Auctions vs. Sequential Mechanisms When Resale is Allowed*

Xiaogang Che† Tilman Klumpp‡

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Abstract

We examine the problem of selling an object to a stream of potential buyers with independent private values and participation costs. If the object can be resold in the future, and resellers can make posted price offers, the original seller may prefer to deal with potential buyers sequentially instead of holding an auction. The reason is that resale opportunities compress the dispersion of buyers' willingness to pay for the object, which lowers the surplus each buyer expects to receive in the auction. This effect may reduce participation in the initial auction to just one buyer, in which case the seller obtains zero revenue. We show that a simple form of sequential mechanism allows the seller to extract positive revenue, and becomes approximately optimal if the resale market is large. Our finding contrasts with the result that sellers usually prefer auctions when resale is not allowed (see Bulow and Klemperer 2009).

Keywords: Sequential mechanisms, auctions, participation costs, sequential entry, resale.

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†Department of Economics, City University of London. Northampton Square, London, EC1V 0HB, United Kingdom. E-mail: xiaogang.che@city.ac.uk.

‡Department of Economics, University of Alberta. 9-20 Tory Building, Edmonton, AB, Canada T6G 2H4. E-mail: klumpp@ualberta.ca.

1 Introduction

Consider the problem of a seller who faces a group of potential buyers. Assume the buyers arrive in sequence and that each buyer can learn his private valuation by paying a cost. The seller must sell the item and does not have any information about the distributions of bidder valuations and entry costs. Therefore, she is constrained from using reserve price and/or subsidizing bidder entry. In this case, the seller can run a standard auction in which buyers first decide to enter, and those who enter then submit their bids simultaneously. Alternatively, the seller can deal with each buyer sequentially, by giving each buyer the opportunity to bid before inviting the next buyer to participate.

Bulow and Klemperer (2009, henceforth "BK") showed that sellers usually prefer to use the auction. Simply put, auctions are "more competitive" than sequential mechanisms. In the latter, bidders can place jump bids (i.e., publicly observable price commitments) that signal the presence of a high-value competitor and deter entry by other bidders. Buyers cannot send such signals in auctions, because bidding commences only after entry has stopped. Thus, in expectation there is more entry in the auction, resulting in more aggressive bids and a higher seller revenue. Of course, the sequential mechanism can lead to a higher price *ex post*, if the bidders who enter in the auction have relatively low valuations. However, BK demonstrate that this potential advantage is unlikely to dominate the competitiveness of the auction *ex ante*, as this requires a large set of potential bidders and a carefully chosen value distribution and entry cost.

In this paper, we examine the same choice between auctions and sequential mechanisms, but allow the buyer who acquires the object from the original seller to resell it to other buyers.[1] We show that the original seller may no longer prefer to use the auction. The reason is the following: The opportunity to resell introduces a common value element to both the auction and the sequential mechanisms. All buyers—even those with low private values—now have a willingness to pay that is at least equal to the item's value in the resale market. This reduces the information rents that buyers can earn and, thus, reduces entry in the auction. If the resale value is high enough, only one buyer will find it profitable to enter the initial auction, in which case the original seller's revenue in the auction must be zero. Whether this extreme outcome happens depends on the expected size of the resale market. We present an example in which entry by the second buyer is deterred if there are only 2.33 potential buyers on expectation.[2]

Thus, it is precisely the competitiveness of the auction that works against it in the presence of resale opportunities. However, even a sequential mechanism may be too competitive. In particular, the mechanism studied by BK has, at every stage, two bidders competing in an ascending auction before the survivor makes an additional jump bid. But if a standard second-price auction with no reserve price leaves insufficient rents for more than one buyer to compete profitably, then the same must be true for an ascending auction that has a non-negative starting price and whose winner may have to compete again in the future. Therefore, the first buyer in the sequential mechanism can deter all future entry by placing a zero (or epsilon) jump bid, leaving the seller with no revenue once again.

To avoid this problem, we introduce a modified sequential selling mechanism in which any simultaneous head-to-head competition between buyers is removed. Each bidder is permitted to make exactly one publicly observable price offer (i.e., a jump bid) immediately after entry, which he commits to paying if accepted by the seller. If, later, another bidder enters, he observes the previous offers and is

[1] Resale is a common feature in real auctions. For example, Orange, a telecom firm, participated in the UK mobile spectrum auction in 2000. Orange won and then immediately sold its license to France Telecom.

[2] See Table 1 in Section 5.3.

then permitted to make exactly one offer himself. We call this process a *fully sequential mechanism*. In the fully sequential mechanism, a bidder does not care about the valuation of previous bidders—he only cares about their bids. Because bids cannot be revised later, in order to deter entry by subsequent bidders a buyer's jump bid must be sufficiently high to begin with. In other words, the fully sequential mechanism utilizes the preemptive nature of jump bids, but shuts down any signaling role. We show that this mechanism becomes approximately revenue-optimal as the number of buyers grows, whereas the auction and the conventional sequential mechanism become worst. Thus, the conclusion in BK—that preemptive jump bidding hurts the seller's revenue while simultaneous competition between bidders helps—is exactly reversed when resale is possible and the number of buyers is large enough (we provide a detailed discussion in Section 6).

An important implication of this result is that the fully sequential mechanism allows the initial seller to approximate the performance of an optimal, but unavailable, mechanism. In general, the seller-optimal mechanism involves the use of reserve prices and entry fees, and we assume that only resellers, but not the original seller, can use such instruments. More precisely, we assume that resellers can (but do not necessarily have to) make take-it-or-leave-it offers. This asymmetry reflects situations in which the original seller is unable or unwilling to commit to not sell below a certain price, or in which the original seller does not have enough information to compute reserve prices. Such constraints are realistic in certain applications, for at least two reasons. First, the initial seller may not want to use an optimal mechanism because of its objective. For example, consider a government agency that wants to privatize a public asset. This agency may have efficiency concerns that override revenue concerns, such as a concern for maximizing the overall economic benefits associated with the transaction. If the asset is the right to operate a railroad in a given area, for instance, an outcome in which rail service is suspended because a reserve price has not been met may be unacceptable.

In addition, the initial seller may be at an informational disadvantage. In general, constructing an optimal mechanism requires knowledge of the distributions of bidder valuations and, in environments with costly participation, knowledge of buyers' entry costs (see, e.g., Myerson 1981; Zheng 2002). Our analysis applies to cases where the initial seller has no knowledge of these variables required to run an optimal mechanism. This implies that the initial seller is restricted to choose from a smaller set of mechanisms that are not, generally, revenue-optimal. The assumption that selling mechanisms not depend on the details of the selling environment, such as the distribution of buyers' valuations, is sometimes referred to as the *Wilson doctrine* (see, e.g., Krishna 2002, p. 75). The doctrine reflects the criterion that "practical mechanisms should be simple and designed without assuming that the designer has very precise knowledge about the economic environment in which the mechanism will operate" (Milgrom 2004, p. 23). On the other hand, buyers may be more likely to possess this knowledge. For example, a government seller may face corporate buyers that are better informed than the government about the commercial value of an asset and or about costs in an industry, enabling them (but not the government seller) to compute optimal reserve prices or otherwise construct an optimal mechanism. Our paper, therefore, applies to environments in which the initial seller, but not the reseller, is constrained by the Wilson doctrine. We show that an initial selling mechanism exists that is approximately revenue-optimal and which does comply with the doctrine.

One way for the seller to deal with the aforementioned constraints is to enlist the help of some buyers to sell the object to others. For such a scheme to be successful, there must be gains from trade after the initial sale—that is, the initial mechanism must be inefficient. This is part of the reason why the seller should not let all buyers compete in an auction without a reserve price, and why both the auction and the BK sequential mechanism may fail to generate enough entry to raise positive revenue in our environment. The original seller's goal, rather, is to construct an alternative mechanism that sells with a high probability but not always to the "correct" buyer, and still leverages competition among potential resellers in a way that allows the original seller to extract some of the resale profits that an unconstrained seller can generate. In the context of the privatization example, this implies that post-privatization trade among private parties does not necessarily imply that money has been "left on the table," provided the government seller can find a way to extract some of the post-privatization gains at the initial selling stage. This is precisely what our fully sequential mechanism accomplishes. In fact, we show that the fully sequential mechanism asymptotically extracts *all* resale profits, and does so without having to use any of the additional instruments available to resellers.

We proceed as follows. In Section 1.1 below, we review the related literature on auctions with resale and auctions with endogenous participation. In Section 2 we present a simple three-bidder example that illustrates the performance of auctions and sequential mechanisms when resale is allowed. In Section 3 we present our basic formal model and in Section 4 we introduce the resale market to that model. In Section 5 we analyze entry and bidding behavior in the auction and the (conventional) sequential mechanism, and show that both fail to generate revenue if there are too many potential buyers. In Section 6 we introduce the fully sequential mechanism and show that it is approximately optimal with a large number of bidders. Section 7 concludes. Most proofs are in the Appendix.

1.1 Related literature

This paper is related to two strands of previous work: The literature on endogenous participation in auctions, and the literature on auctions with resale. To the best of our knowledge, ours is the first model to intersect these two areas. In the following, we briefly review both literatures and then highlight the contributions of our paper to each.

Endogenous participation in auctions. The literature on endogenous participation in auctions can be divided into two branches. The first branch originates with Samuelson (1985) and assumes that potential bidders know their private values before making their costly entry decisions.[3] The entry cost paid by a bidder reflects either entry fees charged by the seller, or the cost for preparing and delivering a formal bid. The second branch originates with McAfee and McMillan (1987) and Levin and Smith (1996) and assumes that potential bidders learn their private values only after making their entry decision. Here, the participation cost can be interpreted as resources spent by a bidder to investigate the item and determine his willingness to pay. Within this branch, the outcome of a selling mechanisms depends crucially on what is assumed about the timing of entry and bidding decisions: If bidding is possible before entry is completed, bidders can deter the participation of future competitors either by preemptive jump bidding (an effect first demonstrated by Fishman 1988) or by coordinated bidding (see Che and Klumpp 2016). Bulow and Klemperer (2009, "BK") later showed that, because of the possibility to deter entry, sellers generally receive more revenue if they use auctions in which bids cannot be submitted until all entry is complete.[4]

[3]See also Stegeman (1996); Tan and Yilankaya (2006); Lu (2009); Cao and Tian (2010); Moreno and Wooders (2011); Shi (2012); Lu and Ye (2012, 2014).

[4]There is also a strand of literature that characterizes optimal selling mechanisms assuming that the seller has control over the set of participating bidders (see Crémer *et al.* 2007, 2009). In such settings, the seller designs both a selling mechanism *and* a search procedure by which participating bidders are selected from a given universe of potential buyers, and which operates in parallel to the selling mechanism. This is not the type of environment considered here. Like BK, we assume that potential buyers arrive

Auctions with resale. If bidders in an auction draw their valuations from asymmetric value distributions, common auction formats may fail to allocate efficiently. In this case, the opportunity for post-auction trade arises naturally. Gupta and Lebrun (1999) characterize the equilibrium of the first-price auction with two asymmetric private-value bidders and resale, assuming that the bidders' private values are common knowledge at the resale stage and resale price is given by a bargaining solution (e.g., the Nash bargaining solution). Hafalir and Krishna (2008) show that, when resale occurs via a take-it-or-leave-it offer, the original seller obtains a higher expected revenue in the first-price auction than in the second-price auction. Virág (2013) extends this result to the case of more than two bidders and to the case where resale occurs via a second-price auction. If the original seller can set a reserve price, however, Virág (2016) shows that this revenue ranking can be reversed. Zheng (2002) examines the design of seller-optimal auctions with resale. He shows that, in order to achieve the optimal allocation of Myerson (1981), resale must occur with positive probability. On the other hand, if buyers' valuations are symmetrically distributed, common auction formats allocate to the buyer with the highest valuation. Thus, additional assumptions on the economic environment are necessary to generate post-auction resale. Haile (2000, 2003) develops models in which bidders face residual uncertainty about their private values, which is resolved after the initial auction. In this case, the winning bidder may attempt to resell to buyers who turn out to have higher private valuations *ex post*. Bose and Deltas (1999) and Haile (2001) assume that some bidders cannot participate in the initial auction. In this case, the winning bidder may attempt to resell to one of the excluded bidders. In either case, the anticipation of future resale opportunities affects bidders' willingness to pay in the initial auction, and depending on the initial auction format, this feedback can increase or decrease the expected revenue of the original seller, relative to the no-resale case.

Our model belongs to the "pay to learn your value" class of endogenous participation models and to the "symmetric values" class of resale models, and it contributes to each. As BK, we compare two types of mechanisms: Simultaneous auctions and sequential processes that permit placing of preemptive bids (within the latter, we now introduce a distinction between fully sequential mechanisms in which *only* preemptive bids are permitted, and mechanisms in which an element of simultaneous competition remains). Regardless of the mechanism, only some buyers participate. Thus, as in Bose and Deltas (1999) and Haile (2001), the reason why resale occurs in our model is that some buyers do not participate in the initial sale. However, instead of being exogenously excluded, these buyers chose not to participate in the initial sale because participation was not optimal. Our results also reflects an important finding in Zheng (2002): When resale is possible and cannot be banned, the seller benefits from selling the item to a buyer who then resells it. In contrast to Zheng (2002), we assume a more restricted set of mechanisms available to the original seller; however, these mechanisms not require knowledge of the distribution of bidder valuations and entry costs.

