Solutions to assignment 9

1. The mass and critical temperature of lead are related by $M^\alpha T_\mathrm{c} = \text{const.}$ with $\alpha = 0.49$. From the periodic table, we find that the atomic weight of naturally occurring lead is 207.19 u; from Table 10-6, we get the corresponding critical temperature, $T_\mathrm{c} = 7.196\,\mathrm{K}$. Hence,

$$\text{const.} = (207.19\,\mathrm{u})^{0.49}(71.96\,\mathrm{K}) = 98.20\,\mathrm{u}^{0.49}\,\mathrm{K}.$$

   It follows that the T_c values for $^{206}\mathrm{Pb}$, $^{207}\mathrm{Pb}$, and $^{208}\mathrm{Pb}$ are

$$^{206}T_\mathrm{c} = 98.20\,\mathrm{K}/(205.974)^{0.49} = 7.217\,\mathrm{K}$$
$$^{207}T_\mathrm{c} = 98.20\,\mathrm{K}/(206.976)^{0.49} = 7.200\,\mathrm{K}$$
$$^{208}T_\mathrm{c} = 98.20\,\mathrm{K}/(207.977)^{0.49} = 7.183\,\mathrm{K}$$

2. (a) Equation 10-71 tells us that the gap is related to the critical temperature by $E_\mathrm{g} = 3.5k_BT_\mathrm{c}$. Since T_c for indium is 3.408 K, the gap is

$$\begin{aligned} E_\mathrm{g} &= 3.5(8.617\times10^{-5}\,\mathrm{eV/K})(3.408\,\mathrm{K}) \\ &= 1.028\times10^{-3}\,\mathrm{eV}. \end{aligned}$$

   (b) A sufficiently energetic photon with wavelength λ has energy $E_g = hc/\lambda$.

$$\lambda = \frac{hc}{E_\mathrm{g}} = \frac{1240\,\mathrm{eV\,nm}}{1.028\times10^{-3}\,\mathrm{eV}} = 1.206\,\mathrm{mm}.$$

3. At $T/T_\mathrm{c} = 0.5$, $E_\mathrm{g}(T)/E_\mathrm{g}(T_\mathrm{c}) \approx 0.95$. Since, $E_\mathrm{g} = 3.5k_BT_\mathrm{c}$, it follows that

$$E_\mathrm{g}(T) = 0.95(3.5)k_BT_\mathrm{c} = 3.325k_BT_\mathrm{c}.$$

   (a) For Sn:

$$\begin{aligned} E_\mathrm{g}(T) &= 3.325(8.617\times10^{-5}\,\mathrm{eV/K})(3.722\,K) \\ &= 1.07\times10^{-3}\,\mathrm{eV}. \end{aligned}$$

   (b) For Nb:

$$\begin{aligned} E_\mathrm{g}(T) &= 3.325(8.617\times10^{-5}\,\mathrm{eV/K})(9.25\,K) \\ &= 2.65\times10^{-3}\,\mathrm{eV}. \end{aligned}$$

   (c) For Al:

$$\begin{aligned} E_\mathrm{g}(T) &= 3.325(8.617\times10^{-5}\,\mathrm{eV/K})(1.175\,K) \\ &= 3.37\times10^{-4}\,\mathrm{eV}. \end{aligned}$$

   (d) For Zn:

$$\begin{aligned} E_\mathrm{g}(T) &= 3.325(8.617\times10^{-5}\,\mathrm{eV/K})(0.85\,K) \\ &= 2.4\times10^{-4}\,\mathrm{eV}. \end{aligned}$$

4. The London equation says that the current $\mathbf{j}$ is proportional to the vector potential $\mathbf{A}$ with constant of proportionality $-1/(\mu_0\lambda_L^2)$. (a) Taking the time derivative of both sides gives

$$\frac{\partial\mathbf{j}}{\partial t} = -\frac{1}{\mu_0\lambda_L^2}\frac{\partial\mathbf{A}}{\partial t}.$$

   We know from Faraday's law and from the definition $\mathbf{B} = \nabla\times\mathbf{A}$ that

$$\nabla\times\mathbf{E} = -\frac{\partial\mathbf{B}}{\partial t} = -\frac{\partial(\nabla\times\mathbf{A})}{\partial t} = \nabla\times\frac{\partial\mathbf{A}}{\partial t}$$

   and hence

$$\nabla\times\left(\mathbf{E} - \frac{\partial\mathbf{A}}{\partial t}\right) = 0.$$

   Up to some circulation-free constant function, we have that $\mathbf{E} = \partial\mathbf{A}/\partial t$. Putting this into our expression for $\partial\mathbf{j}/\partial t$ gives

$$\frac{\partial\mathbf{j}}{\partial t} = \frac{1}{\mu_0\lambda_L^2}\mathbf{E}.$$

   (b) The current represents the motion of charge and is thus understood to be $\mathbf{j} = nq\mathbf{v}$, where n is the number density of carriers, q is their charge, and $\mathbf{v}$ is their velocity. The force felt by the carriers in an electric field is $\mathbf{F} = q\mathbf{E}$. Therefore, Newton's law says that $\mathbf{F} = m\mathbf{a} = md\mathbf{v}/dt = q\mathbf{E}$. Finally,

$$\frac{\partial\mathbf{j}}{\partial t} = nq\frac{\partial\mathbf{v}}{\partial t} = nq\times\frac{q\mathbf{E}}{m} = \frac{nq^2}{m}\mathbf{E} = \frac{1}{\mu_0\lambda_L^2}\mathbf{E}$$

   and $\lambda_L^2 = m/\mu_0nq^2 = \epsilon_0mc^2/nq^2$.

   (c) In a superconductor the carriers are loosely bound pairs of electrons. Hence, $q = -2e$ and $m = 2m_e$.