Supply Ambiguity and Market Fragility*

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ABSTRACT

This paper examines how ambiguity about asset supply affects the pricing, trades, and stability of equilibria. Ambiguity-averse investors learn less from the market price that equilibrates demand and supply. As this makes it more difficult to coordinate equilibrium beliefs, new equilibria emerge in which investors behave like almost ignoring information in price. As a result, the price reflects less supply fluctuation and thus the market paradoxically looks liquid. However, such equilibria are unstable as ignorance turns a blind eye to the real problem. With order fragmentation, equity markets are increasingly becoming subject to these issues. I illustrate these points empirically.

Uncertainty, as well as risk, influences decision making in financial markets. While finance theory has extensively analyzed the trade-off between risk and return, it is not well understood how uncertainty about the sources of risk, or ambiguity for short, can alter such a relation. This paper aims to show that ambiguity about the supply of assets can have a first-order impact on asset pricing, investor trading, and market fragility.

The uncertainty under consideration is one about the process that generates risk, and has a clear distinction from the risk itself. An event is risky if it has a probabilistic distribution. When the probabilistic distribution is uncertain, the event is said to be ambiguous.[1] We are particularly interested in ambiguity about the supply of assets, which has attracted little attention to date. Consider the case in which supply not only is random, but also originates from an uncertain source. For example, investors often lack information about their counterparty, or who is supplying equity to them. They may be accommodating the demand of rational informed traders, strategic liquidity traders, mechanical noise traders, or even an illegal spoofer. If investors are averse to ambiguity, decision making will depend on the extent to which these scenarios are known or unknown, or supply is ambiguous or unambiguous.

The ambiguity can be modeled as parameter uncertainty. If an investor is certain that his counterparty is a noise trader, he can expect the supply he absorbs to average zero. In this case, he might model the random supply to have a zero mean and known variance (no ambiguity, some risk). However, if the investor suspects that insiders may also be trading, there is another scenario in which he would wish to accommodate a range of non-zero supply (some ambiguity, plus risk). Notice that the set of his subjective belief about the mean of supply has expanded from a point (zero) to a domain over which a probabilistic distribution is defined. This increase in the belief set reflects the loss of confidence in the nature of his counterparty. As a result, if the investor is averse to ambiguity, he will require an additional compensation to close the trade on top of the ordinary risk premium. Such an ambiguity premium is delivered as an extra discount in the current price that equilibrates demand and supply. This discount becomes larger as the demand curve gets steeper.

[1] In this paper, I use the terms ambiguity (Ellsberg, 1961) and (Knightian) uncertainty (Knight, 1921) interchangeably.

The steepening of the demand curve also has important implications on the stability of an equilibrium and the liquidity of the market. The demand schedule one submits represents a trading strategy under his subjective belief. The stability of the equilibrium depends on how an investor responds to a change in other investors' beliefs and hence strategies. Suppose an investor steepens his demand curve due to ambiguity about his counterparty. This will cause a downward pressure in the market price, which other investors will perceive if they learn from the price. As a result, they may likewise become aware of possible insider trading and respond by steepening their demand schedules. Confirming his concern, the original investor will demand even less at the same price. This diverging loop implies that the equilibrium may collapse upon a small perturbation of beliefs. This process involves an illiquidity spiral, because an asset traded on a steeper demand curve is more sensitive to a unit change in supply and hence more illiquid. The resulting market is unstable and fragile. All these effects can look anomalous because theory without ambiguity cannot explain these phenomena. Note that there may be no insider after all. It is the loss of investor confidence that could shut down the market.[2]

To analyze these issues, I propose an equilibrium model with supply ambiguity and investor learning that encompasses prominent rational expectations equilibrium (REE) models as special cases. Supply ambiguity is modeled as uncertainty about the mean of the random supply. Investors have the smooth ambiguity-aversion preference proposed by Klibanoff, Marinacci, and Mukerji (2005). Informed investors receive private signals about the future dividend, while uninformed investors do not. Both types of investors can learn from private endowment as well as the market price that equilibrates demand and supply. Thus, supply ambiguity makes the price signal also ambiguous, which hinders coordination of equilibrium beliefs.

I show that at least one equilibrium always exists except for the knife-edge case in which informed investors receive both perfectly heterogeneous private signals and informative endowment. This equilibrium exhibits high volatility, aggressive trading, low liquidity, and a large price deviation from the fundamental value, implying high supply pressure. As usual with REE models, there may be multiple equilibria. I find that ambiguity-tolerant investors tend to allow

[2]See Ilut and Schneider (2011) for an example in a macroeconomic context.

more liquid, less volatile, but unstable equilibria to co-exist under supply ambiguity. In these unstable equilibria, the price loads less negatively on supply, and therefore is more informative. This facilitates coordination of beliefs among investors through the common price signal, which makes it easier for them to reach an equilibrium. However, the equilibrium is less easy to sustain. The reliance on the common price signal induces strategic complementarity. This makes the equilibrium susceptible to an investors' mistake (or an intended off-equilibrium strategy) and hence less stable.

The existence of unstable equilibria has an important implication on market fragility. The paradoxically high liquidity in the unstable equilibria implies that the price impact of liquidity trades is small. Thus, discretionary liquidity traders who seek immediacy in execution can reduce their losses. Camouflaging their identity by trade fragmentation not only achieves this goal but also induces the unstable equilibria by making supply ambiguous. To avoid fragility, regulators can set rules to make the market more transparent and reduce ambiguity. Unfortunately, the resulting more stable equilibrium is not strictly better than the unstable equilibria, because it can exhibit higher volatility, more aggressive informational trade, and lower liquidity. This trade-off calls for careful planning to maximize social welfare.

The above consideration raises a natural question as to whether the equilibrium of last resort, which almost always exists, is guaranteed to be stable. Unfortunately, the answer is no. Strikingly, the unique stable equilibrium in a popular REE setting *always* becomes unstable with large enough aversion to supply ambiguity. This popular setting is the REE with perfectly heterogeneous private signals and no endowment learning, as pioneered in the seminal work of Hellwig (1980) and Diamond and Verrecchia (1981) as well as a multi-security extension of the former by Admati (1985). These studies collectively inspired numerous further extensions including the finite-horizon multiperiod models of Brennan and Cao (1996, 1997) and the infinite-horizon overlapping-generations model of Watanabe (2008). When the equilibrium of last resort becomes unstable, the market can stop functioning upon a small perturbation of beliefs, leading to a trading halt or even a market crash.

When investors are ambiguity neutral, their subjective probability assessment coincides with the objective probability law and our model reduces to the one with rational expectations. Since the literature on REE is vast, we provide only a limited review of its noisy version in which prices partially reveal payoffs under random supply. These so-called noisy REE (NREE) models include those cited in the preceding paragraph in which informed investors receive diverse private signals. The polar case is the model with a common private signal, initially developed almost concurrently by Grossman and Stiglitz (1980). With two classes of investors, informed and uninformed, this allows analyzing information acquisition in a parsimonious manner. Employing this type of information asymmetry in the case with diverse private signals, Ganguli and Yang (2009) show that learning from both a perfectly heterogeneous private signal and endowment promotes complementarity in information acquisition and helps attain multiple equilibria. For many years, researchers have focused on one of these two extreme settings, perhaps because of their tractability. Recently, Manzano and Vives (2011) construct a unified model that encompasses the whole spectrum of information structures that fall between the two polar cases. They find that when the private learning channel through private signals and endowment is strong relative to the public learning channel through market prices, strong strategic complementarity and potentially multiple equilibria obtain. To this general NREE model we introduce investors' aversion to supply ambiguity, whose effect is endogenously determined in equilibrium due to the reliance of price on supply. One can view the current agenda as changing course on investor preference rather than the information structure or trading environment that the existing literature has extensively analyzed.

Although Knight (1921) emphasized the distinction between uncertainty and risk almost a century ago, rigorous applications of uncertainty or ambiguity are relatively new. Starting from axioms, researchers are still proposing formulations to best model it. A prominent one is the max-min expected utility model of Gilboa and Schmeidler (1989). In this framework, the uncertain scenarios would correspond to multiple priors from which the agent would choose to prepare for the worst case. This extremely pessimistic behavior, however, results in kinked indifference curves that are often intractable. To avoid such difficulty, I adopt the smooth ambiguity-aversion preference of Klibanoff, Marinacci, and Mukerji (2005), which can be thought of as modeling aversion to the weighted average of uncertain scenarios, rather than to the worst scenario. With smooth indifference curves, this approach allows us to apply the well-established concepts of equilibrium and stability to the analysis of ambiguity.