Two other papers show that sequential mechanisms can have an advantage over auctions. Roberts and Sweeting (2013) assume that potential buyers first receive noisy signals about their valuations, then decide whether to enter, and then learn their actual valuations if they enter. In this environment, a bidder's entry decision depends not only on the number of previous entrants and their bids, but also on each bidder's signal. Roberts and Sweeting (2013) show that this facilitates entry by high value bidders in both the auction and the sequential mechanism. However, the selection effect is stronger in the latter and may result in a larger expected revenue compared to the auction. Davis *et al.* (2013) test BK's result in a laboratory experiment and find that, contrary to the theoretical prediction, sequential mechanisms

according to an exogenous arrival process over which the seller has no control, and can enter the mechanism as long as they pay their participation costs.

tend to perform better than auctions. The authors show that this could be explained by random shocks to each bidder's (perceived) entry cost. In contrast to these models, ours retains BK's assumptions about information or payoffs and instead modifies the trading possibilities that are available to agents.

Finally, several other papers examine auctions with resale; however, the assumptions and research questions in these studies are quite different from ours. Garratt and Tröger (2006) examine speculation and resale in standard auctions, showing that for second-price (but not first-price) auctions, the speculator can profits from resale. Pagnozzi (2007) shows that resale may encourage low-value bidders to enter while inducing high-value bidders to drop out in order to buy the object in the resale market instead, with seller revenue being increased as a result. Pagnozzi (2010) examines how resale can attract speculators in uniform-price auctions and finds that high-value bidders prefer buy from a speculator in the resale market. Garratt *et al.* (2009) consider collusion through inter-bidder resale in English auctions and find that collusive equilibria interim Pareto-dominate the standard value-bidding equilibrium. Che *et al.* (2013) examine the effect of resale on entry strategies in second price auctions and show entry strategies depend on the reseller's bargaining power.[5]

2 A Simple Motivating Example

In this section, we present a motivating example with a fixed set of three buyers and binary private values. In our main model, we will assume a general stochastic buyer arrival process and a continuous distribution of values. The simple example below nevertheless foreshadows our main results: If resale is allowed, the auction (and BK's sequential mechanism) may fail to generate positive revenue, while the fully sequential mechanism will perform well.

Example 1. A seller has a zero value for an item. There are three potential buyers whose valuations are independent of each other and either $v_i = 0$ or $v_i = 1$ $(i = 1, 2, 3)$ with equal probabilities. Buyers are initially uninformed about their valuations, but each buyer i can learn v_i at a cost of $c = 0.2$.

The seller uses a standard second-price auction, which proceeds as follows. Buyers sequentially decide whether to participate in the auction. If buyer i decides to participate, he pays c and learns v_i. After all entry decisions are made, the participating buyers simultaneously submit their bids. The highest bidder wins and pays the second-highest bid (the seller flips a coin if more than one buyer submitted the highest bid).

Consider first a scenario in which no resale market exists. Clearly, each participating buyer's dominant strategy is to bid his valuation, $b_i = v_i$. Note that, with these strategies, a participating buyer obtains a surplus of 1 if and only if his own valuation is 1 and the valuation of every one of his competitors is 0. Therefore, the expected payoff to a buyer in an auction with n participants is $V(n) = (1/2)^n - c$, or

$$V(1) = 0.3, \;\; V(2) = 0.05, \;\; V(3) = -0.075.$$

In equilibrium, the first two buyers enter and the seller's expected revenue is $(1/4) \cdot 1 = 0.25$.

Now suppose that the winner of the auction can resell the item to buyer 3, using take-it-or-leave-it offer r. Buyer 3 observes r and can then decide whether to pay c to learn v_3 and participate in the resale

[5] Other studies of auctions with resale include Mylovanov and Tröger (2009); Lebrun (2012); Cheng (2011); Zhang and Wang (2013); Carroll and Segal (2019).

market. Buyer 3 participates if and only if

$$\frac{1}{2}(1-r) - c \;\geq\; 0,$$

which implies that the optimal resale offer is $\hat{r} = 0.6$. This offer is accepted if $v_3 = 1$, which has probability $1/2$. Thus, conditional on offering the item for resale, the expected resale revenue is $z^* = \hat{r}/2 = 0.3$. This implies that a buyer with private valuation $v_i = 0$ should be willing to pay up to z^* for the item in the initial auction. If two buyers participate in the auction, and both use the bidding strategy $b_i = 1$ if $v_i = 1$ and $b_i = z^*$ if $v_i = 0$, the expected payoff to each is

$$V(\,2 \,|\, \text{resale allowed}\,) = \frac{1}{4}(1 - z^*) - c \;=\; -0.025.$$

Therefore, when resale is possible two buyers can no longer profitably participate in the initial auction. Instead, only a single buyer will enter now, and the seller's revenue is zero.

Later in the paper, we will show that the same is true if the seller uses the sequential mechanism described in BK. The reason is that this mechanism also involves a second-price auction between the first and the second buyer (the only difference is that the auction is ascending instead of sealed-bid, and buyer 1 gets to make an initial jump bid before the auction commences). However, suppose the seller uses the following selling procedure, called the fully sequential mechanism: First, she asks buyer 1 to make an offer p (which is publicly observable). She then asks whether buyer 2 wants to match it. If the offer is matched, she sells the object to buyer 2 for a price of p; otherwise, she sells the object to buyer 1 for a price of p. As before, the winning buyer may then try to resell the item to buyer 3, using a take-it-or-leave-it offer.[6]

Conditional on having entered, for buyer 2 it is optimal to match any offer $p \leq z^*$ if $v_2 = 0$, and any offer $p \leq 1$ if $v_2 = 1$. Moreover, it is strictly optimal for 2 to enter if and only if

$$\frac{1}{2}(1-p) - c \;>\; 0,$$

or $p < \hat{r} \equiv 0.6$. Thus, by offering 0.6 or more, buyer 1 can deter entry and prevent potentially ruinous competition with the second buyer. It is now straightforward to verify that the following is a subgame perfect equilibrium of the fully sequential mechanism:

(i) Buyer 1 offers $p = 0.3$ if $v_1 = 0$, and $p = 0.6$ if $v_1 = 1$.

(ii) Buyer 2 enters if and only if $p < 0.6$.

(iii) Conditional on entry, buyer 2 matches buyer 1's offer if and only if either $v_2 = 1$ and $p \leq 1$, or $v_2 = 0$ and $p \leq 0.3$.

In the equilibrium, buyer 1 gets exactly a zero expected payoff, buyer 2's expected payoff is 0.15, and the seller's revenue is $(1/2) \cdot \hat{r} + (1/2) \cdot z^* = 0.45$ on expectation.[7]

Note that if the initial seller were able to make take-it-or-leave-it offers herself, she could offer the item to buyer 1 at price $\hat{r}$, which is just small enough to induce buyer 1 to enter. If $v_1 = 0$ then buyer 1 rejects

[6] We assume here that resale cannot take place between the buyers who already participate in the initial mechanism. This assumption will be discussed in more detail in Section 6.2.

[7] There are other subgame perfect equilibria in which the seller's revenue is less than 0.45. However, the seller's expected revenue is at least $z^* = 0.3$ in every subgame perfect equilibrium.

the offer, and the seller would make the same offer to buyer 2, and eventually to buyer 3 if necessary. In this scenario, the seller's expected revenue is $(1-(1/2)^3)\cdot\hat{r}=0.525$, which is the highest value that an individually rational and incentive compatible mechanism can generate.[8] We will later show that the revenue of the fully sequential mechanism under resale converges to this theoretical maximum as the number of potential buyers grows.

3 Basic Model

3.1 Demand structure

Following Bulow and Klemperer (2009), we consider the following setup: The owner of an item faces a (possibly random) queue of risk-neutral potential buyers, indexed $i=1,2,\ldots$. The probability that at least n buyers are present, conditional on $n-1$ buyers being present, is $\rho_n\in[0,1]$. We assume that $\rho_1=\rho_2=1$ and $1\geq\rho_3\geq\rho_4\geq\ldots$, with at least one inequality strict. We denote by

$$\mathcal{N} \;=\; \rho_1\big(1+\rho_2\big(1+\rho_3\big(1+\ldots\big)\big)\big) \;\in\; \big[2,\infty\big)$$

the expected number of buyers, which is our measure of the market's size. Note that this model of buyer arrival subsumes the case of a fixed number N of bidders, which corresponds to $\rho_1=\ldots=\rho_N=1$ and $\rho_{N+1}=\rho_{N+2}=\ldots=0$. In this case, the arrival sequence represents merely an exogenous ordering in which buyers are treated by the seller.

Buyer i, if he exists, must pay $c>0$ in order to learn his private valuation v_i. This valuation is drawn from an atomless distribution F with support $[0,\overline{v}]$ and independent of v_j for all $j\neq i$. We assume that $c<\int_0^{\overline{v}}(1-F(v))F(v)dv$, i.e., at least two buyers could profitably enter a standard second-price auction. It will also be useful to define the (unique) value $0<\hat{r}<\overline{v}$ that satisfies the condition

$$\int_{\hat{r}}^{\overline{v}}(v-\hat{r})dF(v)-c \;=\; 0. \tag{1}$$

If the item was offered at a posted price, then $\hat{r}$ is the highest such price at which a buyer is willing to pay the entry cost, learn his valuation, and buy the item if and only if his valuation exceeds the posted price.[9]

3.2 Initial selling mechanism

The owner's valuation for the item is zero. The owner has to sell the item and can choose among two types of initial selling mechanism: Auctions and sequential mechanisms. In both cases, bidders arrive in sequence and, when they arrive, must decide whether to enter or not. In auctions, bidding takes place only after the entry of all participants has stopped, whereas in sequential mechanisms, buyers can place bids on which later buyers can condition their entry decisions. BK compare a standard second-price auction to the sequential mechanism originally introduced by Fishman (1988), which they generalize to the case of more than two buyers.

[8] This can be shown by applying the arguments in Crémer *et al.* (2009). With non-binary valuations, the mechanism that achieves this maximum value would be more complicated (it would involve all buyers paying an entry fee to the seller initially, followed by two differentiated take-it-or-leave-it offers to buyers 1 and 2, followed (if necessary) by a second price auction among all three buyers.

[9] Our term $\hat{r}$ is the same as the term V_K in BK.

Second-price auction. When buyer 1 arrives he decides whether or not to enter. If 1 enters he pays c and learns his valuation v_1, and the seller gives buyer 2 the opportunity to enter. If 2 enters, he pays c and learns his valuation v_2, and the seller waits for buyer 3 to arrive. If 3 arrives he is given the opportunity to enter. If 3 enters he pays c, learns valuation v_3, and the seller waits for buyer 4 to arrive. This process continues until either the arrival of new buyers comes to a halt or a new buyer arrives and decides not to enter for the first time. Previous entrants do not observe which of the two possibilities is the case.[10] At this time, all buyers i who did enter simultaneously submit bids $b_i \geq 0$. The bidder who submitted the highest bid wins and pays the second-highest bid. If only a single bid is placed, the winner pays zero.

Fishman/BK sequential mechanism. The initial price is $p^0 = 0$. When buyer 1 arrives he decides whether to enter. If he enters, 1 pays c, learns v_1, and can place a bid of $p^1 \geq 0$. This is a *jump bid*, i.e., a public commitment to pay p^1 if 1 wins. When bidder 2 arrives, he observes p^1 and decides whether to enter. If he enters he pays c and learns v_2. Bidders 1 and 2 then simultaneously raise the price until one of them quits. The survivor is the new high bidder, and can make an additional discrete jump bid. The price at the end of the second stage is p^2. Now bidder 3 arrives, observes the previous price history, and decides to enter. If he enters he pays c, learns v_3, and then competes with the survivor of the previous stage by simultaneously raising the price until one bidder quits. The survivor is the new high bidder and can place an additional jump bid, after which the price is p^3. This process continues until either the arrival of buyers stops, or until the first buyer arrives and decides not to enter. At this time, the current high bidder wins and pays the current price.

We compare the same two selling mechanisms, and then consider a third. The third mechanism is a sequential mechanism similar to the Fishman/BK mechanism, with one exception: The new entrant and the current high bidder do not simultaneously raise the price before the survivor makes a jump bid:

Fully sequential mechanism. The fully sequential mechanism proceeds as follows. The initial price is $p^0 = 0$. When bidder 1 arrives he decides whether to enter. If he enters, 1 pays c, learns v_1, and can make a jump bid of $p^1 \geq 0$. When bidder 2 arrives, he observes p^1 and decides whether to enter. If he enters he pays c and learns v_2. He can then either quit, or submit a jump bid $p^2 > p^1$ to become the new high bidder. Now bidder 3 arrives, observes p^2, and decides to enter. If he enters he pays c, learns v_3, and then either quits or bids $p^3 > p^2$ to become the new high bidder. This process continues until either the arrival of buyers stops, or until the first buyer arrives and decides not to enter. At this time, the current high bidder wins and pays the current price.

After the initial selling mechanism has concluded, the successful buyer may, if he wishes, attempt to resell the object. In Section 4, we formally introduce and analyze the resale stage of our model.

