrm{g}}$, $E_{\mathrm{ph}} - 3E_{\mathrm{g}}$, and so on. Thus, the maximum number of particle-hole pairs that can be produced is

$$N = E_{\mathrm{ph}}/E_{\mathrm{g}} = 6.6 \times 10^5/0.72 = 9.17 \times 10^5.$$

   (b) Since scattering is a random, probabilistic process, the law of large numbers tells us that the actual number of particle-hole pairs produced will vary (roughly) between $N - \sqrt{N}$ and N. The resolution of the detector will go as

$$\frac{\Delta N}{N} = \frac{\sqrt{N}}{N} = \frac{1}{\sqrt{N}} = \frac{1}{956} = 0.10\%.$$

4. According to the Drude theory of conduction, the mean free path is given by

$$l = \frac{m^*\langle v\rangle}{n\rho e^2}.$$

   In this instance, the charge carriers are electrons in the conduction band of Si. These carriers have an effective mass $m^* = 0.2m_e$ and constitute a dilute, semi-classical gas with mean (Boltzman-distributed) velocity

$$\begin{aligned}\langle v\rangle &= (8k_BT/\pi m^*)^{1/2}\\ &= \left[\frac{8(1.38 \times 10^{23}\,\mathrm{J/K})(300\,\mathrm{K})}{0.2\pi(9.11 \times 10^{-31}\,\mathrm{kg})}\right]^{1/2}\\ &= 2.41 \times 10^5\,\mathrm{m/s}.\end{aligned}$$

   Their mean free path is

$$\begin{aligned}l &= \frac{0.2(9.11 \times 10^{-31}\,\mathrm{kg})(2.41 \times 10^5\,\mathrm{m/s})}{(10^{22}\,\mathrm{m^{-3}})(5 \times 10^{-3}\,\Omega\,\mathrm{m})(1.60 \times 10^{-19}\,\mathrm{C})^2}\\ &= 34\,\mathrm{nm}.\end{aligned}$$

   For Cu, the relevant velocity is the *Fermi velocity*:

$$\begin{aligned}u_F &= (2E_F/m_e)^{1/2} = \left[\frac{2(7.06\,\mathrm{eV})}{9.11 \times 10^{-31}\,\mathrm{kg}}\right]^{1/2}\\ &= 1.57 \times 10^6\,\mathrm{m/s}.\end{aligned}$$

   The Fermi energy is taken from Table 10-3. The electronic density and resistivity

$$n = 8.47 \times 10^{28}\,\mathrm{m^{-3}}, \quad \rho = 1.7 \times 10^{-8}\,\Omega\,\mathrm{m}$$

   are given in Example 10-6. These values produce $l = 39\,\mathrm{nm}$. The mean free paths in Si and Cu are nearly equal.

5. (a) The Taylor expansion $e^x = 1 + x + \cdots$ is good for small x. Thus, when the bias voltage $V_b \ll k_BT/e$,

$$\begin{aligned}I &= I_0(e^{eV_b/k_BT} - 1)\\ &= I_0(1 + eV_b/k_BT + \cdots - 1)\\ &= I_0eV_b/k_BT.\end{aligned}$$

   Making the identification with Ohm's law, $I = V_b/R$, we find that the resistance of the diode is

$$\begin{aligned}R &= \frac{V_b}{I} = \frac{k_BT}{eI_0} = \frac{0.025\,\mathrm{eV}}{e \times 10^{-9}\,\mathrm{A}}\\ &= \frac{0.025\,\mathrm{V}}{10^{-9}\,\mathrm{A}} = 25\mathrm{M}\Omega.\end{aligned}$$

   (b) For reverse bias $V_b = -0.5\,\mathrm{V}$,

$$R = \frac{V_b}{I} = \frac{0.5\,\mathrm{V}}{10^9\,\mathrm{A}} = 500\,\mathrm{M}\Omega.$$

   (c) For forward bias $V_b = +0.5\,\mathrm{V}$, the current is no longer the saturation current but instead

$$\begin{aligned}I &= I_0(e^{eV_b/k_BT} - 1)\\ &= (10^{-9}\,\mathrm{A})(e^{0.5/0.0025} - 1)\\ &= 0.485\,\mathrm{A}.\end{aligned}$$

This gives a dc resistance

$$R = \frac{V_b}{I} = \frac{k_B T}{eI} = \frac{0.025\,\mathrm{V}}{0.485\,\mathrm{A}} = 1.03\,\Omega.$$

(d) The ac resistance is defined as the derivative dV_b/dI (rather than the ratio V_b/I). Since,

$$\frac{dI}{dV_b} = \frac{eI_0}{k_B T} e^{eV_b/k_B T},$$

it follows that

$$\begin{aligned} R_{\mathrm{ac}} &= \frac{k_B T}{eI_0} e^{-eV_b/k_B T} \\ &= \frac{0.025\,\mathrm{eV}}{e \times 10^{-9}\,\mathrm{A}} e^{-0.5/0.025} \\ &= 0.0515\,\Omega. \end{aligned}$$