Finance implications of ambiguity aversion are addressed in the seminal work of Epstein and Wang (1994), and more recently in the studies by Epstein and Schneider (2008), Illeditsch (2011), Easley, O'Hara, and Yang (2013), and Garlappi, Giammarino, and Lazrak (2014). In particular, coupling the smooth ambiguity-averse preference with the Grossman-Stiglitz (1980) information structure, Caskey (2009) shows that ambiguity-averse investors may prefer an aggregate signal to a set of more ambiguous, disaggregate signals at the cost of informational loss. While he employs a common private signal with uncertain noise mean, it is the quantity supplied that is ambiguous here. Hirshleifer, Huang, and Teoh (2017) show that an index fund can encourage ambiguity-averse investors to fully participate in the market. They further derive a CAPM-like pricing relation in such a setting. Although their main analysis features a max-min type utility and ambiguous supply precision, they state that their main result extends to the smooth ambiguity-averse preference and ambiguity about any parameter. Our focus quite differs from Caskey (2009) and Hirshleifer, Huang, and Teoh (2017): We address the stability of multiple equilibria and market fragility. Notably, the development of the finance literature on ambiguity has been mainly on the theoretical side so far, except for a few pieces of experimental evidence (Bossaerts et al. (2010)). I provide empirical illustrations of derived results.

The rest of the paper is organized as follows: Section 1 sets out the model, solves for equilibria, and examines their characteristics including stability. It also provides empirical illustrations. The final section concludes with future agenda.

1. A Model of Supply Ambiguity

1.1 Set-up

The economy is set up in two periods. In the initial period, two securities are traded in the financial markets. The first security is a riskless bond and pays a sure unit dividend in the terminal period. It is in infinitely elastic supply and always sells for the price of unity, implying that the riskless interest rate is normalized at 0 without loss of generality. The second security, called a stock, is risky and pays a random dividend in the terminal period. The dividend is distributed normally with zero mean and variance σ_d^2, denoted as $d \sim N(0, \sigma_d^2)$. The stock's supply, n, is random and distributed normally with an unknown mean, θ, and a known variance, σ_n^2. The randomness of supply can arise from a variety of reasons. For example, it may represent the trade of a counterparty in the equity market. At a macroeconomic level, it may result from the adjustment of a firm's productive capacity. θ is distributed normally with mean μ_θ and variance σ_θ^2, and represents ambiguity about the mean of supply. This implies that the marginal distribution of supply is also normal with mean μ_θ and variance $\sigma_n^2 + \sigma_\theta^2$ to an econometrician who observes neither n nor θ. However, the two components of the marginal variance, σ_n^2 and σ_θ^2, will be evaluated differently by an investor who cares about both risk and ambiguity. This is the source of the pricing of ambiguity, in addition to risk, and forms the central theme of this paper.

An investor, indexed by j, possesses negative exponential utility,

$$\begin{aligned} u(w_j) &\equiv -\exp(-\gamma w_j), \\ w_j &= (d-p)x_j + pn_j, \end{aligned} \tag{1}$$

where w_j is his terminal wealth, γ is the constant absolute-risk aversion (CARA) coefficient, p is the stock price, x_j is his demand for the stock, and n_j is his personal endowment.[3] Investors are averse to ambiguity about the mean of supply, θ. For tractability, I adopt the smooth ambiguity aversion preference proposed by Klibanoff, Marinacci, and Mukerji (2005). Specifically, the

[3]To derive the expression for w_j, write down the budget constraint

$$px_j + 1 \cdot b = pn_j,$$

where b is the bond position. Solve for b and substitute the result into the expression for the terminal wealth,

$$w_j = dx_j + 1 \cdot b.$$

Above, we have assumed that the bond endowment is zero. A non-zero bond endowment does not change the equilibrium property in any way.

investor chooses the demand x_j that solves the utility maximization problem,

$$\max_{x_j} E[h(E[u(w_j)|\mathbf{s}_j,\theta])|\mathbf{s}_j] = E[-(E[\exp(-\gamma w_j)|\mathbf{s}_j,\theta])^g|\mathbf{s}_j], \tag{2}$$
$$h(E[u]) \equiv -(-E[u])^g,$$

where $\mathbf{s}_j$ is the vector of signals comprising his information set and $g \geq 1$ is a measure of constant relative ambiguity aversion.[4] Intuitively, the inner expectation of (2) gives the ordinary, risk-averse expected utility given ambiguity about θ. The ambiguity aversion parameter g enhances its curvature. Taking the outer expectation with respect to the source of ambiguity yields the expected utility of an investor who is averse to both ambiguity and risk.

While there are several alternatives to model ambiguity aversion, I take advantage of the tractability of the smooth ambiguity-averse preference in the normal-CARA framework. For its general properties and axiomatic derivation, refer to Klibanoff, Marinacci, and Mukerji (2005). When $g = 1$, the above problem reduces to the usual expected utility maximization with no ambiguity aversion, with θ contributing to the randomness of future wealth as perceived risk, rather than ambiguity, via its marginal variance, $\sigma_n^2 + \sigma_\theta^2$. To highlight the effect of ambiguity, all investors are endowed with common risk aversion, γ, and common ambiguity aversion, g. However, introducing heterogeneity in either type of aversion is straightforward.

There are two types of investors, the informed (belonging to set I) and the uninformed (set U), with mass $\omega > 0$ and $1 - \omega$, respectively. The strict positivity of ω reflects our primary interest in the cases in which information asymmetry plays an active role. However, $\omega = 0$ does admit a trivial equilibrium with no private information, and it is straightforward to modify the formulas to follow when they involve division by ω. For an informed investor, we write $j \in I$, or simply replace subscript j by I abusing notation, and similarly for set U.

An informed investor receives a private signal about the terminal dividend,

$$s_j = d + \sqrt{\rho}\varepsilon + \sqrt{1-\rho}\varepsilon_j, \quad j \in I,$$

[4] The coefficient of (absolute) ambiguity aversion in Corollary 3 of Klibanoff, Marinacci, and Mukerji (2005) is $-h''(x)/h'(x) = (g-1)/(-x) > 0$ for $x = -E[\exp(-\gamma w_j)] < 0$. The coefficient of relative ambiguity aversion for $x < 0$ can be defined as $-|x|h''(x)/h'(x) = g - 1$.

where $0 \leq \rho \leq 1$, $\varepsilon \sim N(0, \sigma_\varepsilon^2)$, and $\varepsilon_j \sim N(0, \sigma_{\varepsilon,j}^2)$, with the ε noises independent among themselves for all j and with any other shocks. ε without the j subscript represents the common component in the noise of private signals, and the subscripted ε_j does the heterogeneous component.

The strong law of large numbers implies that the heterogeneous ε_j component disappears upon aggregation over any strictly positive mass. For example, over the whole population of informed traders,

$$\int_{j \in I} s_j dj = d + \sqrt{\rho}\varepsilon.$$

Thus, if $\rho = 1$, the signal error noise is perfectly correlated across informed agents as in Grossman and Stiglitz (1980). If $\rho = 0$, it is perfectly uncorrelated as in Hellwig (1980), Diamond and Verrecchia (1981), and Admati (1985). The j subscript on $\sigma_{\varepsilon,j}^2$ signifies that the signal precision could differ between subsets of traders as in Brennan and Cao (1996, 1997) and Watanabe (2008), making some informed investors better informed than others. Our numerical examples will focus on the case with common private-signal variance, $\sigma_{\varepsilon,j}^2 \equiv \sigma_\varepsilon^2$, for all j. In this case, changing ρ will vary the contribution of the common and heterogeneous noise components while keeping the signal-to-noise ratio (or the total noise variance) constant. Nevertheless, our formulas are fully consistent with heterogeneous $\sigma_{\varepsilon,j}^2$ across investors, in which case setting $\omega = 1$ (all investors are informed) permits an equilibrium with nondegenerate trades in a parsimonious manner.

All investors are endowed with n_j shares of the stock, where

$$n_j = n + \zeta_j, \quad \zeta_j \sim N(0, \sigma_\zeta^2). \tag{3}$$

Therefore, the personal endowment provides some information about the supply and hence the dividend. We will first solve the updating problem of investors..