3.3 Strategies and beliefs

In all initial selling mechanisms we consider, an agent's strategy has two components: An entry strategy and a bidding strategy. An entry strategy is a mapping from the information available to a potential buyer at the time the buyer arrives to that player's entry decision. This information varies depending on

[10] This assumption matters for the equilibrium of the auction insofar as, if entry stops due to a lack of potential buyers, there will be no resale market. If existing buyers knew that this was the case, their bidding behavior would be different from the case in which there was a further potential buyer who decided not to enter (in the latter case, existing buyers would know that there is at least one buyer to whom the object could be resold).

the mechanism considered; however, since buyers learn their valuations only after having entered, the decision to enter can never be conditioned on a buyer's private valuation. Likewise, a bidding strategy is a mapping from the information available to a buyer after entry in the initial mechanism to the buyer's bid(s), including jump bids. This information, too, varies depending on the mechanism considered, but since only buyers who have entered can bid, bidding strategies can always be conditioned on a buyer's private valuation.

To predict the outcome of a mechanism, and to derive implications of a particular of mechanism on seller revenue, one must identify mutually optimal entry and bidding strategies in the games induced by these mechanisms. Since these games are Bayesian games, it is convenient to derive the optimality of strategies given a player's beliefs and derive beliefs from strategies and observable information through Bayes' rule whenever possible. The resulting strategy-belief profile then constitutes a weak perfect Bayesian Nash equilibrium (and the strategy profile alone constitutes a Nash Equilibrium). When we describe an "equilibrium" of a mechanism, therefore, we mean a profile of strategies and beliefs that constitute a weak perfect Bayesian Nash equilibrium. For each mechanism we consider, appropriate formalizations of the relevant beliefs, as well as of the relevant information sets on which strategies are conditioned, will be introduced in Sections 5 and 6.

4 The Resale Market

The winner of the initial mechanism announces a resale mechanism after the initial mechanism has ended, but before any subsequent buyers arrive. Buyers who have already entered the initial mechanism are free to participate in the resale mechanism, and any potential buyer who arrives later must pay the entry cost c to participate. Potential buyers who arrive but do not enter cannot return at a later stage. We assume that the item can be resold at most once, that is, the winner of the resale mechanism consumes the item.

We now state the formal assumptions underpinning the resale mechanisms in our model; this is done in Section 4.1. We then develop a number of results concerning the item's resale value (and, hence, buyers' willingness to pay at the initial selling stage) in Section 4.2. Finally, in Section 4.3 we discuss the role of our assumptions for these results.

4.1 Resale mechanisms: Assumptions and notation

The reseller can choose among some class of available resale mechanisms. A resale mechanism is a complete description of the rules governing sale to any set of potential buyers. Regarding the set of mechanisms available to the reseller, we make the following assumptions:

A1. The set of available resale mechanisms does not depend on the original selling mechanism, on the identity of the seller, or on how many bidders have already arrived or entered.

A2. The set of available resale mechanisms includes the option to not resell the object, and it includes resale via take-it-or-leave-it offers to individual buyers.

A3. All resale mechanisms are *ex ante* and *interim* individually rational. That is, a potential buyer cannot be forced to enter and participate in the mechanism, and if he enters and learns his valuation, he can walk away from the mechanism at no additional cost.

Once a resale mechanism is announced, buyers interact with each other by playing an (appropriately defined) equilibrium of the game induced by the mechanism. Instead of characterizing this equilibrium for every resale mechanism, we summarize its outcome as follows. Fix a resale mechanism, its associated equilibrium, the sequence $\{\rho_n\}_{n=1}^{\infty}$, the entry cost c, and the value distribution F, and suppose n bidders participated in the initial mechanism. Let $Q_n \in [0,1]$ be the probability that the resale mechanism results in a sale to some buyer $n' > n$. Let X_n be the expected total payment made by buyers $n' > n$ in the resale mechanism.[11] The set of available resale mechanisms can then be summarized by the set of sequences $(Q, X) = (Q_n, X_n)_{n=1,2,\ldots}$ associated with each mechanism and its equilibrium. We call this set Ω and make the following technical assumption:

A4. $\{(Q_n, X_n) : (Q, X) \in \Omega\} \in \mathbb{R}^2$ is compact for each n.

4.2 Resale revenue and effective valuations

Buyers' willingness to pay for the object at the initial selling stage will depend, in part, on the revenue they can expect to receive if they were to sell the object in the resale market. We now examine this relationship between expected resale revenue and the surplus a buyer obtains when he acquires the object in the initial mechanism. We begin by establishing an upper bound on resale revenue, which is driven by the individual rationality assumption A3:

Lemma 1. *The expected revenue of any resale mechanism that satisfies assumption A3 and that sells with probability q is at most $q\hat{r}$, where $\hat{r}$ was defined in* (1).

Suppose that n buyers participate in the initial mechanism and that buyer $i \le n$ is the winner. Assume that the initial mechanism allocates to the buyer with the highest private valuation (this will be the case in both the auction and the Fishman/BK sequential mechanism). This implies that, if buyer i resells the object to buyer j, then $j > n$. For each n and v_i, we can then define

$$z_n(v_i) \;=\; \max_{(Q,X)\in\Omega} \big\{X_n + (1 - Q_n)v_i\big\} \tag{2}$$

to be i's *effective valuation* in the initial selling mechanism with n participants, that is, the possible surplus that i can attain on expectation if he wins the object. Note that $z_n(0)$ is the "pure resale value" of the object. In the following result, we collect several properties of effective valuations to be used later:

Lemma 2. *Suppose the set of resale mechanisms satisfies assumptions A1–A4. For given n, the effective valuation function z_n is well-defined and has the following properties:*

(*i*) *z_n is strictly increasing and weakly convex.*

(*ii*) $z_n(v_i) \ge \max\{z_n(0), v_i\}$.

[11] For example, suppose $v_i \sim U[0,1]$ and $\rho_i = \alpha$ for all $i \le N$ and $\rho_i = 0$ for $i > N$. As a resale mechanism, consider a sequence of take-it-or-leave-it offers $p \in (0,1)$ that are made by the reseller to every buyer, until either the offer is accepted or the arrival of new buyers stops. For $n = 1, \ldots, N-1$ we have

$$Q_n = \alpha\big[p + (1-p)\alpha\big[p + (1-p)\alpha[\;\ldots\;]\big]\big] = \alpha p \frac{1 - \big[(1-p)\alpha\big]^{N-n}}{1-(1-p)\alpha} \quad \text{and} \quad X_n = Q_n p,$$

and for $n \ge N$ we have $Q_n = X_n = 0$.

(*iii*) $z_n(v_i) \leq \max\{\hat{r}, v_i\}$, *where* $\hat{r} < \overline{v}$ *was defined in* (1).

(*iv*) $z_n(v_i) - z_n(v_i') \leq v_i - v_i'$ *for all* $v_i > v_i'$.

Because the lowest effective valuation is $z_n(0)$, the difference $z_n(v_i) - z_n(0)$ limits the information rent buyer i can hope to earn in the initial mechanism. We will now show that this rent is small when the resale market is large.

To formalize the notion of a large resale market, recall that the number of potential buyers to which the item can be resold to depends (stochastically) on the sequence (ρ_i) that specifies the conditional arrival probabilities of buyers. We endow the space of all such sequences with the product topology; that is, one sequence converges to another if it converges pointwise. Now consider a sequence of sequences $(\rho^t) = ((\rho_i)^t_{i=1,2,\ldots})^{t=1,2,\cdots}$, where t indexes the sequence and i indexes buyers in the sequence. Suppose that $\rho^t \to (1, 1, 1, \ldots)$. This includes two important cases: When there is a finite number of buyers and this number grows (e.g., $\rho_i^t = 1\ \forall i \leq t$ and $\rho_i^t = 0\ \forall i > t$); and when there is a potentially infinite number of bidders whose arrival probabilities increase (e.g., $\rho_i^t = \alpha^t < 1\ \forall i, t$ and $\alpha^t \to 1$). Intuitively, in both cases the set of resale opportunities expands as $t \to \infty$. At the same time, if $\mathcal{N}^t$ is the expected number of buyers associated with ρ^t, then $\mathcal{N}^t \to \infty$ as $t \to \infty$.[12]

Thus, from now on when we say there is a "sufficiently large number of buyers" or "sufficiently large resale market," we mean that $\mathcal{N}$ is large (but still finite), which is equivalent to (ρ_i) being close, but not equal, to $(1, 1, 1, \ldots)$. The following result describes what happens to effective valuations in this case:

Lemma 3. *Suppose the set of resale mechanisms satisfies assumptions A1–A4. Fix F and c. Take any sequence of arrival probability sequences (ρ^t), such that $\rho^t \to (1, 1, 1, \ldots)$ pointwise. Let $z_n^t : [0, \overline{v}] \to [0, \overline{v}]$ be the effective value function associated with sequence ρ^t, for given n. As $t \to \infty$,*

(*i*) $z_n^t(0) \to \hat{r}$;

(*ii*) $z_n^t(v_i) \to \max\{\hat{r}, v_i\}$ *uniformly;*

(*iii*) $E\big[z_n^t(v_i) - z_n(0)\big] \to c$.

4.3 Discussion of assumptions

Our model of the resale market encompasses a variety of ways in which resellers interact with buyers, subject to the assumptions spelled out in Section 4.1. These assumptions are that the object can be resold only once, as well as the requirements in assumptions A1–A4 on the set of resale mechanisms available to the reseller.

The assumption that repeated resale is not possible is made in order to rule out Ponzi schemes in the resale market, that is, situations in which the object is sold indefinitely for an unbounded total profit. Suppose buyer i buys the object from the previous buyer $i-1$ for some price, say p^{i-1}, and then resells it to the next buyer $i+1$ for a higher price p^i. If $p^i > (c + p^{i-1})/\rho_{i+1}$, the scheme compensates buyer i for the entry cost c, the payment p^{i-1} to the previous buyer, and the risk that the next buyer does not arrive. As long as $\rho_i > 0\ \forall i$, every buyer who arrives would earn a positive expected profit from trading, which is uncoupled from any buyer's consumption value of the traded item. In anticipation of such speculative

[12] This follows from the assumption that the ρ-sequences are monotonic.

bubbles in the resale market, buyers would have a potentially unlimited willingness to pay to acquire the object in the initial mechanism. We need to rule out this possibility, and the simplest way of doing so is to assume that the object can only be resold once.

Of the remaining assumptions A1–A4, assumption A1 is made for convenience and can be relaxed at the expense of additional notation. Assumption A4 is a technical requirement that ensures that the reseller's choice of resale mechanism is well-defined. The substantive assumptions are A2 and A3. Assumption A2 serves to put a lower bound on a buyer's effective valuation, and A3 serves to put an upper bound on a buyer's effective valuation. With a large number of buyers these two bounds lie very close together, and our characterization of the outcomes of the initial selling mechanism for large $\mathcal{N}$ (presented in the following sections) stems from this fact. Therefore, these two assumptions are central for our results.

The first part of A2 is uncontroversial: It allows the winner of the initial mechanism to consume the object if expected resale revenue is below the initial winner's private valuation. The second part of A2—the ability to make take-it-or-leave-it offers—is the primary source of asymmetry between the original seller and resellers in our model. Note that we are not requiring resellers to actually sell via take-it-or-leave-it offers; we only assume that such offers *can* be made.[13] With an infinite stream of potential buyers, an optimal individually rational resale mechanism is, in fact, a sequence of take-it-or-leave-it offers at price $\hat{r}$. However, this is not generally the case if $\mathcal{N} < \infty$, and resellers may well prefer to use other mechanisms, if available (e.g., auctions with buyer-specific or time-specific reserve prices). The ability to make take-it-or-leave-it offers only implies that any mechanism the reseller actually uses generates at least the same expected revenue that a sequence of such offers would generate.

Assumption A3 imposes two individual rationality constraints on the reseller. The first is an *ex ante* constraint, which implies that a buyer who enters receives, on expectation, sufficient surplus from the mechanism to cover the entry cost c. This, in turn, limits the revenue the reseller can extract from each buyer—in particular, the proof of Lemma 1 depends only on the *ex ante* constraint. The second constraint is an interim individual rationality constraint, which means that the buyer can walk away from the mechanism at no cost (other than the entry cost c) after learning his valuation. Interim individual rationality precludes pre-entry contracts between resellers and potential buyers that provide the buyer with enough surplus on expectation to induce entry, but oblige the buyer to receive a negative surplus for some realizations of v_i. We exclude such resale mechanisms from our model in order to guarantee that effective valuations are strictly monotone in private values (see the proof of Lemma 2). This, in turn, guarantees that both the auction and the BK/Fishman sequential mechanism allocate the object to the buyer with the highest private value in the initial mechanism, as they do in the no-resale case.

5 Equilibrium in the Auction and Sequential Mechanism

In this section, we focus on the two mechanisms examined in BK: The second-price auction, and the Fishman/BK sequential mechanism.

[13]The previous literature on auctions with resale has assumed a variety of resale mechanisms, including take-it-or-leave-it offers (e.g., Halafir and Krishna 2008; Che *et al.* 2013), re-auctioning (e.g., Zheng 2002), and Nash bargaining (e.g., Haile 2000, 2003; Pagnozzi 2007). In this paper we do not rule out any of these mechanisms, but merely assume that take-it-or-leave-it offers are possible.

5.1 Second-price auction

In the auction, when a buyer arrives he faces a decision whether to enter or not if and only if all previous buyers have entered. Thus, buyer i's entry decision is a binary variable $e_i \in \{0, 1\}$, where $e_i = 1$ means that i enters if he arrives to the auction. Once either buyer arrival stops or the first arriving buyer decides not to enter, the game proceeds to the bidding stage. In the bidding stage, every participating buyer will know his private valuation, v_i, and the total number of buyers in the auction, n. Thus, a bidding strategy for buyer i is a mapping $s_i : [0, \overline{v}] \times \{1, 2, \ldots\} \to [0, \infty)$, where $s_i(v_i, n)$ is the bid submitted by participating bidder i with private valuation v_i in the second-price auction with n participants.