1.2 Updating

Investors learn about the dividend from the price. They conjecture a price function linear in state variables,

$$p = a_d(d + \sqrt{\rho}\varepsilon) + a_n n + a_0 \equiv a_d \xi + a_0, \tag{4}$$

$$\xi \equiv d + \sqrt{\rho}\varepsilon + \lambda n = (p - a_0)/a_d, \tag{5}$$

$$\lambda \equiv \frac{a_n}{a_d} < 0,$$

where $a_d > 0$, $a_n < 0$, and a_0 are coefficients to be determined in equilibrium. The signs of parameters are motivated economically; $a_d > 0$ because the stock price should increase in the dividend and $a_n < 0$ to discount supply risk, or for the demand curve to slope down. ξ represents the informational content of the price. Following the standard practice, we confine our attention to a linear price function as in Equation (4). By Equation (5), investors can infer ξ from p in equilibrium, but not its components d, ε, or n separately. Thus, ξ provides partial information about the dividend, d. Moreover, combined with ξ, the endowment n_j also serves as a private signal about the dividend. This can be represented in the following form:[5]

$$\phi_j \equiv \xi - \lambda n_j = d + \sqrt{\rho}\varepsilon - \lambda \zeta_j,$$

where the last equality follows from (3). As with ξ, investors can infer ϕ_j in its entirety, and not its components separately.

To derive investors' posterior beliefs, define the vector $\mathbf{s}_I$ of signals comprising the informa-

[5] In some existing literature, the informational content of individual endowment is made devoid. See Footnote 12 of Watanabe (2008). Setting $\sigma_\zeta^2 = \infty$ corresponds to such a case.

tion set of the informed investors as

$$\mathbf{s}_I \equiv \begin{cases} \begin{pmatrix} s_j \\ \phi_j \\ \xi \end{pmatrix} = \begin{pmatrix} d + \sqrt{\rho}\varepsilon + \sqrt{1-\rho}\varepsilon_j \\ d + \sqrt{\rho}\varepsilon - \lambda\zeta_j \\ d + \sqrt{\rho}\varepsilon + \lambda n \end{pmatrix} & \text{if } 0 \le \rho < 1, \\ \begin{pmatrix} s_j \end{pmatrix} = \begin{pmatrix} d + \varepsilon \end{pmatrix} & \text{if } \rho = 1. \end{cases} \tag{6}$$

The last line recognizes the fact that signals ϕ_j and ξ will not provide any additional information beyond s_j when $\rho = 1$, because $d + \varepsilon$ is the most accurate of all signals about d. For the uninformed investors, the vector of their signals, $\mathbf{s}_U$, lacks private information,

$$\mathbf{s}_U \equiv \begin{pmatrix} \phi_j \\ \xi \end{pmatrix} = \begin{pmatrix} d + \sqrt{\rho}\varepsilon - \lambda\zeta_j \\ d + \sqrt{\rho}\varepsilon + \lambda n \end{pmatrix} \text{ for any } 0 \le \rho \le 1. \tag{7}$$

A straightforward application of normal projection theory outlined in Appendix A.1 gives the posterior distribution of d given the informed investors' information set in Equation (6) for any $0 \le \rho \le 1$ as

$$d|\mathbf{s}_I \sim N(\mu_{d|\mathbf{s}_I}, \sigma^2_{d|\mathbf{s}_I}),$$

$$\sigma^{-2}_{d|\mathbf{s}_I} = \frac{1}{\sigma^2_d} + \frac{1}{\rho\sigma^2_\varepsilon + (1-\rho)\hat{\sigma}^2_{eI}(\lambda)},$$

$$\hat{\sigma}^{-2}_{eI}(\lambda) \equiv \sigma^{-2}_{\varepsilon,j} + (1-\rho)\lambda^{-2}[\sigma^{-2}_\zeta + (\sigma^2_n + \sigma^2_\theta)^{-1}], \tag{8}$$

$$\mu_{d|\mathbf{s}_I} = \sigma^2_{d|\mathbf{s}_I} \frac{\sigma^{-2}_{\varepsilon,j} s_j + (1-\rho)\lambda^{-2}[\sigma^{-2}_\zeta \overbrace{\phi_j}^{=\xi-\lambda n_j} + (\sigma^2_n + \sigma^2_\theta)^{-1}(\xi - \lambda\mu_\theta)]}{1 - \rho + \rho\sigma^2_\varepsilon \hat{\sigma}^{-2}_{eI}(\lambda)}, \tag{9}$$

where $\hat{\sigma}^{-2}_{eI}(\lambda)$ signifies its general dependence on λ except for the important special case of $\rho = 1$. These formulas show that the conditional precision, $\sigma^{-2}_{d|\mathbf{s}_I}$, increases as investors get more information, and the conditional mean, $\mu_{d|\mathbf{s}_I}$, is the precision-weighted average of prior means and signals.

Similarly, the posterior belief of the uninformed investors is

$$\begin{aligned}
d|\mathbf{s}_U &\sim N(\mu_{d|\mathbf{s}_U}, \sigma^2_{d|\mathbf{s}_U}),\\
\sigma^{-2}_{d|\mathbf{s}_U} &= \frac{1}{\sigma^2_d} + \frac{1}{\rho\sigma^2_\varepsilon + \lambda^2/[\sigma_\zeta^{-2} + (\sigma^2_n + \sigma^2_\theta)^{-1}]},\\
\mu_{d|\mathbf{s}_U} &= \sigma^2_{d|\mathbf{s}_U} \frac{\lambda^{-2}[\sigma_\zeta^{-2} \overbrace{\phi_j}^{=\xi-\lambda n_j} + (\sigma^2_n + \sigma^2_\theta)^{-1}(\xi - \lambda\mu_\theta)]}{1 + \rho\sigma^2_\varepsilon\lambda^{-2}[\sigma_\zeta^{-2} + (\sigma^2_n + \sigma^2_\theta)^{-1}]}.
\end{aligned} \tag{10}$$

Armed with these expressions, we can now the tackle investors' utility maximization problem.

1.3 Equilibrium

With the updated beliefs about the terminal dividend, in Appendix A.2 we explicitly calculate the expected utility in (2) for the two types of investors:

$$\begin{aligned}
&\max_{x_j} E[h(E[u(w_j)|\mathbf{s}_j, \theta])|\mathbf{s}_j]\\
&= -\exp\left(-g\gamma\left[(\mu_{d|\mathbf{s}_j} - p)x_j - \frac{\gamma}{2}\underbrace{\sigma^2_{d|\mathbf{s}_j}\left\{g + (1-g)\frac{\sigma^2_{d|\mathbf{s}_j\theta}}{\sigma^2_{d|\mathbf{s}_j}}\right\}}_{=\sigma^2_{d|\mathbf{s}_j a} \text{ in (13)}} x_j^2 + pn_j\right]\right),
\end{aligned} \tag{11}$$

where $\sigma^2_{d|\mathbf{s}_j} \geq \sigma^2_{d|\mathbf{s}_j\theta}$ with the equality holding only when $\sigma^2_\theta = 0$; not knowing θ exposes the investor to ambiguity and generally results in a higher posterior variance. The certainty equivalent in the square bracket has the mean-variance form, with the penalty proportional to $g\sigma^2_{d|\mathbf{s}_j} + (1-g)\sigma^2_{d|\mathbf{s}_j\theta}$. Since $\sigma^2_{d|\mathbf{s}_j} > \sigma^2_{d|\mathbf{s}_j\theta}$ as long as $\sigma^2_\theta > 0$, this weighted average of conditional variances increases in ambiguity aversion, g. When $g = 1$, it recovers the standard, ambiguity-neutral case. The preservation of the mean-variance form is a benefit of the smooth ambiguity-aversion preference of Klibanoff, Marinacci, and Mukerji (2005).

The first order condition for maximizing the certainty equivalent in the square bracket of Equation (11) gives the optimal demand in the following form common to both types of investors:

$$x_j = \frac{\mu_{d|\mathbf{s}_j} - p}{\gamma \sigma^2_{d|\mathbf{s}_j a}} \equiv b_{sj} s_j + b_{nj} n_j + b_{pj} p + b_{0j}, \tag{12}$$

$$\sigma^2_{d|\mathbf{s}_j a} \equiv \sigma^2_{d|\mathbf{s}_j} \left\{ g + (1-g) \frac{\sigma^2_{d|\mathbf{s}_j \theta}}{\sigma^2_{d|\mathbf{s}_j}} \right\} = g \sigma^2_{d|\mathbf{s}_j} + (1-g) \sigma^2_{d|\mathbf{s}_j \theta}, \quad j = I, U, \tag{13}$$

where the b coefficients are to be determined. Equation (12) writes the demand as a linear function of state variables, s_j, n_j, and p. By convention, we set $b_{sU} \equiv 0$, because the uninformed investors do not receive the private signal, s_j. To determine the b coefficients, in Appendix A.3 we substitute the conditional moments in Equations (9) and (10) into (12) and compare the coefficients on the three state variables. The expressions for b_{sI}, b_{nI}, and b_{nU} turn out to be functions of only λ and no other conjectured coefficients (see Equations (A3), (A4), and (A8)). This defines a vector-valued mapping $\varphi : \Re \mapsto \Re^3$ from the ratio of price coefficients, $\lambda = a_n / a_d$, to the vector of investors' strategies, $b \equiv \begin{pmatrix} b_{sI} & b_{nI} & b_{nU} \end{pmatrix}'$:

$$\varphi(\lambda) = b, \tag{14}$$

where $\varphi(\lambda)$ is a 3 by 1 vector with elements given in Equations (A3), (A4), and (A8) in Appendix A.3.