No participating bidder i observes the private values of the other $n - 1$ participants, and knows only that each of these values is independent and follows distribution F. In addition, i does not observe whether the auction contains n bidders because only n bidders exist, or because bidder $n + 1$ exists but decided not to enter. This information matters to i, because a resale market exists only in the latter case but not in the former. Thus, we let $\mu_i(n) \in [0, 1]$ denote bidder i's belief that buyer $n + 1$ exists. This belief is not conditioned on v_i because i's private valuation is independent of with whether bidder $n + 1$ exists; however, it is a conditioned on the number of participating bidders, n.

Suppose, for the time being, that there is a Bayesian Nash equilibrium in which the first $n^* \geq 1$ buyers enter in the auction, if n^* or more buyers exist. That is, we consider the following profile of entry strategies:

$$e_i \;=\; \begin{cases} 1 & \text{if } i \leq n^*, \\ 0 & \text{otherwise.} \end{cases} \tag{3}$$

Now consider the bidding stage, where the *actual* number of entrants, n, is observed by all participating bidders. What should participating bidder i's beliefs be $(i = 1, \ldots, n)$? There are three cases to consider:

(1) $n < n^*$ and $\rho_{n+1} < 1$. In this case, each of the n participating buyers must infer that entry stopped at n bidders because buyer $n + 1$ does not exist. This means that Bayesian beliefs are $\mu_i(n) = 0$.

(2) $n = n^*$. In this case, entry stopped at n^* bidders either because buyer $n + 1$ arrived and decided not to enter (probability ρ_{n^*+1}), or because buyer $n + 1$ does not exist (probability $1 - \rho_{n^*+1}$). This means that Bayesian beliefs are $\mu_i(n) = \rho_{n^*+1}$.

(3) All other possibilities, i.e.: $n < n^*$ and $\rho_{n+1} = 1$, or $n > n^*$. In this case, having n bidders participate is an out-of-equilibrium event that have zero probability under the hypothesized entry strategies (in the first case, bidder $n + 1$ arrived but did not enter despite his entry strategy prescribing entry; in the second case bidders $n^* + 1, \ldots, n$ arrived and entered despite their strategies prescribing non-entry.) In each case, out-of-equilbrium beliefs μ_{n+1} are non-Bayesian. Because the mistaken entry decision that led to the out-of-equilibrium event could not have been conditioned on the existence of buyer $n + 1$, we assign the prior probability that buyer $n + 1$ exists: $\mu_i(n) = \rho_{n+1}$.[14]

The arguments above imply the following equilibrium beliefs (about the existence of buyer $n + 1$) for each participating bidder $i = 1, \ldots, n$:

$$\mu_i(n) \;=\; \begin{cases} 0 & \text{if } n < n^* \text{ and } \rho_{n+1} < 1, \\ \rho_{n+1} & \text{otherwise.} \end{cases} \tag{4}$$

[14] For example, these are the only out-of-equilibrium beliefs that are structurally consistent with the game in the spirit of Kreps and Wilson (1982).

In the case where $\mu_i(n) = 0$, the continuation game at the bidding stage amounts to a standard second-price auction with independent private values, and it follows that bidding private value v_i is a dominant strategy for each buyer i. On the other hand, if $\mu_i(n) = \rho_{n+1}$, as long as the auction allocates efficiently, the participating bidders expect a resale market with probability ρ_{n+1}. We show in the Appendix that, in this case, each bidder i in the initial mechanism should bid his effective valuation $z_n(v_i)$. Thus, we following result:

Proposition 4. *Suppose that, in the equilibrium of the auction game, entry strategies are given by* (3)*. Let n be the actual number of bidders who participate in the auction. Then, at the subgame in which bidding commences, the following is a continuation equilibrium in bidding strategies for $i = 1, \dots, n$:*

$$s_i(v_i) \;=\; \begin{cases} v_i & \text{if } n < n^* \text{ and } \rho_{n+1} < 1, \\ z_n(v_i) & \text{otherwise,} \end{cases} \tag{5}$$

where z_n is defined in (2)*. The beliefs about the existence of buyer $n + 1$ that support these bidding strategies are given by* (4)*.*

We now turn to the entry stage. If n is the actual number of entrants, and these entrants bid as described in Proposition 4, the expected surplus each of them receives is

$$\pi^{AU}(n, n^*) \;=\; \begin{cases} \displaystyle\int_0^{\overline{v}} \int_0^{v} (v - w)\, dF(w)^{n-1} dF(v) - c & \text{if } n < n^*, \\ \displaystyle\int_0^{\overline{v}} \int_0^{v} (z_n(v) - z_n(w)) dF(w)^{n-1} dF(v) - c & \text{if } n \geq n^*. \end{cases} \tag{6}$$

When bidder n^* arrives, he compares $\pi^{AU}(n^*, n^*)$ to the expected "outside payoff" from not entering. Because z_n is bounded for all n, $\pi^{AU}(n^*, n^*)$ must be negative for all large enough n^*. On the other hand, the outside payoff is at least zero (but may be positive if there is a positive probability that a bidder who does not enter purchases the good on the resale market later). Thus, the equilibrium value for n^* is given by the largest integer such that $\pi^{AU}(n^*, n^*)$ is at least equal to the outside payoff.[15]

The following result shows that, if the set of resale opportunities is large in the sense defined in Section 4, then $n^* = 1$. That is, exactly one bidder enters in the equilibrium.

Proposition 5. *Fix F and c. Assume the initial seller uses a second-price auction and consider the equilibrium described by entry strategies* (3)*, beliefs* (4)*, and bidding strategies* (5)*. If $\mathcal{N}$ is sufficiently large, the equilibrium number of entrants in the auction is $n^* = 1$, and the initial seller receives a zero price.*

Thus, if the original seller uses the auction and there are a sufficiently many resale opportunities available to the winning bidder, the original seller will receive a zero price in the equilibrium.

[15] To confirm that bidders $n < n^*$ also have an incentive to enter, note that if $n < n^*$ enters and entry stops at n^* bidders, n receives $\pi^{AU}(n^*, n^*)$, which is at least as large as the payoff from not entering (by definition of n^*). If n enters and entry stops at n' bidders with $n \leq n' < n^*$, there can be no resale market and the expected surplus from not entering is zero. In this event, n receives an expected surplus of $\pi^{AU}(n'|n^*)$ in the auction, which (using Lemma 2 (iv) and $n' < n^*$) is larger than $\pi^{AU}(n^*|n^*)$, which in turn is at least zero. Thus, if n^* has an incentive to enter, $n < n^*$ must have an incentive to enter as well.

5.2 Fishman/BK sequential mechanism

In the sequential mechanism, entry and bidding decisions are intertwined: Buyer i's entry decision depends on the bid history prior to i's arrival, and i's bidding strategy will affect the entry decisions of future buyers.

To define these strategies formally, recall that a buyer can place two kinds of bids in the sequential mechanism. We refer to these as "continuous bids" and "jump bids," respectively. A continuous bid is the maximum value to which the buyer is willing to raise the price in the ascending auction, at any stage of the mechanism. A jump bid represents the value to which a buyer openly raises the price, should he win the ascending auction. Following the original BK model, we assume that both continuous bids and jump bids depend on a buyer's private value and the stage of the mechanism. Thus, the continuous bid component of n's bidding strategy is a mapping

$$s_n : [0, \overline{v}] \times \{n, n+1, \ldots\} \to [0, \infty).$$

The interpretation is that $s_n(v_n, n')$ is the maximum price at which buyer n with private value v_n would remain in the ascending auction at stage n' of the mechanism, if buyer n was competing at that stage. Likewise, the jump bid component is a mapping

$$j_n : [0, \overline{v}] \times \{n, n+1, \ldots\} \to [0, \infty)).$$

The interpretation is that $j_n(v_n, n')$ is the value to which buyer n with private value v_n would raise the price following the ascending auction at stage n' of the mechanism, if buyer n competed at that stage, won the ascending auction, and the price was below $j_n(v_n, n')$. The choice of *not* making a jump bid at stage n' can simply be represented by $j_n(v_n, n') = 0$.

Now consider buyer n's entry strategy. As in the original BK model, we assume that the buyer's entry decision depends on the current price of the mechanism, p_{n-1} and on whether this price was reached as the outcome of the ascending auction or a jump bid. We denote the first case by "C" (for "continuous bid") and the second case by "J" (for "jump bid.") Therefore, an entry strategy for buyer $n > 1$ is a mapping

$$e_n : [0, \infty) \times \{\mathrm{C}, \mathrm{J}\} \to \{0, 1\}.$$

For instance, $e_n(p_{n-1}, \mathrm{J}\} = 1$ means that buyer n enters if he observes a jump bid of p_{n-1} at the end of stage $n-1$. For the first buyer, $n = 1$, the entry strategy is simply a decision $e_1 \in \{0, 1\}$.

Finally, in the context of the sequential mechanism the relevant beliefs are those entertained by a newly arriving buyer n about the private value of the incumbent (denoted v^I). Thus, we let $\mu_n(v^I | p_{n-1}, \mathrm{C})$ denote a cumulative density function on $[0, \overline{v}]$ that represents the belief of buyer $n > 1$ about v^I if the current price is p_{n-1} and the result of a continuous bid. Similarly, we let $\mu_n(v | p_{n-1}, \mathrm{J})$ denote a cumulative density function on $[0, \overline{v}]$ that represents the belief of buyer $n > 1$ about v^I if the current price is p_{n-1} and the result of a jump bid.

Before proceeding to characterize the equilibrium of the sequential mechanism with resale, it will be helpful to review the outcome of the mechanism when there is no resale.

Recap of the outcome of the sequential mechanism without resale. BK showed that the sequential mechanism has equilibria of the following form: Bidders enter at low enough prices. After entering, a bidder whose valuation is below some cutoff v^* raises the price against his rivals (i.e., against the previous high bidder and, should he prevail, against future entrants) until there is no more entry and he wins, or until the price reaches his valuation and he leaves. Such bidders never place jump bids. A bidder with valuation v^* or higher bids in the same way first, but when he becomes the high bidder he places an additional jump bid, which deters all further entry. Multiple equilibria of this form exist that differ in their threshold v^*. The largest cutoff value is $\hat{r}$ ($= V_K$ in BK), and the smallest is a value V_S that satisfies the condition

$$\frac{1}{1-F(V_S)}\int_{V_S}^{\overline{v}}\int_{V_S}^{v}(v-w)\,dF(w)dF(v)-c \;=\; 0. \tag{7}$$

To understand (7), note that if the current high-bidder's value was known to be at least V_S, then paying the entry cost to compete against this bidder yields exactly a zero surplus. A jump bid signals that the current bidder's valuation is at least v^*, and since $v^* \geq V_S$ entry is deterred. The precise value of the entry-deterring jump bid, j_n^*, is determined by the requirement that only bidders above v^* want to use this costly signal.[16]

BK show that only the $v^* = V_S$ equilibrium satisfies a forward induction criterion, perfect sequentiality. In the other equilibria, buyers with valuations in $[v^*, \overline{v}]$ can deviate and place lower jump bids that, if interpreted "correctly" by potential entrants, also deter entry and give the deviating player a strictly larger payoff.

If resale is possible, the basic logic behind the BK equilibria continues to hold, but the characterization of the lower entry deterrence threshold (V_S) becomes much more tedious. The reason is that this threshold now depends on the period to which it applies.[17] We steer clear of this complication by proving an asymptotic result: If the number of potential buyers is large, then an equilibrium exists in which $v^* = 0$ is an entry-deterring threshold in the initial stage of the mechanism.

We begin with an intermediate result that establishes a condition under which the sequential mechanism ends, which is independent of the equilibrium being played (the proof is in the Appendix):

Lemma 6. *Fix F and c and assume the initial seller uses the Fishman/BK sequential mechanism. Consider any stage n of the mechanism. For $\mathcal{N}$ sufficiently large, the following holds in every Perfect Bayesian Equilibrium of the mechanism: If*

- (i) *the price at the end of stage n of the mechanism is $p_n \geq z_n(0)$; and*
- (ii) *buyer $n+1$ arrives and, upon observing p_n, believes that the distribution of the current incumbent's private valuation is $\mu_{n+1}(v^I|\cdot) = F(v^I|v^I \geq y)$ for some $y \in [0, \overline{v}]$,*

then buyer $n+1$ does not enter.

Lemma 6 can be used to construct an equilibrium of the sequential mechanism in which all types of buyer 1 deters entry by buyer 2, provided $\mathcal{N}$ is large.

[16] Because the expected payoff from *not* using the signal (i.e., continuing to compete in the mechanism) depends on how high the current auction price is, the value of the entry-deterring jump bid changes as the sequential mechanism progresses, even if the threshold v^* stays constant.

[17] This is because, unlike the private value v_i, a bidder's effective valuation $z_n(v_i)$ is time-dependent. That is, the function z_n is generally not the same as the function $z_{n'}$. The exception is the case in which new buyers arrive with a constant probability in every period (i.e., $\rho_3 = \rho_4 = \ldots$). In this case, the z_n-functions are the same for all n, and the lower bound V_S becomes stationary.

Suppose buyer 1 enters the mechanism but does not submit a jump bid (or, equivalently, submits a jump bid of zero). Since buyer learns nothing from seeing a stage-1 price of zero, 2's Bayesian beliefs about 1's type are

$$\mu_2(v_1|0,\mathrm{C}) \;=\; \mu_2(v_1|0,\mathrm{J}) \;=\; F(v_1).$$

If buyer 2 enters at stage 2, he competes against buyer 1 in an ascending auction. Assume, for a moment, that buyer 3 (if he exists) does *not* enter at the third stage. Thus, bidders 1 and 2 will raise the price until it reaches $z_2(v_1)$ or $z_2(v_2)$, whichever comes first. If there is no further jump bid, the price at the end of stage 2 equals

$$p_2 \;=\; \min\{z_2(v_1), z_2(v_2)\} \;\geq\; z_2(0).$$

Moreover, buyer 3 will believe that the incumbent's valuation is at least $z_2^{-1}(p_2)$; that is,

$$\mu_3(v^I|p_{n-1},\mathrm{C}) \;=\; F(v^I|v^I \geq z_2^{-1}(p_2)).$$

Provided $\mathcal{N}$ is sufficiently large, Lemma 6 implies that bidder 3 does not enter, as was hypothesized. Going back to player 2's entry decision, the expected surplus that buyer 2 achieves by entering against incumbent 1 is given by

$$\begin{aligned}
\pi^{SM}(2) \;&=\; \int_0^{\overline{v}}\int_0^{v_1} (z_2(v_2) - z_2(v_1))\, dF(v_2) d\mu_2(v_1) - c \\
&=\; \int_0^{\overline{v}}\int_0^{v_1} (z_2(v_2) - z_2(v_1))\, dF(v_2) dF(v_1) - c \;=\; \pi^{AU}(2,2).
\end{aligned} \tag{8}$$

For $\mathcal{N}$ sufficiently large, Proposition 5 implies that $\pi^{AU}(2,2) < 0$. Since buyer 2's expected surplus from not entering is at least zero, he prefers not to enter. If, out of equilibrium, buyer 1 were to submit a positive jump bid, we assume that 2's beliefs remain $\mu_2(v_1|p_1,\mathrm{J}) = F(v_1)$ (which is equivalent to saying that buyer 2 believes that all types of buyer 1 are equally likely to have made the positive jump bid). Buyer 2's expected payoff from entering would then be

$$\int_{z_2^{-1}(p_1)}^{\overline{v}}\int_0^{v_1} (z_2(v_2) - z_2(v_1))\, dF(v_2) dF(v_1) - c \;\leq\; \pi^{SM}(2),$$

so buyer 2 would still prefer not to enter. An analogous argument can be made if 2 entered and a positive jump bid was submitted at stage 2.

Thus, we have shown the following:

Proposition 7. *Fix F and c. Assume the initial seller uses the Fishman/BK sequential mechanism. If $\mathcal{N}$ is sufficiently large, there exists a weak perfect Bayesian equilibrium of the sequential mechanism exists in which bidder 1 enters, does not bid, and no further bidders enter. This equilibrium is characterized by the following strategies and beliefs:*

$$\begin{aligned}
&e_1 = 1,\;\; e_n(p_{n-1},\mathrm{C}) = e_n(p_{n-1},J) = 0 \;\;\forall n > 1, \\
&s_n(v_n, n') = z_{n'}(v_n) \;\;\forall n,\; \forall n' \geq n, \\
&j_n(v_n, n') = 0 \;\;\forall n,\; \forall n' \geq n, \\
&\mu_n(v^I|p_{n-1},\mathrm{C}) = \mu_n(v^I|p_{n-1},J) = F(v^I) \;\;\forall n,\; \forall p_{n-1}.
\end{aligned}$$

The seller's revenue in the equilibrium is zero.

The equilibrium in Proposition 7 corresponds to the V_S-equilibrium in BK, i.e., when resale is possible and $\mathcal{N}$ is large, then $V_S = 0$. (Note that the expected surplus term in (7) for the no-resale case mirrors the the corresponding term in (8) for the resale case). Furthermore, like the V_S-equilibrium in BK, the one in Proposition 7 satisfies the perfect sequentiality criterion. To understand the criterion, it is helpful to consider those equilibria in BK that do *not* satisfy it. Specifically, suppose that, in equilibrium, only those buyers with valuations $v_1 \geq v^* > 0$ deter entry, by placing jump bid $j_1^* > 0$. This implies that all jump bids $j_1 < j_1^*$ must lead to entry (as, otherwise, buyers who want to deter entry would not use j_1^*). Now imagine that bidder 1 actually did place jump bid j_1 just slightly below j_1^*. This bid is clearly a mistake for all types of buyer 1, as player 2 enters in response to such a bid. Thus, for j_1 not to be a mistake, buyer 1 must anticipate that a bid of j_1 would deter entry (contrary to the equilibrium strategy of buyer 2). BK show that, as long as $v^* > V_S$, the types of buyer 1 for whom a bid of j_1 would be preferable to the equilibrium bid *if it deterred entry* are buyers whose valuations are above some threshold $\underline{v}_1(j_1) < v^*$. Thus, if buyer 2 were to attempt to explain the unexpected bid j_1 as optimal behavior on part of buyer 1, he would conclude that $v_1 \geq \underline{v}_1(j_1)$. BK then show that, with these beliefs, player 2 should *not* enter following jump bid $j_1 \in (0, j_1^*)$. On the other hand, the V_S-equilibrium cannot be broken in this way.

In our case, the equilibrium in Proposition 7 trivially satisfies the refinement since all types of buyer 1 deter entry by buyer 2 with a zero jump bid, which means all types of buyer 1 receive the maximum possible payoff the mechanism permits. Thus, there is no scope for out-of-equilibrium signalling by placing an unexpected jump bid.

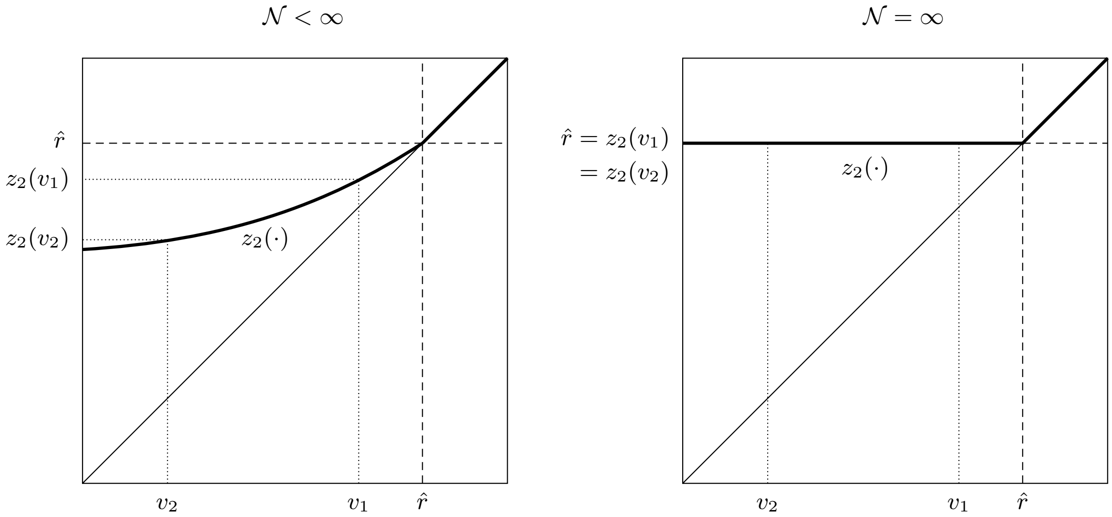
5.3 Intuition and numerical example

We showed that only one bidder enters in the equilibrium of both the auction and the sequential mechanism if resale is allowed and the expected number of bidders in the market is sufficiently large. To understand why, note that the introduction of a resale market has two consequences.

The first consequence is that even if a buyer draws a low private value, he can still resell the item if he wins it. Thus, every buyer's willingness to pay for the object at the initial selling stage (i.e., the buyer's effective valuation) increases. This does not necessarily translate into increased revenue for the initial seller, however, because buyers enter the initial mechanism only if they expect to earn a sufficiently high surplus from participating. The surplus the winning buyer receives in both the auction and the sequential mechanism is an information rent. A buyer's expected information rent may be small even if his expected willingness to pay is large, which happens, in particular, when *every* buyer's willingness to pay is large. The second consequence of having a resale market is precisely that it compresses the distribution of buyers' effective valuations: Every buyer has access to the same resale market; therefore, the resale profit one buyer can achieve is the same as the resale profit another buyer can achieve.

Figure 1 illustrates this effect. Suppose two buyers participate in the initial mechanism and buyers 1 and 2 are the buyers with the highest and second highest private valuation, respectively. Without resale, buyer 1's information rent in the initial mechanism would be the difference $v_1 - v_2$. The left diagram in Figure 1 shows the effective valuation function $z_n(\cdot)$ when resale is allowed: For private values less than $\hat{r}$, the effective valuation function is above the $45°$-line (the first effect of resale) and the slope of the effective valuation function is strictly less than one (the second effect of resale). Thus, the winner's information rent $z_n(v_1) - z_n(v_2)$ is strictly less than what it would be without resale. If $\mathcal{N}$ increases, effective valuations increase and compress more, and as $\mathcal{N} \to \infty$ the z_2-function approaches the limit

Figure 1: Resale and compression of effective valuations.



shown in the right diagram (this is formally stated in Lemma 3). For $\mathcal{N}$ large enough, a buyer's expected information rent will fall below the entry cost c. Once this happens, only a single buyer will enter and the initial seller's revenue drops to zero.

Our results did not address the question *how* large the resale market needs to be for only one buyer to enter at the initial selling stage. We now compute this threshold in an example. We demonstrate that even for moderate $\mathcal{N}$, entry by all but a single buyer can be deterred in the equilibrium of the initial mechanism.

Example 2. We assume that $\rho_n = \alpha < 1$ for $n \geq 3$. The set of available resale mechanisms consists of take-it-or-leave-it offers. We consider four different distributions of private values on the unit interval: The uniform distribution ($F(v) = v$), a distribution that shifts mass toward lower values relative to the uniform case ($F(v) = 2v - v^2$), and two distribution that shift mass toward higher valuations ($F(v) = v^2$ and $F(v) = v^3$). Moreover, we consider four different values for the entry cost: $c = 0.1$, $c = 0.075$, $c = 0.05$, and $c = 0.025$.

Note that, once the first two buyers have arrived, the resale environment becomes stationary: If r is an optimal posted price offer in period $n \geq 3$ and the offer is not accepted by buyer n, the reseller makes the same posted price offer r in the next period to buyer $n+1$. If a reseller with private valuation v posts price r in every period $n \geq 3$, he obtains the following expected payoff (not counting his own entry cost and payment to the original seller):

$$(1-\alpha)v \;+\; \alpha\Big[\big(1-F(r)\big)r \;+\; F(r)\Big[(1-\alpha)v \;+\; \alpha\Big[\big(1-F(r)\big)r \;+\; \dots$$
$$= \frac{(1-\alpha)v \;+\; \alpha\big(1-F(r)\big)r}{1-\alpha F(r)}.$$

Thus, the effective valuation function for reseller $n \geq 2$ is

$$z_n(v) \;=\; \begin{cases} \max\limits_{r \leq \hat{r}} \dfrac{(1-\alpha)v \;+\; \alpha\big(1-F(r)\big)r}{1-\alpha F(r)} & \text{if } v < \hat{r}, \\ \qquad\qquad v & \text{if } v \geq \hat{r}. \end{cases} \tag{9}$$

If only one buyer enters the initial mechanism, he can resell the object in period 2 already, and buyer 2 exists with probability 1. Thus, the effective valuation for reseller $n = 1$ is

$$z_1(v) \;=\; \begin{cases} \max\limits_{r \leq \hat{r}} \big(1-F(r)\big)r + F(r)z_3(v) & \text{if } v < \hat{r}, \\ \qquad\qquad v & \text{if } v \geq \hat{r}. \end{cases} \tag{10}$$

Given F, α, and c, one can obtain a numerical solution for the effective valuation functions as well as the optimal posted prices in each period. We use these solutions to compute the second buyer's expected payoff from entering the initial mechanism, as well as the expected payoff from not entering the initial mechanism and instead entering the resale market if and only if the first buyer offers the item for resale. For all twelve possible combinations of F and c, Table 1 below shows the values of α (and thus $\mathcal{N}$) for which the payoff from not entering just exceeds the payoff from entering. Note that in all cases shown in the table, at least two bidders would enter in the auction if resale was not allowed.

Table 1: Values for α and $\mathcal{N}$ such that only one bidder enters the initial auction/ sequential mechanism in Example 2.