To close the loop, find a mapping from the investors' strategy vector b to the ratio of price coefficients, λ, by imposing market clearing:

$$\begin{aligned} n &= \omega \int_{j \in I} x_j dj + (1-\omega) \int_{j \in U} x_j dj \\ &= \omega b_{sI} (d + \sqrt{\rho} \varepsilon) + [\omega b_{nI} + (1-\omega) b_{nU}] n \\ &\quad + [\omega b_{pI} + (1-\omega) b_{pU}] p + \omega b_{0I} + (1-\omega) b_{0U}, \end{aligned}$$

where we have substituted the expressions for demand in Equation (12) in the second equality.

Solving for p yields

$$p = \frac{-\omega b_{sI}(d + \sqrt{\rho}\varepsilon) + [1 - \omega b_{nI} - (1-\omega)b_{nU}]n - \omega b_{0I} - (1-\omega)b_{0U}}{\omega b_{pI} + (1-\omega)b_{pU}}.$$

This relation must hold almost surely for all realizations of the random variables. So, equate the coefficients on $d + \sqrt{\rho}\varepsilon$, n, and the constant term, respectively, in the original price conjecture (4) and the above expression:

$$a_d = -\frac{\omega b_{sI}}{\omega b_{pI} + (1-\omega)b_{pU}}, \tag{15}$$

$$a_n = \frac{1 - \omega b_{nI} - (1-\omega)b_{nU}}{\omega b_{pI} + (1-\omega)b_{pU}}, \tag{16}$$

$$a_0 = -\frac{\omega b_{0I} + (1-\omega)b_{0U}}{\omega b_{pI} + (1-\omega)b_{pU}}. \tag{17}$$

Taking the ratio of the second equation to the first gives

$$\lambda = \frac{\omega b_{nI} + (1-\omega)b_{nU} - 1}{\omega b_{sI}} \equiv \psi(b), \tag{18}$$

where we have defined mapping $\psi : \mathfrak{R}^3 \mapsto \mathfrak{R}$ from b to λ. This completes the loop and defines an equilibrium as follows:

Definition 1. *(Equilibrium) A linear equilibrium is a set of demand that maximizes each individual investor's utility in (11) and the price conjecture in (4) that solves the vector-valued fixed-point problem,*

$$b = \varphi \circ \psi(b). \tag{19}$$

Equation (19) defines an equilibrium b as the fixed point of the best-response function, $\varphi \circ \psi$. To interpret this function economically, consider an agent following a strategy, b. This implies a belief about price coefficients consistent with the strategy, $\psi(b) = \lambda$. This further implies a strategy consistent with the price conjecture, $\varphi \circ \psi(b)$, which must coincide with the original strategy, b, in equilibrium. This feedback loop can have multiple fixed points corresponding to multiple equilibria. If there is no fixed point, there is no equilibrium. Once we find a fixed point $b = \begin{pmatrix} b_{sI} & b_{nI} & b_{nU} \end{pmatrix}'$ that satisfies Equation (19), we can determine all the remaining coefficients as described in Appendix A.4.

We call an equilibrium *stable* if it does not diverge against a small perturbation in investors' strategies. This notion of stability can be defined in terms of a first-order discretization of Equation (19). Specifically, an equilibrium is stable if the Jacobian of the fixed point mapping $\varphi \circ \psi$ has all eigenvalues smaller than one in magnitude:

Definition 2. *(Stability) An equilibrium is stable if the absolute value of all eigenvalues of the Jacobian matrix* $\frac{\partial \varphi}{\partial \lambda} \frac{\partial \psi}{\partial b'}$ *is less than one:*

$$\max \left| eig \left(\frac{\partial \varphi}{\partial \lambda} \frac{\partial \psi}{\partial b'} \right) \right| < 1, \tag{20}$$

where $\max |eig(\cdot)|$ *denotes the maximum eigenvalue of the argument matrix.*

Here, by (18), we can explicitly calculate

$$\frac{\partial \psi}{\partial b'} = \frac{1}{b_{sI}} \begin{pmatrix} \underbrace{-\psi(b)}_{=-\lambda} & 1 & (1-\omega)/\omega \end{pmatrix}.$$

The next lemma makes the stability condition in (20) more operational by transforming the vector problem to a scalar problem:

Lemma 1. *(Existence and stability in the price space) The necessary and sufficient conditions for the existence and stability of an equilibrium are given by the following scalar restrictions:*

$$\textit{Existence: } \lambda = \psi \circ \varphi(\lambda) \equiv f(\lambda), \tag{21}$$

$$\textit{Stability: } |f'(\lambda)| < 1. \tag{22}$$

Proof. See Appendix B for all proofs. ■

The economic interpretation of Equation (21) is analogous to that of Equation (19), but the fixed point-loop here starts with a belief about (the ratio of) price coefficients, λ, rather than a strategy, b. The lemma proves that the loop will diverge in the price space if it does in the strategy space, and vice versa. This result implies that the stability condition in Equation (20) or equivalently (22) is closely related to the notions of stability proposed in the existing literature. In particular, when there is only one type of investors ($\omega = 1$ or $\omega = 0$), it is a sufficient condition for strong rationality of Guesnerie (1992, 2002) and other weaker conditions discussed therein.[6] In a likewise general model but without ambiguity, Manzano and Vives (2011) directly define scalar conditions for existence and stability similar to those in Lemma 1 above.

Noting that $\frac{b_{nI}}{b_{sI}} = -\frac{(1-\rho)\sigma^2_{\varepsilon,j}}{\sigma^2_\zeta \lambda}$, the function $f(\cdot)$ in Equation (21) can be explicitly written as

$$f(\lambda) = -\frac{(1-\rho)\sigma^2_{\varepsilon,j}}{\sigma^2_\zeta \lambda} + \overbrace{\frac{(1-\omega)b_{nU} - 1}{\omega b_{sI}}}^{\equiv f_{UI}(\lambda)}, \tag{23}$$
$$f_{UI}(\lambda) \equiv \frac{(1-\omega)b_{nU} - 1}{\omega b_{sI}},$$

where the subscript UI signifies that f_{UI} contains both b_{nU} and b_{sI} and therefore is determined through the interaction between the informed and uninformed traders when $\omega < 0 < 1$. We first present a result on the existence of an equilibrium.

Proposition 1. *(Existence) When $\rho > 0$, there is at least one equilibrium. When $\rho = 0$, a sufficient condition for existence is that*

$$f_{UI}(\lambda_\#) \leq 2\lambda_\#, \quad \lambda_\# \equiv -\sqrt{\frac{(1-\rho)\sigma^2_{\varepsilon,j}}{\sigma^2_\zeta}} \tag{24}$$

[6] To see this, note that the sufficient condition for strong nationality in Guesnerie (2002, p.453, Point (i)) in our notation is that $\omega \left\| \frac{\partial\varphi_I}{\partial\lambda}\frac{\partial\psi_I}{\partial b'_I} \right\| + (1-\omega) \left\| \frac{\partial\varphi_U}{\partial\lambda}\frac{\partial\psi_U}{\partial b'_U} \right\| < 1$, where $\|\cdot\|$ denotes the norm of the argument matrix induced by the Euclidean norm and φ_j, ψ_j, and b_j , $j = I, U$, denote the elements of φ, ψ, and b corresponding to the informed and uninformed investors' demand, respectively. As in his Point (ii), a norm induced by the Euclidean norm equals the largest absolute eigenvalue, and the above condition reads that the weighted average of the largest absolute eigenvalues of the Jacobian matrices for the informed and uninformed investors are less than one: $\omega \max \left| eig\left(\frac{\partial\varphi_I}{\partial\lambda}\frac{\partial\psi_I}{\partial b'_I} \right) \right| + (1-\omega) \left\| \frac{\partial\varphi_U}{\partial\lambda}\frac{\partial\psi_U}{\partial b'_U} \right\| < 1$. Lemma 1 further translates it to the scalar condition, $\omega \left| \frac{\partial\psi_I}{\partial b'_I}\frac{\partial\varphi_I}{\partial\lambda} \right| + (1-\omega) \left| \frac{\partial\varphi_U}{\partial\lambda}\frac{\partial\psi_U}{\partial b'_U} \right| < 1$. On the other hand, Equation (22) can be expressed as $\left| \frac{\partial\psi_I}{\partial b'_I}\frac{\partial\varphi_I}{\partial\lambda} + \frac{\partial\varphi_U}{\partial\lambda}\frac{\partial\psi_U}{\partial b'_U} \right| < 1$. It is straightforward to see that these conditions are equivalent when $\omega = 1$ or $\omega = 0$.

with at least two equilibria if the inequality holds strictly.

The result invokes two facts: for $\rho > 0$, $f(\lambda)$ converges to a finite negative number as λ tends to $-\infty$, and stays negative and bounded away from zero in the other limit as λ approaches 0 from the left (see Equations (A14) and (A15) in Appendix B). Thus, its graph intersects the 45° degree line at least once when $\rho > 0$. If $\rho = 0$, however, $f(\lambda)$ diverges to positive infinity in the right limit, and therefore existence is not guaranteed. Equation (24) is a sufficient condition for existence in such a case.

Figure 1 depicts the fixed-point problems for the case with positive ρ ($\rho = 0.01 > 0$) for selected values of g under the parameter values shown in the figure caption. The small value of ρ is chosen to demonstrate that even a tiny deviation from $\rho = 0$ guarantees the existence. These parameter values will be repeatedly used in the rest of the paper as the benchmark case, although their representativeness is admittedly an open question. However, my main objective here is to demonstrate that supply ambiguity and investors' tolerance to it *can* allow unstable equilibria to co-exist, which can disappear upon a small perturbation of beliefs and leads to market fragility. In the figure, an equilibrium is represented by an intersection of the graphs of $f(\lambda)$ and λ (the 45° degree line). There is at least one equilibrium for all the three values of g.

1.4 Ambiguity aversion and stability of equilibria

We now analyze the effect of ambiguity aversion on the stability of possibly multiple equilibria.

Proposition 2. *(Ambiguity aversion and stability) When one or more equilibria exist, number them in the ascending order of λ (the first equilibrium has the most negative λ). Even-numbered equilibria are always unstable. As g increases, the absolute value of λ increases in odd-numbered equilibria and decreases in even-numbered equilibria. In the end, all equilibria but the first will disappear.*

To understand this proposition, again consider the fixed-point loop. As investors become more averse to supply ambiguity (g increases), they will scale back their positions, i.e., the absolute value of the b coefficients decreases. This is consistent with a "pessimistic" belief about price coefficients, resulting in a larger magnitude of $\psi(b) = \lambda$. This further induces a less aggressive strategy $\varphi \circ \psi(b)$, which must coincide with the original strategy, b. By Lemma 1, an equivalent iteration in the price space yields a larger absolute $\psi \circ \varphi(\lambda) = f(\lambda)$, which must match the original λ in equilibrium. That is, the graph of $f(\lambda)$ shifts down ($f(\lambda)$ becomes negative and larger in magnitude), which produces the changes in equilibrium λ, or the fixed point, described in the proposition. This is proved in Appendix B.3.

Figure 1 also illustrates the claims in Proposition 2. Recall that $f(\lambda)$ converges to a finite negative number as λ tends to $-\infty$ (which is evident in the figure). This implies that its graph always intersects the 45° degree line from above at odd-numbered equilibria, while it does from below at even-numbered equilibria. Therefore, the slope of $f(\lambda)$ in an even-numbered equilibrium is always positive and exceeds one, i.e. the equilibrium is unstable by Lemma 1. This can be seen for the case with $g = 1$, which has three equilibria. Numbering the equilibria by a subscript, a numerical method gives the fixed-point solutions $\lambda_1^* = -0.75$, $\lambda_2^* = -0.21$, and $\lambda_3^* = -0.045$, where the star superscript signifies an equilibrium value. As g increases, the graph of $f(\lambda)$ shifts downward. The second and third equilibria almost coincide at $g = 2.19$ and disappear at a higher value, $g = 3$. It is straightforward to see that the equilibrium values λ_1^* and λ_3^* decrease (become negative and larger in magnitude) in g, while λ_2^* increases (becomes negative and smaller in magnitude).

Panel (A) of Figure 2 depicts the behavior of λ^* in the three equilibria as g varies. For g approximately above 2.2, there is only one equilibrium. Panel (B) plots the derivative of $f(\lambda)$ at the fixed points, $f'(\lambda^*)$, which is computed numerically over a small perturbation of λ around λ^*.[7] The magnitude of the derivative indicates that only the first equilibrium is stable in this example; $|f'(\lambda_1^*)| < 1$ while both $|f'(\lambda_2^*)| > 1$ and $|f'(\lambda_3^*)| > 1$ throughout. Panels (C) and (D) show the price coefficients on the dividend (a_d) and supply (a_n), respectively.

[7] Specifically, I compute the numerical derivative as

$$f'(\lambda^*) = \frac{f(\lambda^*(1 + 10^{-6})) - f(\lambda^*(1 - 10^{-6}))}{2 \cdot 10^{-6}\lambda^*}.$$

In the second and third equilibria, a_d is larger and a_n is smaller in magnitude than they are in the first equilibrium. The smaller absolute a_n implies that the price is more informative, which facilitates the coordination of beliefs. This allows for strong strategic complementarity, as indicated by positive $f'(\lambda_2^*)$ in the second equilibrium. This connection between strategic complementarity and positive f' is implied by Lemma 1. Its proof establishes the equivalence of $f'(\lambda)$ and the only non-zero eigenvalue of the Jacobian of the vector-valued best-response function, $\varphi \circ \psi(b)$. Therefore, when $f'(\lambda) > 0$, an investor tends to respond in the same direction as other investors. The more positive and larger $f'(\lambda)$ is, the stronger this strategic complementarity. The facilitated coordination of beliefs makes it easier to reach an equilibrium. Graphically, the positive derivative allows the graph of $f(\lambda)$ to reach the 45° line more often (see Figure 1), producing more equilibria.

The proposition raises a natural question as to whether the first equilibrium that exists under a large enough value of g is stable. While it is difficult to prove the general case, in a popular setting extensively employed in the literature, the answer turns out to be no.

Proposition 3. *(Stability under perfectly heterogeneous private signals and no endowment learning) When the informed investors' private signals do not contain a common noise component and there is no learning from endowment ($\rho = \sigma_\zeta^{-2} = 0$), the equilibrium is always stable if investors are ambiguity-neutral ($g = 1$). As investors become more ambiguity-averse, however, the equilibrium will become always unstable for a large enough g; specifically,* $\lim_{g\to\infty} f'(\lambda) = -2$.

This point is illustrated in Figure 3, which uses the same parameter values as Figure 1 except that $\rho = 0$ and $\sigma_\zeta^{-2} = 10^{-20}$, which sets the precision of the endowment signal virtually zero. The graph of $f(\lambda)$ with $g = 1$ is flat, implying that the equilibrium is stable when investors are ambiguity neutral; the proposition asserts that this is always true as long as $\rho = \sigma_\zeta^{-2} = 0$. The figure shows that $f(\lambda)$ bends down and becomes downward-sloping as g increases. Both the value of λ at the fixed-point, λ^*, and the slope of the graph there, $f'(\lambda^*)$, become negative and larger in magnitude, as depicted in Figure 4. The slope converges to -2 in the limit as $g \to \infty$ (not depicted in the figure). This result is striking, as the equilibrium that is universally stable under the traditional setting without ambiguity can *always* be made unstable by simply making investors averse enough to supply ambiguity. The setting with $\rho = \sigma_\zeta^{-2} = 0$ is employed in the seminal work of Hellwig (1980) and Admati (1985), and their numerous extensions including the multi-period models of Brennan and Cao (1996, 1997).

1.5 Properties of equilibria

To analyze the equilibrium characteristics, define the unconditional volatility of dollar return by

$$\begin{aligned}\sigma_{d-p} &= \sqrt{var(d-p)} = \sqrt{var((1-a_d)d - a_d\sqrt{\rho}\varepsilon - a_n n - a_0)} \\ &= \sqrt{(a_d-1)^2\sigma_d^2 + a_d^2\rho\sigma_\varepsilon^2 + a_n^2(\sigma_n^2+\sigma_\theta^2)}.\end{aligned}$$

Expected volume can be computed by aggregating the absolute change in the investors' equity holdings. To this end, define an investor's demand in excess of his endowment,

$$\Delta x_j \equiv x_j - n_j. \tag{25}$$

Then, trading volume is given by

$$V = E\left[\frac{1}{2}\int_j |\Delta x_j|\right] = \omega\sqrt{\frac{1}{2\pi}var(\Delta x_I)} + (1-\omega)\sqrt{\frac{1}{2\pi}var(\Delta x_U)}, \tag{26}$$

which is explicitly calculated in Appendix A.5. Finally, define liquidity as the inverse slope of the aggregate demand curve:

$$L = \frac{1}{|a_n|}. \tag{27}$$

A steeper demand curve implies that the price will move more as the exogenous supply, n, changes (consider intersecting the demand curve with a vertical supply curve). This means that, if the supply change is caused by liquidity trading, its price impact will be higher. Therefore, the slope of the demand curve is an inverse measure of market liquidity. This notion of liquidity is general and can be defined even in a competitive endowment economy, as proposed by Johnson

(2006).