		Entry cost: $c =$			
Distribution:		0.100	0.075	0.050	0.025
$F(v) = v$	$\alpha >$	.8325	.9240	.9720	.9942
	$\mathcal{N} >$	6.97	14.16	36.71	173.41
$F(v) = 2v - v^2$	$\alpha >$	.7470	.9075	.9726	.9958
	$\mathcal{N} >$	4.95	11.81	37.50	239.10
$F(v) = v^2$	$\alpha >$	.6079	.8262	.9363	.9866
	$\mathcal{N} >$	3.55	6.75	16.70	75.63
$F(v) = v^3$	$\alpha >$	.2487	.7221	.9073	.9819
	$\mathcal{N} >$	2.33	4.60	11.79	56.25

For example, consider the uniform values case $F(v) = v$ and assume an entry cost of $c = 0.1$. If $\alpha > .8325$—or if at least 6.97 buyers are in the market on average—only one bidder enters in the auction or sequential mechanism. If resale was not allowed, one can show that three bidders would have entered in the auction (assuming that the third bidder exists). For the non-uniform distributions, the thresholds for α and $\mathcal{N}$ are, with a few exceptions, lower than for the uniform distributions. For example, in the final (cubic) case, $\mathcal{N}$ needs to be just slightly larger than 2 for the second bidder to stay out.[18]

[18]The intuition is that the uniform distribution is the maximum entropy distribution over a given interval. This means that a buyer's private information is, on expectation, most valuable, and the resulting expected information rent highest, in the uniform case. Thus, it is generally less difficult to deter entry in the non-uniform case, where information rents are lower to begin with due to the prior distribution being more informative. There is, however, a countervailing effect that depends critically on the possibility

6 The Fully Sequential Mechanism

We have shown that auctions may attract only one buyer when resale is possible and the expected number of potential buyers is large, resulting in a zero revenue for the initial seller. The BK/Fishman sequential mechanism may generate positive revenue in these circumstances, but it also has a perfect sequential equilibrium in which zero revenue is generated. We will now show that a variant of the sequential mechanism, the *fully sequential mechanism*, can robustly generate positive revenue. Moreover, when the number of potential buyers is large, the fully sequential mechanism is approximately optimal.

The mechanism we describe requires commitment on part of the initial seller to not "come back" to a buyer who was outbid by a rival. It also requires that the initial seller is able to restrict, via contract, resale among the participating buyers (but not resale to buyers who do not participate in the initial mechanism). We will discuss these assumptions and requirements in more detail in Section 6.2 below. At the same time, the fully sequential mechanism does *not* use a reserve price, minimum bid, posted price, entry subsidy, or the like. It relies solely on competition between the buyers to generate revenue, but organizes this competition in a way that leaves a positive (if small) expected information rent to every buyer who does enter.

For simplicity, we consider here a very simple mechanism in which the original seller involves only the first two buyers. The seller first invites the first buyer to make an offer, j_1. If this buyer does not enter, the seller retains the object. If buyer 1 enters and bids j_1, the bid is publicly announced he second buyer is invited to make an offer, j_2. If the second buyer enters and submits offer $j_2 \geq j_1$, he wins the object and pays j_2.[19] He can then consume the object or resell it to any buyer other than buyer 1. In this case, the first buyer has no opportunity to come back and react to the second buyer's offer or to compete with the second buyer head-to-head, as would be the case in the Fishman/BK mechanism. If the second buyer does not enter, or if he enters and submits offer $j_2 < j_1$, the first buyer wins and pays j_1. He can then consume the object or resell it (to any buyer).

6.1 Equilibrium

Strategies are simple: Bidder 1 must make an entry decision $e_1 \in \{0, 1\}$ and, after entry, a jump bidding decision $j_1(v_1)$ that is a function of bidder 1's valuation. Buyer 2 must make an entry decision $e_2(j_1)$ that is conditioned on buyer 1's jump bid, and a bidding decision $j_2(v_2, j_1)$ that is conditioned on buyer 2's valuation and buyer 1's bid. Unlike in the BK/Fishman sequential mechanism, strategies do not include any continuous components $s_1(\cdot)$ or $s_2(\cdot)$.

Also unlike in the BK/Fishman mechanism, buyer 2 does not care about the distribution of v_1 conditional on observing 1's bid. This is because buyer 1 cannot react to whatever bid buyer 2 makes, and buyer 2 cannot resell the object to buyer 1 should he win. Thus, we do not need to consider buyer 2's belief buyer 1's type in the fully sequential mechanism, and subgame perfect equilibrium is the appropriate solution concept.

of resale. When buyers are likely to have low private valuations (e.g., if $F(v) = 2v - v^2$), the resale market is less profitable, and the winner of the initial mechanism becomes more inclined to consume the object instead of reselling it. This effect slows down the rate at which effective valuations become compressed when α increases. It is then possible that entry is deterred for larger α-values than in the uniform case, even though information rents are higher in the uniform case without resale. In Table 1, this happens in the last two columns for $F(v) = 2v - v^2$.

[19] The only reason we assume that buyer 2 wins if he offers at least the amount buyer 1 offered (instead of strictly more) is that in this case buyer 2 always has a well-defined best response to 1's offer. The assumption could easily be relaxed, either by permitting almost-best responses in equilibrium or by requiring offers to be integer multiples of some small currency unit.

Proposition 8. *Assume the initial seller uses the fully sequential mechanism in which only the first two buyers get to participate. Suppose further that the initial seller can restrict resale from buyer 2 to buyer 1, should buyer 2 win. The following holds in subgame perfect equilibrium of this mechanism (regardless of F, c, and $\mathcal{N}$):*

(i) *Buyer 1 enters ($e_1 = 1$) and bids some amount $j_1(v_1) \in [z_2(0), \hat{r}]$.*

(ii) *Buyer 2 enters if and only if $j_1 < \hat{r}$:*

$$e_2(j_1) \;=\; \begin{cases} 1 & \textit{if } j_1 < \hat{r}, \\ 0 & \textit{otherwise.} \end{cases}$$

If buyer 2 enters, he bids

$$j_2(v_2, j_1) \;=\; \begin{cases} j_1 & \textit{if } z_2(v_2) > j_1, \\ 0 & \textit{otherwise.} \end{cases}$$

The initial seller's revenue is at least $z_2(0)$, and is approximately equal to $\hat{r}$ for large $\mathcal{N}$.

To prove the result consider buyer 2's strategy first. Note that, since 2 can resell to buyers $3, 4, \ldots$ but not to buyer 1, the most buyer 2 is willing to pay for the object is $z_2(v_2)$; thus, 2's bidding strategy is optimal.[20] Now turn to buyer 2's entry decision. At the time 2 must make this decision, he effectively faces a take-it-or-leave-it offer j_1 but does not know v_2 yet. The definition of $\hat{r}$ and Lemma 2 imply that buyer 2 strictly prefers entry if and only if $j_1 < \hat{r}$. Thus, buyer 2's entry strategy is optimal.

Now consider 1's decision, anticipating 2's strategy. Note first that buyer 1 enters: If he does not enter, 1's payoff is zero. If he enters, he can obtain a positive net surplus if he makes the following bid:[21]

$$j_1 \;=\; \begin{cases} \hat{r} & \text{if } v_1 \geq \hat{r}, \\ \dfrac{z_2(0) + z_2(v_1)}{2} & \text{otherwise.} \end{cases} \tag{11}$$

If $v_1 \geq \hat{r}$ and buyer 1 bids $\hat{r}$, then buyer 2 does not enter and 1 wins for a payment of $\hat{r}$. By definition of $\hat{r}$, this possibility is sufficient to yield exactly a zero net profit from entering. If $v_1 < \hat{r}$, then the bid $[z_2(0) + z_2(v_2)]/2$ induces entry by buyer 2; however, buyer 1 still wins with positive probability and, if he wins, pays a price that is strictly less than his effective valuation. Thus, strategy (11) generates an expected positive net surplus from entering, and it follows that buyer 1 enters. Conditional on buyer 1 entering, we observe that buyer 1 never bids $j_1 < z_2(0)$, for any v_1. The reason is that buyer 2 will enter and, with probability one, will win. At the same time, buyer 1 also never bids $j_1 > \hat{r}$: Any bid of at least $\hat{r}$ deters entry by bidder 2, and since buyer 1 must pay his bid, any offer $j_1 > \hat{r}$ is strictly dominated by a jump bid of $j_1' = \hat{r}$. Thus, it follows that $j_1(v_1) \in [z_2(0), \hat{r}]$.

Finally, since buyer 1 enters with probability 1 and makes jump bid $j_1(v_1) \in [z_2(0), \hat{r}]$, the initial seller's revenue is at least $z_2(0)$; $z_2(0) \approx \hat{r}$ for large $\mathcal{N}$ by Lemma 3. $\square$

[20] Buyer 2 may want to bid zero and wait and see if buyer 1 attempts to resell the object. But buyer 1 would not have made an offer to buy the object for j_1 if his resale strategy was to resell the object for an expected price of j_1 or less. Thus, the expected surplus buyer 2 can achieve by waiting for the resale market must be strictly less than $v_2 - j_1$, which in turn is less than $z_2(v_2) - j_1$.

[21] This is not necessarily buyer 1's equilibrium bidding strategy; strategy (11) is examined only to demonstrate that entry is profitable for buyer 1 conditional on buyer 2's strategy.

6.2 Discussion

The result in Proposition 8 should not be confused with BK's finding that, in the no-resale case, the sequential mechanism may generate more revenue than the auction if the set of potential buyers is large. Without resale, the potential advantage of dealing with buyers sequentially is that it permits the flexibility to generate additional entry if previous buyers have low valuations. This effect is mechanical and translates into an *expected* revenue advantage over the auction only under contrived assumptions about the value distribution and the entry cost (see BK, p. 1558). If resale is possible and there is a large stream of potential buyers, the fully sequential mechanism beats the auction for strategic reasons, and for all value distributions and all entry costs.

The advantage of the fully sequential mechanism stems from the fact that no buyer has the opportunity to revise an earlier offer. This means that, conditional on entry, the first buyer must bid at least $z_2(0)$ to have a chance at winning, so the seller can be assured to receive at least one jump bid for an amount $z_2(0)$ or higher. Table 2 displays the value $z_2(0)$ as a percentage of $\hat{r}$, for the parameter combinations in Example 2 and assuming that the buyer arrival rate α is equal to the corresponding thresholds at which entry by the second bidder would be deterred in the auction or sequential mechanism (see Table 1). Therefore, the numbers in Table 2 are lower bounds on the performance of the fully sequential mechanism in those cases where the auction or sequential mechanism result in a zero revenue.[22]

For the fully sequential mechanism to yield this revenue, two requirements must be fulfilled, which we now discuss. First, the initial seller must commit to not give any buyer the opportunity to revise an offer once a better offer by a competing buyer is received. There are several ways in which such commitment can be achieved in practice. The seller may try to develop a reputation for not renegotiating previous offers. If the seller is a large organization, it may explicitly instruct its agents to only deal with one buyer at a time; or it could limit the number of agents authorized to negotiate with buyers, which makes repeated bargaining with the same buyers less likely.

Table 2: Lower bounds on the fully sequential mechanism's revenue (as a percentage of $\hat{r}$) in Example 2, in cases where the auction/sequential mechanism yield zero revenue.

	Entry cost: $c =$			
Distribution:	0.100	0.075	0.050	0.025
$F(v) = v$	69.0%	82.5%	91.7%	97.5%
$F(v) = 2v - v^2$	56.6%	78.2%	90.9%	97.7%
$F(v) = v^2$	46.4%	69.9%	85.7%	95.6%
$F(v) = v^3$	14.6%	52.7%	77.5%	93.2%

[22]The numbers are lower bounds for two reasons: First, Proposition 8 states that the initial seller receives at least $z_2(0)$. In general, revenue will be more than that, as was illustrated already in our motivating Example 1 earlier in the paper. Second, the numbers are computed for α being exactly equal to the entry deterrence thresholds in Table 1. If α is larger than this threshold, $z_2(0)$ will increase.

Second, if the initial seller awards the objects to buyer 2, this sale must involve the condition that buyer 2 does not resell the object to buyer 1. To see why, note that in equilibrium buyer 2's entry is deterred if buyer 1 places a jump bid of $\hat{r}$. Moreover, as shown above, the ability to deter 2's entry with a jump bid of $\hat{r}$ makes it (just) profitable for buyer 1 to enter. If buyer 2 could resell the object to 1, this would no longer be the case. The reason is the following: Since only types $v_1 \geq \hat{r}$ would bid $\hat{r}$ or higher, buyer 2, upon seeing $j_1 = \hat{r}$, must believe that $v_1 \geq \hat{r}$. Buyer 2 could now enter, purchase the object for $\hat{r}$, and attempt to sell it (via a take-it-or-leave-it offer) to buyer 1. Since buyer 1 has already paid the entry cost c and has already revealed that his valuation is above $\hat{r}$, buyer 2 will receive a positive expected net surplus from entry (whereas, without the resale opportunity, he would receive exactly a zero net surplus from entering). In turn, buyer 1 is no longer able to guarantee to win the object for a price of $\hat{r}$, and without this ability buyer 1's expected surplus from entering will be negative if $\mathcal{N}$ is large.

Thus, in order to receive the gains from any potential resale to buyers who do not participate in the mechanism, the initial seller must put some restrictions on resale among those buyers who do participate. The purpose of the restriction is to protect buyer 1 from opportunistic entry by buyer 2 and, therefore, preserve buyer 1's ability to make a preemptive bid that guarantees a win for a surplus just high enough to make entry by buyer 1 profitable. Whether the initial seller can impose such a condition depends on the nature of the asset being sold and on the legal environment. Seller-imposed restrictions on resale are common, for instance, in manufacturer-dealer relationships and in the licensing of intellectual property.

Notwithstanding these requirements, the fully sequential mechanism has several practical advantages over alternative selling mechanisms. Note that the initial seller could decide to use either the auction or the sequential mechanism, but subsidize entry to ensure that at least two buyers participate. However, in order to compute the entry subsidy the initial seller would have to know at least the value of the entry cost c, and possibly also the distribution of buyer valuations and their arrival probabilities. The fully sequential mechanism, on the other hand, does not require knowledge of these parameters on part of the initial seller. The fully sequential mechanism is also robust in another sense: It yields at least revenue $z_2(0)$, which is the pure resale value of the item. Thus, unless the number of buyers is exactly two, the mechanism generates positive revenue for the initial seller regardless of $\mathcal{N}$. While the auction and the sequential mechanism may generate higher revenue *on expectation*, they run the risk of not resulting in any revenue.

7 Conclusion

As discussed in Section 5.3, our results are driven by the effect that the possibility of resale compresses the distribution of each buyer's willingness to pay: Even those buyers with very low private values will be willing to pay at least the item's expected resale value. If all buyers have access to the resale market, this effect reduces the information rents that can be earned by competing buyers in the initial auction or sequential mechanism, and hence tends to reduce entry to the point where only a single buyer enters. By adopting the fully sequential mechanism, the initial seller can leverage competition between buyers in a way that still leaves enough surplus for the second buyer to enter.

In principle, similar results would obtain in any situation in which the willingness-to-pay distribution was sufficiently compressed and the seller was restricted to choose among auctions and sequential mechanisms—a resale market is only one application in which such a compressed distribution might arise. A different application might be the case in which the buyers are a group of similar firms that compete for an input—say, logging rights—and that face the same demand curve and are well (but not perfectly) informed about each other's production costs. If the seller—say, a government selling logging rights—is comparatively less well informed, it may be restricted to non-optimal mechanisms, which do not entail reserve prices and other variables that depend on the value distribution or entry costs. It is then possible, for the reasons as we analyzed in this paper, that an auctions or sequential mechanisms would fail to generate revenue.

The above notwithstanding, the possibility of resale is an important application as a resale market is a natural assumption in many settings in which the underlying value distribution is not *a priori* compressed. These include several scenarios not examined explicitly in this paper. For example, while we considered only the case where buyers learn their private value fully after entry but have no information about it prior to entry, in certain applications it may be more realistic to assume that potential buyers receive informative signals about their values before deciding to enter (see, e.g., Roberts and Sweeting 2013 for a model with this information structure). However, even buyers with the most pessimistic of signals will be willing to pay at least the pure resale value, $z_n(0)$, in the initial mechanism. If the resale market is large (in the sense that $\mathcal{N}$ is large), this pure resale value will be relatively high—and just like in the model examined here, this will result in a compression of the willingness-to-pay distribution in the initial mechanism.

On the other hand, there are also situations in which resale may *not* result in the same compression effect. For example, if bidders' valuations v_i are correlated, a buyer's private value provides a signal about the private value of other buyers, and hence about the object's expected resale value. If this correlation is positive, then a buyer with a low private value would expect other buyers to be more likely to also have low private values, which implies a low expected value in the resale market (and vice versa for buyers with high values). In this case, increasing the number of buyers who participate in the resale market on expectation would no longer guarantee that the effective valuation functions $z_n(\cdot)$ converge to the limiting function shown in the right graph in Figure 1. Consequently, an outcome in which the entry of all but one buyer is deterred (in the auction or sequential mechanism) may not arise, even if resale is possible and the resale market is large. Thus, there may be no need for the seller to adopt a fully sequential mechanism.

Proofs

Proof of Lemma 1

The part of Assumption 4 that matters for this result is *ex ante* individual rationality (EA-IR). Take any EA-IR resale mechanism that results in resale with probability q and has expected revenue R. By the revelation principle, without loss of generality we can assume that the mechanism is an EA-IR and incentive compatible (IC) direct revelation mechanism. We first show that a direct revelation mechanism exists that is EA-IR but not necessarily IC, is such that losing bidders pay exactly zero, sells with probability q, and generates revenue R if buyers report their valuations truthfully (Step 1 below). Second, we show that any direct mechanism that is EA-IR, is such that losing bidders pay exactly zero, and sells with probability q generates at most $q\hat{r}$ in expected revenue if buyers report their valuations truthfully (Step 2 below). The result that $R \leq q\hat{r}$ then follows.

Step 1. Existence of the direct revelation mechanism

Consider a buyer j who arrives, observes information set I_j and then enters and learns valuation v_j. Denote by $q_j(I_j, v_j)$ the probability with which j wins the object conditional on his post-entry information (I_j, v_j), and let $m_j(I_j, v_j)$ be the expected payment j makes conditional on (I_j, v_j) (net of any entry fees charged, or entry subsidies paid, by the reseller). Since the mechanism is EA-IR, we have for all j and I_j

$$\int_0^{\overline{v}} q_j(I_j, v_j)v_j - m_j(I_j, v_j)dF(v_j) \;\geq\; c.$$

We will assume that $\int_0^{\overline{v}} q_j(I_j, v_j)dF(v_j) > 0$ for every buyer j who enters. This is without loss of generality: A buyer who expects a zero probability of winning would only enter if he paid $-c$ or less on expectation. We could then replace the mechanism with one in which this buyer pays zero and does not enter,[23] which would increase the reseller's expected revenue. Since our goal is to establish an upper bound on the reseller's revenue, we may assume that every buyer who does enter has a strictly positive probability of winning the object (conditional on the buyer's pre-entry information).

We now construct a new direct revelation mechanism with the same allocation rule as the first mechanism. The payment that bidder j makes in the new mechanism depends on (I_j, v_j) and whether j wins or loses, but not on any other variables, and is given by

$$x_j(I_j, v_j) \;=\; \begin{cases} 0 & \text{if } j \text{ loses,} \\ \displaystyle\int_0^{\overline{v}} m_j(I_j, t)dF(t) \Big/ \int_0^{\overline{v}} q_j(I_j, t)dF(t) & \text{if } j \text{ wins.} \end{cases}$$

(This is well-defined because $\int_0^{\overline{v}} q_j(I_j, v_j)dF(v_j) > 0$ for every buyer j who enters.) Conditional on the pre-entry information I_j, a bidder's expected payoff from entering is

$$\begin{aligned} \int_0^{\overline{v}} q_j(I_j, v_j)\left[v_j - x_j(I_j, v_j)\right]dF(v_j) \;&=\; \int_0^{\overline{v}} q_j(I_j, v_j)\left[v_j - \frac{\int_0^{\overline{v}} m_j(I_j, t)dF(t)}{\int_0^{\overline{v}} q_j(I_j, t)dF(t)}\right]dF(v_j) \\ &=\; \int_0^{\overline{v}} q_j(I_j, v_j)v_j - m_j(I_j, v_j)dF(v_j), \end{aligned}$$

which is the same as in the first mechanism. Hence, the new mechanism is still EA-IR and results in the same entry decisions as the first mechanism; and because the allocation rule is the same it results in selling probability q. Since the expected payments entering bidders make are the same as well, this new mechanism will have expected revenue R if buyers report their valuations truthfully.[24]

It follows that for every EA-IR and IC direct revelation resale mechanism that sells with probability q and yields expected revenue R, there exists an EA-IR (but not necessarily IC) direct revelation resale mechanism which sells with probability q, generates expected revenue R if buyers report their valuations truthfully, and in which losing bidder pay exactly zero.

[23] If it is necessary for the operation of the mechanism, the reseller can always "simulate" the presence of buyer j by drawing a random value v_j from F.

[24] Since the mechanism is not IC, this revenue is hypothetical: If the mechanism was actually used, buyers would misrepresent their information strategically and thereby reduce the revenue earned by the seller.

Step 2. Revenue bound in the direct revelation mechanism

Next, we show that every EA-IR direct revelation mechanism that sells with probability q and in which losing bidders pay exactly zero can at most have revenue $q\hat{r}$ if buyers report their valuations truthfully. Consider buyer j's entry decision, having information I_j but not knowing his valuation v_j. Let λ be the probability with which j expects to win if he enters, and let τ be the expected payment j makes if he enters and wins. By EA-IR, buyer j enters if and only if

$$\lambda\big(E\big[v_j\big|I_j, j \text{ wins}\big] - \tau\big) \;\geq\; c. \tag{12}$$

For given λ, $E\big[v_j\big|I_j, j \text{ wins}\big]$ is maximized if the allocation rule of the mechanism is such that j wins the object whenever $v_j \geq F^{-1}(1-\lambda)$. In this case

$$\lambda E\big[v_j\big|I_j, j \text{ wins}\big] \;=\; \int_{F^{-1}(1-\lambda)}^{\overline{v}} v dF(v),$$

and it follows that, for given λ, the maximum τ that satisfies condition (12) is

$$\tau(\lambda) \;=\; \frac{1}{\lambda}\left[\int_{F^{-1}(1-\lambda)}^{\overline{v}} v dF(v) - c\right]. \tag{13}$$

We need the following result (which we state as a formal Lemma for later reference):

Lemma 9. *The expression in* (13) *is no larger than* $\hat{r}$, *where* $\hat{r}$ *is defined in* (1).

Proof. The first-order condition for a maximum of (13) is

$$\begin{aligned}\tau'(\lambda) \;&=\; -\frac{1}{\lambda^2}\left[\int_{F^{-1}(1-\lambda)}^{\overline{v}} v dF(v) - c\right] - \frac{1}{\lambda}\left[F^{-1}(1-\lambda) f\big(F^{-1}(1-\lambda)\big)\frac{d}{d\lambda}F^{-1}(1-\lambda)\right] \\ &=\; -\frac{1}{\lambda^2}\left[\int_{F^{-1}(1-\lambda)}^{\overline{v}} v dF(v) - c\right] + \frac{1}{\lambda}F^{-1}(1-\lambda) \;=\; 0,\end{aligned}$$

which can be rearranged to

$$\int_{F^{-1}(1-\lambda)}^{\overline{v}} v - F^{-1}(1-\lambda)\, dF(v) - c \;=\; 0. \tag{14}$$

Using (1), (14) implies $F^{-1}(1-\lambda) = \hat{r}$, or $\lambda = 1 - F(\hat{r})$. Plugging this back into (13), and using (1) again, we have

$$\tau(1 - F(\hat{r})) \;=\; \frac{1}{1 - F(\hat{r})}\left[\int_{\hat{r}}^{\overline{v}} v dF(v) - c\right] \;=\; \frac{1}{1 - F(\hat{r})}\int_{\hat{r}}^{\overline{v}} \hat{r} dF(v) \;=\; \hat{r}. \qquad \square$$

Returning now to the task of proving Lemma 1, we conclude: Conditional on winning, the expected payment made by a buyer in this mechanism is at most $\hat{r}$. Since losing buyers do not pay, the expected revenue of this mechanism is at most $q\hat{r}$. $\square$

Proof of Lemma 2

We begin by translating the definition of effective valuations (2) into a more convenient form.

Note that for any $(Q, X) \in \Omega$ and all n, if $Q_n = 0$ then $X_n \leq 0$. This follows from the EA-IR constraint in assumption A4: Buyers would not pay the entry cost c to participate in a mechanism in which they had a zero chance of winning, unless $X_n < 0$. However, since $(0,0) \in \Omega$ by assumption A3, if the reseller chooses a mechanism with $Q_n = 0$ he would never set $X_n < 0$. Hence we can assume, without loss of generality, that $Q_n = 0$ implies $X_n = 0$. Now define

$$Y_n \;\equiv\; \begin{cases} X_n/Q_n & \text{if } Q_n > 0, \\ 0 & \text{if } Q_n = 0, \end{cases}$$

and let $\widetilde{\Omega} \equiv \big\{(Q, Y) : (Q, X) \in \Omega\big\}$. We can then restate (2) as follows:

$$\begin{aligned} z_n(v_i) &= \max_{(Q,X)\in\Omega} \big\{X_n + (1 - Q_n)v_i)\big\} && \text{original definition (2)} \\ &= \max_{(Q,Y)\in\widetilde{\Omega}} \big\{Q_n Y_n + (1 - Q_n)v_i)\big\}. && (15) \end{aligned}$$

In other words, we can focus (without loss of generality) on resale mechanisms that generate non-zero revenue only in the event that the object is resold. Assumption A4 guarantees that $\max_{(Q,X)\in\Omega}\{X_n + (1 - Q_n)v_i\}$ exists for all n; hence $z_n(v_i)$ is well-defined.

With this simplification, we now proceed to establish properties (i)–(iv) of the result.

Property (i). Take some $v_i' \in [0, \overline{v}]$ and let

$$(Q', Y') \;=\; \arg\max_{(Q,Y)} \big\{Q_n Y_n + (1 - Q_n)v_i'\big\}.$$

If v_i is another valuation, with $v_i > v_i'$, then we have

$$\begin{aligned} z_n(v_i) \;=\; \max_{(Q,Y)} \big\{Q_n Y_n + (1 - Q_n)v_i\big\} \;&\geq\; Q_n' Y_n' + (1 - Q_n')v_i \\ &\geq\; Q_n' Y_n' + (1 - Q_n')v_i' \;=\; \max_{(Q,Y)} \big\{Q_n Y_n + (1 - Q_n)v_i'\big\} \;=\; z_n(v_i'), \quad (16) \end{aligned}$$

which shows that z_n is weakly increasing. To show that it is strictly increasing, we will argue that $Q' < 1$ for all $v_i' > 0$. (This implies that the second inequality in (16) is strict whenever $v_i > v_i' > 0$; since we already know that z_n is weakly increasing over $[0, \overline{v}]$, strict monotonicity follows.)