Figure 5 plots these quantities as well as investors' demand coefficients for the setting in Figure 1. Panels (A) and (B) show the informed investors' demand coefficients on the private signal (b_{sI}) and endowment (b_{nI}), respectively. The first equilibrium has a lower value of b_{nI} in Panel (B) than the other two equilibria, because this equilibrium has a large absolute value of λ (see Panel (A) of Figure 2) and therefore the price signal, and hence the endowment signal, are both relatively uninformative. This results in a larger value of b_{sI} in Panel (A) as the informed investors rely more on their private signals. Panel (C) plots the uninformed investors' shadow demand coefficient on endowment, b_{nU}. It is a "shadow" demand because their mass is zero ($\omega = 1$) in this example. However, the shape of the graphs is similar to that of b_{nI} in Panel (B), and their market participation due to a small change in ω around 1 will not change the equilibrium characteristics materially in this setting.

Volatility in Panel (D) is highest in the first equilibrium. Recall that the absolute price coefficient on supply (the magnitude of a_n in Panel (D) of Figure 2) was highest in this equilibrium. The large loading on a non-fundamental shock implies excessive volatility, and directly translates to low liquidity (Panel (F)) in this equilibrium. Like volatility, volume in Panel (E) is also high, suggesting that the informed investors rebalance their equity position more actively given the three signals.

The analysis has an important implication on market fragility. First, the less volatile, more liquid equilibria (Equilibria 2 and 3) unfortunately turn out to be less stable, and in fact unstable in the current setting; a small perturbation in the belief of either the price or demand coefficients (recall the equivalence of the stability in terms of these coefficients in Lemma 1) will move us away from such equilibria. Moreover, these equilibria do not survive a large increase in ambiguity aversion. When g exceeds the threshold for their existence, the equilibrium will jump to the only remaining equilibrium (Equilibrium 1) that is volatile, illiquid and has high volume. Since large absolute a_n implies a large price discount, there will be a big price drop reminiscent of a market crash. Moreover, there is another nuisance parameter: supply ambiguity. We will now analyze its effect on equilibrium stability and characteristics.

1.6 Supply ambiguity and market fragility

Figure 6 depicts the fixed-point problems for various values of supply ambiguity, σ_θ^2, again using the benchmark values for other parameters. Unlike g, changing σ_θ^2 does not universally shifts up or down the graph of $f(\lambda)$ everywhere. When $\sigma_\theta^2 = 2$, there are three equilibria, two of which eventually disappear as σ_θ^2 declines. This is highlighted in Panel (A) of Figure 7. According to Panel (B), however, the second and third equilibria are again unstable with a large absolute slope of the graph ($|f'(\lambda)| > 1$), a high dividend loading, and a low absolute loading on supply (Panels (C) and (D)).

Large supply ambiguity allows these unstable equilibria to co-exist. This has an important implication on market fragility from the perspectives of both traders and regulators. For example, liquidity traders may wish to reduce expected losses. Suppose the supply n is provided by liquidity traders.[8] Since their position is its complement, $-n$, their expected profits can be calculated as

$$\min_{\sigma_\theta^2} E_L[(d-p)(-n)] = E_L[\{(a_d - 1)d + a_d\sqrt{\rho}\varepsilon + a_n n + a_0\}n] = a_n n^2 < 0,$$

where $E_L[]$ denotes the expectation of liquidity traders given their information set, which contains n but not d or ε. Here, their losses are proportional to the magnitude of a_n, or the price impact of their trades. They would strive to reduce this inverse measure of liquidity (see Equation (27)). One way to achieve this is to split their trades to look like noise. That is, they will make their trades more ambiguous. As a result, σ_θ^2 increases, and according to Panel (D) of Figure 7, the two unstable equilibria will emerge. These are exactly where the liquidity traders will achieve their objective to reduce $|a_n|$ and hence their losses; notice that $|a_n|$ is smaller in the two unstable equilibria than in Equilibrium 1. Therefore, the loss-minimizing behavior of discretionary liquidity traders induces market fragility.

To avoid fragility, regulators may wish to make the market transparent by setting rules that reduce the ambiguity of trades, or of supply to marginal, price-setting investors. Such a policy

[8] This is the case in which learning about n from personal endowment n_j may be inappropriate. Setting $\sigma_\zeta^2 = \infty$ turns off endowment learning. See Footnote 5.

would directly aim at reducing σ_θ^2. Unfortunately, this does not necessarily lead to a strictly "better" market. According to Panel (A) of Figure 8, the informed investors' demand will rely more heavily on their private signals s_j in the most stable equilibrium in blue. Panels (B) and (C) show that the loadings of the demand on endowment n_j are similar between the informed and uninformed investors. As a result, trading occurs primarily due to an informational motive rather than an allocative motive in this numerical examine, and more strongly so in the blue equilibrium. The stronger reliance on private information in that equilibrium leads to higher volatility in Panel (D) and more aggressive trades or larger volume in Panel (E). Since the trades are primarily information based, the large volume does not imply a liquid market; rather, liquidity in the blue equilibrium is lower in Panel (F) than in the two unstable equilibria.

To summarize, investors' incentive to camouflage their trades tends to make the supply more ambiguous and the market more fragile. Regulators' effort to reduce ambiguity can stabilize the market, but possibly at the cost of higher volatility, more aggressive informational trade, and lower liquidity. This trade-off calls for careful planning to maximize social welfare.

1.7 Empirical illustrations

This section provides empirical illustration of the role supply ambiguity plays in the market. We examine two cases in which ambiguity appears to be elevated: insider trading and the flash crash.

1.7.1 Insider trading

The sample contains insider trades from Thomson Financial (Table 1, Forms 3-5) over the period from 1985 through 2012. The insider trades are matched to prices and returns in the CRSP database, and to trades and quotes in the TAQ dataset, of the stocks insiders trade. Using TAQ, I sign individual trades by the Lee and Ready (1991) algorithm and compute signed and unsigned share volume, $SVOL$ and VOL, as the sum of the signed and unsigned trades, respectively, on each day. Normalizing the former by the number of shares outstanding ($SHROUT$) in CRSP gives signed share turnover, $STOV = SVOL/SHROUT$, which measures standardized order imbalance. In the spirit of Barber, Odean, and Zhu (2009), we further calculate the proportion of buys, $PBUY = (SVOL/VOL + 1)/2$, and the proportion of the number of buys, $PNBUY = NBUY/(NBUY + NSELL)$, where $NBUY$ and $NSELL$ are the number of buys and sells, respectively.[9] Trades are sorted into the five trade size bins used by Barber, Odean, and Zhu (2009). The three signed trade measures are aggregated within the smallest TAQ trade-size quintile, denoted as $STOV1$, $PBUY1$, and $PNBUY1$. Denote the day of insider trading as Day 0. On Day -1, separately for insider buys and sells, we form equally weighted portfolios by sorting stocks into quintiles based on $STOV1$, $PBUY1$, or $PNBUY1$ for the smallest trade size bin. CAR is then the alpha from the regression of a portfolio return on the CRSP equally weighted market return, cumulated over days and normalized to 0 on Day -1.

Figure 9 shows a clear difference in the behavior of CAR around insider trading days. For buy orders in Panels (A), (C), and (E), CAR abruptly rises from Day 0 through Day 4. This suggests heightened supply ambiguity and its gradual resolution. Noticing a surge in the market price on Day 0, investors would suspect that an insider may have placed a buy order. However, it is also possible that a large liquidity trader has bought the stock. For example, a mutual fund may be executing customer orders. The informational content of the trade is very different between these two cases. Without knowing the reason behind the price upsurge, investors will collectively accommodate the suspicious trade under supply ambiguity.