Since $\rho_j < 1$ for some j, there is a strictly positive probability that the arrival of buyers breaks down, and hence a strictly positive probability that $v_j < v_i'$ for all existing buyers $j > n$. Consider this event. By the interim individual rationality (I-IR) constraint in assumption A3, conditional on a buyer's entry decision and private valuation, the buyer's expected payment cannot exceed this valuation times the buyer's probability of winning. Thus, expected resale revenue must be strictly less than v_i'. However, a reseller with valuation v_i' would never choose a mechanism that yielded expected revenue strictly below v_i': He would prefer to either not sell the object or make a series of take-it-or-leave it offers at price v_i'; by assumption A2 both are possible. Thus, for a reseller with private valuation $v_i' > 0$, it cannot be optimal to choose a resale mechanism that results in resale with (unconditional) probability one.

To show convexity of z_n, take $\alpha \in (0,1)$ and observe that

$$\begin{aligned}
z_n(\alpha v_i + (1-\alpha)v_i') &= \max_{(Q,Y)} \big\{Q_n Y_n + (1-Q_n)(\alpha v_i + (1-\alpha)v_i')\big\} \\
&\leq \alpha \max_{(Q,Y)} \big\{Q_n Y_n + (1-Q_n)v_i\big\} \\
&\qquad\qquad + (1-\alpha) \max_{(Q,Y)} \big\{Q_n Y_n + (1-Q_n)v_i')\big\} \\
&= \alpha z_n(v_i) + (1-\alpha) z_n(v_i').
\end{aligned}$$

Therefore, z_n is weakly convex.

Property (ii). Note that $Q_n = Y_n = 0$ is a feasible choice (i.e., by assumption A2, a buyer who wins the object has the option to not resell it), which implies $z_n(v_i) \geq v_i$. Since z_n is strictly increasing, we have $z_n(v_i) > z_n(0)$. Thus, $z_n(v_i) \geq \max\{z_n(0), v_i\}$.

Property (iii). Note that Lemma 1 implies $X_n = Q_n Y_n \leq Q_n \hat{r}$ for all n. Thus $Y_n \leq \hat{r}$, which implies that for $v_i > \hat{r}$ the optimal reselling mechanism is such that $Q_n = Y_n = 0$; by assumption A2 such a mechanism is available. It follows that $z_n(v_i) \leq \max\{\hat{r}, v_i\}$.

Property (iv). Recall the following properties: $z_n(v_i) \geq v_i$ (from (ii)), $z_n(\overline{v}) = \overline{v}$ (from (ii) and (iii)), and z_n increasing and convex (from (i)). Together, these properties imply that z_n is a (weak) contraction: $z_n(v_i) - z_n(v_i') \leq v_i - v_i'$ for $v_i > v_i'$. □

Proof of Lemma 3

Suppose $\rho^t \to (1,1,1,\ldots)$ pointwise. For every $\alpha \in (0,1)$ and every integer $k > n$, there exists $T(\alpha,k) < \infty$ such that $\rho_i^t > \alpha$ for all $i < k$ and $t > T(\alpha,k)$. Fix any such $t > T(\alpha,k)$. If the reseller can make a take-it-or-leave-it offer at posted price $\hat{r}$ to every $i > n$ (which is possible by assumption A3), his expected revenue is at least

$$\begin{aligned}
&\overbrace{\alpha\big(1-F(\hat{r})\big)\hat{r} + F(\hat{r})\Big[\alpha\big(1-F(\hat{r})\big)\hat{r} + F(\hat{r})\Big[\alpha\big(1-F(\hat{r})\big)\hat{r} + \ \ldots \ \Big]\Big]}^{k-n \text{ iterations}} \\
&\qquad\qquad\qquad = \hat{r}\alpha\big(1-F(\hat{r})\big)\frac{1-F(\hat{r})^{k-n}}{1-F(\hat{r})} = \hat{r}\alpha\big(1-F(\hat{r})^{k-n}\big).
\end{aligned}$$

At the same time, by Lemma 1 the reseller's expected revenue can never be more than $\hat{r}$. Thus, for $t > T(\alpha,k)$, $\hat{r}\alpha\big(1-F(\hat{r})^k\big) \leq z_n^t(0) \leq \hat{r}$. The term on the left-hand side becomes arbitrarily close to $\hat{r}$ as $\alpha \to 1$ and $k \to \infty$. It follows that $z_n^t(0) \to \hat{r}$ as $t \to \infty$; this establishes (i). Statement (ii) now follows from (i) and the fact that $\max\{z_n(0), v_i\} \leq z_n(v_i) \leq \max\{\hat{r}, v_i\}$. Statement (iii) follows from (i), (ii), and the definition of $\hat{r}$ in (1). □

Proof of Proposition 4

Suppose entry strategies and post-entry beliefs are given by (3) and (4), respectively. Let E be the set of bidders who entered the auction, with $|E| = n$

If $n < n^*$ and $\rho_{n+1} < 1$, the buyers in E believe that it has a zero probability that a potential buyer exists who is not in E already. In this case, the auction is a standard second price auction with symmetrically distributed independent private values, and it is a dominant strategy profile to bid these values. This gives $b_i(v_i, n) = v_i$.

In all other cases, the buyers in E believe that at least one potential buyer not in E exists with probability ρ_{n+1}. Suppose, for a moment, that no resale takes place among agents in E. Fix some $i \leq n$, let $\overline{b}_{-i}$ be the highest bid submitted by bidders $j \neq i$, and let G be the distribution of $\overline{b}_{-i}$. Bidder i's expected payoff from bidding b_i when his valuation is v_i can be expressed as

$$U_i(b_i|v_i) \;=\; \int_0^{b_i} z_n(v_i) - \overline{b}_{-i}\; dG(\overline{b}_{-i}),$$

and this is maximized if i submits bid $b_i = z_n(v_i)$. Since z_n is strictly increasing, if all bidders bid their effective valuations the object is allocated to the bidder with the highest private valuation, which implies that there can be no resale among the buyers in E, as was posited initially. This gives $b_i(v_i, n) = v_i$, subject to one caveat:

Buyer i may still deviate from this bidding strategy *and* change his resale strategy, in a way that involves the bidders in E. We show that this cannot benefit i. Suppose all buyers $j \in E\backslash i$ bid their effective valuations $z_n(v_j)$. If buyer i deviates this strategy and, as a result of the deviation, wins even though he does not have the highest valuation, he pays $\max_{j\in E\backslash i} z_n(v_j)$ to the original seller. But this is the most any bidder in $E\backslash i$ would be willing to pay to acquire the object from i in the aftermarket, so i cannot strictly benefit from the deviation. Similarly, if buyer i deviates and does not win even though he has the highest valuation, and then tries to acquire the object in the aftermarket, he would have to pay at least $\max_{j\in E\backslash i} z_n(v_j)$ to the winner of the auction. But this is the same amount i would have had to pay to the original seller if he had won in the auction, so again i cannot strictly benefit from the deviation either. □

Proof of Proposition 5

Take a sequence $\rho^t \to (1, 1, 1, \ldots)$ pointwise. For a given n, let z_n^t, π_n^t, and $\overline{\pi}_n^t$ be a buyer's effective valuation, expected payoff from participating in the auction with n bidders, and expected payoff from not participating in the auction, associated with the t^{th} sequence. By Lemma 3 (ii), $\lim_t z_n^t(v_i) = \max\{\hat{r}, v_i\}$ uniformly. Since (6) is continuous in z_n, we have

$$\begin{aligned}
\lim_{t\to\infty} \pi_n^t &= \int_0^{\overline{v}} \int_0^{v} \max\{\hat{r}, v\} - \max\{\hat{r}, w\}\, dF(w)^{n-1} dF(v) - c \\
&= \int_{\hat{r}}^{\overline{v}} \left(F(\hat{r})^{n-1}(v - \hat{r}) + \int_{\hat{r}}^{v} v - w\, dF(w)^{n-1} \right) dF(v) - c,
\end{aligned}$$

which is strictly decreasing in n. For $n = 2$ we have

$$\begin{aligned}\lim_{t\to\infty} \pi_2^t &= \int_{\hat{r}}^{\overline{v}} \left(F(\hat{r})(v-\hat{r}) + \int_{\hat{r}}^{v} v - w\, dF(w) \right) dF(v) - c \\ &< \int_{\hat{r}}^{\overline{v}} \big(F(\hat{r})(v-\hat{r}) + (1-F(\hat{r}))(v-\hat{r}) \big) dF(v) - c \\ &= \int_{\hat{r}}^{\overline{v}} v - \hat{r}\, dF(v) - c \;=\; 0,\end{aligned}$$

where the final equality follows from the definition of $\hat{r}$ in (1). This implies that for t sufficiently large, $\pi_n^t < 0 \le \overline{\pi}_n^t$ for all $n > 1$. On the other hand, $\pi_1^t \ge \overline{\pi}_1^t = 0$ for all t ($\pi_1^t > 0$ because bidder 1 wins and pays a zero price; $\overline{\pi}_1^t = 0$ because there is no resale market if 1 does not enter). Stated in equivalent terms: For $\mathcal{N}$ sufficiently large,

$$\pi^{AU}(1,1) \;>\; 0 \;>\; \pi^{AU}(2,2) \;>\; \pi^{AU}(3,3) \;>\; \ldots,$$

so $n^* = 1$, which means the seller receives a zero price. □

Proof of Lemma 6

If bidder $n+1$ enters, then he and incumbent compete in an ascending auction. Let v^I denote the incumbent's private value and suppose bidder $n+1$ beliefs that v^I is distributed according $\mu_{n+1}(v^I) = F(v^I | v^I \ge y)$ for some $y \in [0, \overline{v}]$. Bidder $n+1$ gains at most $z_{n+1}(v_{n+1})$ if he wins against the incumbent in the ascending auction. Furthermore, since the incumbent will raise the price at least to v^I, if $n+1$ wins he must pay at least $\max\{p_n, v^I\} \ge \{z_n(0), v^I\}$ (since $p_n \ge z_n(0)$ by assumption). Thus, an upper bound on the expected surplus that $n+1$ achieves if he enters is

$$\begin{aligned}&\int_{z_{n+1}^{-1}(p_n)}^{\overline{v}} \int_{y}^{v_{n+1}} \Big(z_{n+1}(v_{n+1}) - \max\big\{ z_n(0), v^I \big\} \Big) d\mu_{n+1}(v^I) dF(v_{n+1}) \;-\; c \\ =\;& \frac{1}{1-F(y)} \int_{z_{n+1}^{-1}(p_n)}^{\overline{v}} \int_{y}^{v_{n+1}} \Big(z_{n+1}(v_{n+1}) - \max\big\{ z_n(0), v^I \big\} \Big) dF(v^I) dF(v_{n+1}) \;-\; c \\ \le\;& \int_0^{\overline{v}} \int_0^{v_{n+1}} \Big(z_{n+1}(v_{n+1}) - \max\big\{ z_n(0), v^I \big\} \Big) dF(v^I) dF(v_{n+1}) \;-\; c \\ \le\;& \int_0^{\hat{r}} \int_0^{v_{n+1}} \big(\hat{r} - z_n(0) \big) dF(v^I) dF(v_{n+1}) \;+\; \int_{\hat{r}}^{\overline{v}} \int_0^{\hat{r}} \big(v_{n+1} - z_n(0) \big) dF(v^I) dF(v_{n+1}) \\ &+ \int_{\hat{r}}^{\overline{v}} \int_{\hat{r}}^{v_{n+1}} \big(v_{n+1} - v^I \big) dF(v^I) dF(v_{n+1}) \;-\; c \\ <\;& \int_0^{\hat{r}} \int_0^{\hat{r}} \big(\hat{r} - z_n(0) \big) dF(v^I) dF(v_{n+1}) \;+\; \int_{\hat{r}}^{\overline{v}} \int_0^{\hat{r}} \big(v_{n+1} - z_n(0) \big) dF(v^I) dF(v_{n+1}) \\ &+ \int_{\hat{r}}^{\overline{v}} \int_{\hat{r}}^{\overline{v}} \big(v_{n+1} - \hat{r} \big) dF(v^I) dF(v_{n+1}) \;+\; \int_{\hat{r}}^{\overline{v}} \int_{\hat{r}}^{\overline{v}} \big(\hat{r} - v^I \big) dF(v^I) dF(v_{n+1}) \;-\; c. \qquad (17)\end{aligned}$$

If $\mathcal{N} \to \infty$ then $\rho \to (1, 1, 1, \ldots)$, and Lemma 3 implies that the right-hand side in (17) converges to

$$
\begin{aligned}
& F(\hat{r})^2(\hat{r} - \hat{r}) \;+\; F(\hat{r}) \int_{\hat{r}}^{\overline{v}} (v_{n+1} - \hat{r}) dF(v_{n+1}) \;+\; (1 - F(\hat{r})) \int_{\hat{r}}^{\overline{v}} (v_{n+1} - \hat{r}) dF(v_{n+1}) \\
& \qquad \;+\; \int_{\hat{r}}^{\overline{v}} \int_{\hat{r}}^{\overline{v}} \left(\hat{r} - v^I\right) dF(v^I) dF(v_{n+1}) \;-\; c \\
&= \;\; 0 \;+\; F(\hat{r})c \;+\; (1 - F(\hat{r}))c \;+\; \int_{\hat{r}}^{\overline{v}} \int_{\hat{r}}^{\overline{v}} \left(\hat{r} - v^I\right) dF(v^I) dF(v_{n+1}) \;-\; c \\
&= \;\; \int_{\hat{r}}^{\overline{v}} \int_{\hat{r}}^{\overline{v}} \left(\hat{r} - v^I\right) dF(v^I) dF(v_{n+1}) \;\; < \;\; 0.
\end{aligned}
$$

It follows that bidder $n + 1$ should not enter for $\mathcal{N}$ sufficiently large. □

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