CAR behaves slightly differently around insider sales (Panels (B), (D), and (F)). First, the price hikes until Day 0. Insiders appear to be contrarian traders as they sell after a spell of positive returns, creating the peak in CAR. Second, the price changes course on Day 0 for sell orders, while it does not for buy orders. Third, the decrease in CAR from Day 0 through Day 4 is smaller in magnitude than the increase for buys (note the difference in scale). Finally, CAR continues to noticeably fall after Day 4. To be fair, CAR for buy orders also keeps rising over a longer horizon, as is well known from existing studies (e.g., Seyhun (1986)). Our focus here, however, is the short-term movement of CAR that creates the kinks on Day 0 (for sells)

[9] To derive the formula for $PBUY$, note that the sum of buys are given by $(SVOL + VOL)/2$. Dividing by VOL produces the formula.

or -1 (for buys) and Day 4. What is special about Day 4? A possible answer is the filing requirement and the media effect. According to Rogers, Skinner, and Zechman (2013), the Security and Exchange Commission required insiders to file their trades (Form 4) within two days from trading in 2002, and moreover do so via the EDGAR electronic system in June of 2003. This filing is time-stamped. Thus, starting in June 2003, the details of insider trading, time-stamped to the second, became publicly available on Day 3 in a relatively costless manner. Even if each investor does not access the EDGAR system, the media may have disseminated the news of insider trading after market closure on that day. Therefore, investors may have become certain about insider trading by the end of Day 4, clearing the ambiguity. Since our sample includes the period before this requirement was in place, we are likely to be underestimating this effect.

The plots of CAR suggest that the price is generated through different mechanisms between several days following insider trading and other periods. Prior to insider trading, suppose that the market is in a steady state with a low level of ambiguity. In Figure 7, this corresponds to the left region in each panel where σ_θ^2 is small. There is only one equilibrium as depicted in blue. If investors perceive heightened ambiguity about the trades supplied to them due to possible insider trading, σ_θ^2 rises and we are moving to the right in each panel of the figure. Such a movement allows highly unstable equilibria (Equilibria 2 and 3) to coexist. Note that these equilibria are not inconsistent with the view that insider trading should help the price reflect the fundamental value, because the a_n coefficient in Panel (D) is smaller in magnitude than in Equilibrium 1; recall that a_n measures the deviation of the price from the dividend value (see Equation (4)). However, such a "correction" in price may be short-lived, because the two unstable equilibria are less likely to survive a small perturbation in investors' beliefs. If the equilibrium goes back to the original one, the market is in turmoil. Moreover, it is important to recall that even the original equilibrium may not be stable; the steady state may be fragile. If there is no stable equilibrium, the market may break down upon a perturbation of beliefs caused by the rumor of insider trading. Since we have excluded trading halts from the current data, Figure 9 does not fully warn us against such disastrous outcomes.

1.7.2 Flash crash

While high frequency traders are blamed for the flash crash on May 6, 2010, Kirilenko, Kyle, Samadi, and Tuzun (2016) argue that high frequency traders did not cause the flash crash but exacerbated the downward pressure triggered by a large sell program. An article in *Wall Street Journal* (April 21, 2015) reports a recent charge against an individual trader who operated at his West London home and illustrates the illegal trading strategy called spoofing or layering that allegedly caused the crash.[10] The time it took to identify the causes of the crash and bring a formal charge suggests how ambiguous the market was on the day of the crash.

Panel (A) of Figure 10 plots the quote midpoint (MID, on the left axis) and the cumulative normalized order imbalance ($CSTOV$, on the right axis) of Procter & Gamble (P&G) at one minute intervals on the day of the flash crash. P&G is a constituent of the Dow Jones Industrial Average index that plummeted by almost 37% within minutes. Because of the screen from the previous subsection, which is applied commonly to all trades and quotes at all times, the trough in blue in the panel shows a more modest drop around 2:47pm. However, the negative return in such a short period (specifically, -14% over five minutes) and the subsequent rebound depict a clearly significant event.

Both the trade and price suggest the existence of ambiguity that the flash crash introduced to the market. First, intriguingly, the graph of the cumulative imbalance in pink reverted to the previous level immediately after the flash crash; there is only a tiny dip at 2:47pm. However, it is followed by a sell-off starting at around 3:00pm, which came from apparently confused investors. Moreover, this sell-off is stronger for small trades. Panel (B) plots the cumulative imbalances in selected trade size bins, smallest (Bin 1), medium (Bin 3), and largest (Bin 5). The cumulative imbalance of the smallest trades in blue drops most abruptly. Smallest trades are traditionally interpreted as orders from mostly retail traders (Barber, Odean, and Zhu (2009), Lee and Radhakrishna (2000)), although the distinction between such and fragmented institutional trades is blurred in recent years. However, the time lag in sales makes it less likely that the drop in the cumulative smallest-bin imbalance represents fragmented algorithmic

[10] "'Flash Crash' Charges Filed," *Wall Street Journal*, April 21, 2015, by Aruna Viswanatha, Bradley Hope, and Jenny Strasburg.

trading responding to the crash per se. It also makes sense that a consumer product firm such as P&G may have more retail investors than other firms. Therefore, the sell-off seen in the smallest trades is likely to come from retail traders. In addition, trades of other size, including the largest which is likely institutional, also exhibits a sell pressure.

The price also reflects the apparent confusion by investors. After the crash, MID in Panel (A) has clearly become more volatile and declined in level. An untabulated analysis shows that the average price fell from \$62.3 before the crash (from market opening to 2:40pm) to \$60.7 after the crash (3pm to closing). This is consistent with the discount in price that ambiguity-averse investors require to hold an asset whose supply is ambiguous in nature.

2. Conclusion

This paper highlights the potential threat originating from investors' aversion (or tolerance thereof) to ambiguity about asset supply. Such aversion can make the market unstable and lead to a failure in the extreme. It is ironic that ambiguity tolerance allows the equilibrium to be either liquid, less volatile but unstable, or illiquid, volatile and only possibly stable. The former unstable equilibrium will not survive as investors become averse enough to supply ambiguity. Moreover, the latter equilibrium does not give us a sense of security, as it tends to become less stable with ambiguity aversion. In particular, it will *always* become unstable for large enough ambiguity aversion when investors receive perfectly heterogeneous private signals and do not learn from endowment. In such a case strategic substitutability is at work, and it is extremely difficult for investors to coordinate equilibrium beliefs.

Our analysis leaves promising agenda for future work. First, observe that the informed investors do not perceive ambiguity when their private signals are perfectly correlated ($\rho = 1$) as in Grossman and Stiglitz (1980). This is clear from the bottom branch of Equation (6), where the informed investors condition only on their private signals. In this case they do not use the price or the endowment in updating their beliefs, and are intact from supply ambiguity beyond its contribution to pure risk. The uninformed traders would then have a stronger incentive to acquire information. Thus, I expect the cost of information to be bid up under supply ambiguity.

Second, the issue of disambiguation is left unaddressed. Caskey (2009) shows that ambiguity-averse investors may prefer a single, aggregate, less informative signal to a set of multiple, disaggregate, informative signals, which is unanimously favored by the standard, Savage-type investors who care only about risk. This is guaranteed when the former signal is a sufficient statistic for inferring the dividend. In the normal-CARA case this is when the aggregate signal takes the form of $\mu_{d|\mathbf{s}}$ in Equation (A2), because the conditional mean is the sufficient statistic for normal distribution. Because of continuity, this property would hold for a small perturbation of the signal weights in (A2). This raises another obstacle in achieving an equilibrium when investors are ambiguity averse; they have an incentive not to condition their trades on the price and other signals separately, but on a potentially less informative signal such as a variant of $\mu_{d|\mathbf{s}}$ where the signal weights are manipulated. In the current analysis, investors are not given a choice to receive such an aggregated signal. If they were, an information provider would have an incentive to provide manipulated information. Moreover, the tendency to decondition on the price signal could violate the assumption of learning from prices that is fundamental to REE.

Third, except for my main claims, I have relied on a special case to make my points as in Proposition 3. Proving the general case is an open question. Finally, it will be illuminating to examine stability in real time within a dynamic model, rather than in fictitious time within a static model. One way to explore this direction is with the recursive smooth-ambiguity preference proposed by Ju and Miao (2012).

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Appendix A Derivations

A.1 Deriving the posterior beliefs in Equations (9) and (10)

Since our comprehensive framework allows for various levels of learning, it is convenient to state the following fact on updating that we will invoke repeatedly:

Lemma 2. *(Updating) Suppose there are M signals about $d \sim N(0, \sigma_d^2)$ of the form*

$$\mathbf{s}_m = d + \sqrt{\rho}\varepsilon + e_m, \quad 1 \leq m \leq M,$$

where $\varepsilon \sim N(0, \sigma_\varepsilon^2)$, $e_m \sim N(\mu_{em}, \sigma_{em}^2)$, $\rho \geq 0$ is a constant, and d, ε, and e_m are independent of each other. Then, the conditional precision and mean of d given the vector of signals $\mathbf{s} = \begin{pmatrix} \mathbf{s}_1, & \cdots, & \mathbf{s}_M \end{pmatrix}'$ are given, respectively, by

$$\sigma_{d|\mathbf{s}}^{-2} \equiv var^{-1}(d|\mathbf{s}) = \frac{1}{\sigma_d^2} + \frac{1}{\rho\sigma_\varepsilon^2 + \sigma_e^2}, \tag{A1}$$

$$\mu_{d|\mathbf{s}} \equiv E[d|\mathbf{s}] = \sigma_{d|\mathbf{s}}^2 \frac{1}{1 + \rho\sigma_\varepsilon^2\sigma_e^{-2}} \sum_{m=1}^{M} \frac{\mathbf{s}_m - \mu_{em}}{\sigma_{em}^2}, \tag{A2}$$

where

$$\sigma_e^{-2} \equiv \sum_{m=1}^{M} \sigma_{em}^{-2}$$

is the total precision of the independent component of the signal noise, e_m.

Proof. By the standard formulas for the conditional mean and variance of multivariate normal distribution, with the so-called updating formula (see, e.g., Greene p.822, Equation (A-66)) to invert the covariance matrix. ■

A straightforward application of this lemma gives Equations (9) and (10) in the main text.

It is informative to interpret the economic meaning of the lemma. The case without the common signal component is perhaps familiar to the reader. When $\rho = 0$, Equation (A1) says that the posterior precision is simply the sum of prior dividend and noise precisions. Thus, the posterior belief becomes more accurate as more information becomes available, which is the fundamental feature of Bayes updating. Similarly, if $\rho = 0$, Equation (A2) asserts that the posterior mean is the precision-weighted average of prior means and signals, with weights adding up to 1.[11] The posterior mean shifts from the prior mean (0) by deviations of the realized signals from their prior mean, $\mathbf{s}_m - \mu_{em}$, and more toward precise signals with smaller σ^2_{em}. When $\rho > 0$, the common component of signal noises additionally obscures information, as represented by the $\rho\sigma^2_{\varepsilon}$ term. ■

A.2 Deriving the expected utility in Equation (11)

Using the expressions for wealth in Equation (1), we can calculate the expected utility in (2) as follows:

$$
\begin{aligned}
&\max_{x_j} E[h(E[u(w_j)|\mathbf{s}_j,\theta])|\mathbf{s}_j] \\
&= E[h(E[-\exp(-\gamma\overbrace{\{(d-p)x_j + pn_j\}}^{=w_j})|\mathbf{s}_j,\theta])|\mathbf{s}_j] \\
&= E[-\exp(-g\gamma\{(E[d|\mathbf{s}_j,\theta]-p)x_j - (\gamma/2)\sigma^2_{d|\mathbf{s}_j\theta}x_j^2 + pn_j\})|\mathbf{s}_j] \\
&= -\exp\left(-g\gamma\left[(E[E(d|\mathbf{s}_j,\theta)|\mathbf{s}_j]-p)x_j - \frac{\gamma}{2}\{\sigma^2_{d|\mathbf{s}_j\theta} + gVar(E[d|\mathbf{s}_j,\theta]|\mathbf{s}_j)\}x_j^2 + pn_j\right]\right) \\
&= -\exp\left(-g\gamma\left[(E[d|\mathbf{s}_j]-p)x_j - \frac{\gamma}{2}\{\sigma^2_{d|\mathbf{s}_j\theta} + g(\sigma^2_{d|\mathbf{s}_j} - \sigma^2_{d|\mathbf{s}_j\theta})\}x_j^2 + pn_j\right]\right),
\end{aligned}
$$

where we have used both the law of iterated expectations and the law of total variance in the last equality. This yields Equation (11) in the main text. ■

A.3 Deriving the expressions for the demand coefficients in Equation (12)

Noting that private signals s_j and n_j are contained only in the private assessment of the mean dividend $\mu_{d|\mathbf{s}_j}$ in Equation (12), first compare coefficients on s_j, n_j, p, and the constant term, respectively, using Equation (9) with $\xi = (p - a_0)/a_d$ (see Equation (5)) for informed

[11] As usual, the weights add up to 1. To see this, explicitly incorporate $\mu_d = 0$ in Equation (A2):

$$\mu_{d|\mathbf{s}} \equiv E[d|\mathbf{s}] = \mu_d + \sigma^2_{d|\mathbf{s}}\frac{1}{1+\rho\sigma^2_{\varepsilon}\sigma_e^{-2}}\sum_{m=1}^{M}\frac{\mathbf{s}_m - E[\mathbf{s}_m]}{\sigma^2_{em}},$$

where $E[\mathbf{s}_m] = \mu_d + \sqrt{\rho}\cdot 0 + \mu_{em}$ is the prior mean of the signal. The weights on $\mathbf{s}_m$ and $E[\mathbf{s}_m]$ cancel out, and the weight on μ_d in the first term is 1. The weights are the ratio of the individual precision to the total precision, $\sigma^{-2}_{em}(1+\rho\sigma^2_{\varepsilon}\sigma_e^{-2})^{-1}/\sigma^{-2}_{d|\mathbf{s}}$.

traders:

$$b_{sI} = \frac{\sigma^2_{d|\mathbf{s}_I}}{\gamma\sigma^2_{d|\mathbf{s}_I a}} \frac{\sigma^{-2}_{\varepsilon,j}}{1-\rho+\rho\sigma^2_\varepsilon\hat{\sigma}^{-2}_{eI}(\lambda)}, \tag{A3}$$

$$b_{nI} = \frac{\sigma^2_{d|\mathbf{s}_I}}{\gamma\sigma^2_{d|\mathbf{s}_I a}} \frac{(1-\rho)\lambda^{-2}\sigma^{-2}_\zeta\cdot(-\lambda)}{1-\rho+\rho\sigma^2_\varepsilon\hat{\sigma}^{-2}_{eI}(\lambda)}, \tag{A4}$$

$$b_{pI} = \frac{1}{\gamma\sigma^2_{d|\mathbf{s}_I a}} \left[\sigma^2_{d|\mathbf{s}_I} \frac{(1-\rho)\lambda^{-2}[\sigma^{-2}_\zeta+(\sigma^2_n+\sigma^2_\theta)^{-1}]/a_d}{1-\rho+\rho\sigma^2_\varepsilon\hat{\sigma}^{-2}_{eI}(\lambda)} - 1\right], \tag{A5}$$

$$b_{0I} = \frac{\sigma^2_{d|\mathbf{s}_I}}{\gamma\sigma^2_{d|\mathbf{s}_I a}} \frac{(1-\rho)\lambda^{-2}\{[\sigma^{-2}_\zeta+(\sigma^2_n+\sigma^2_\theta)^{-1}](-a_0/a_d)+(\sigma^2_n+\sigma^2_\theta)^{-1}(-\lambda\mu_\theta)\}}{1-\rho+\rho\sigma^2_\varepsilon\hat{\sigma}^{-2}_{eI}(\lambda)}, \tag{A6}$$

where $\hat{\sigma}^{-2}_{eI}(\lambda)$ is defined in Equation (8), and we have substituted the last expression in (5) for ξ in deriving the relation for b_{pI}. Similarly, for the uninformed traders,

$$b_{sU} = 0, \tag{A7}$$

$$b_{nU} = \frac{\sigma^2_{d|\mathbf{s}_U}}{\gamma\sigma^2_{d|\mathbf{s}_U a}} \frac{\lambda^{-2}\sigma^{-2}_\zeta\cdot(-\lambda)}{1+\rho\sigma^2_\varepsilon\lambda^{-2}[\sigma^{-2}_\zeta+(\sigma^2_n+\sigma^2_\theta)^{-1}]}, \tag{A8}$$

$$b_{pU} = \frac{1}{\gamma\sigma^2_{d|\mathbf{s}_U a}} \left[\sigma^2_{d|\mathbf{s}_U} \frac{\lambda^{-2}[\sigma^{-2}_\zeta+(\sigma^2_n+\sigma^2_\theta)^{-1}]/a_d}{1+\rho\sigma^2_\varepsilon\lambda^{-2}[\sigma^{-2}_\zeta+(\sigma^2_n+\sigma^2_\theta)^{-1}]} - 1\right], \tag{A9}$$

$$b_{0U} = \frac{\sigma^2_{d|\mathbf{s}_U}}{\gamma\sigma^2_{d|\mathbf{s}_U a}} \frac{\lambda^{-2}\{[\sigma^{-2}_\zeta+(\sigma^2_n+\sigma^2_\theta)^{-1}](-a_0/a_d)+(\sigma^2_n+\sigma^2_\theta)^{-1}(-\lambda\mu_\theta)\}}{1+\rho\sigma^2_\varepsilon\lambda^{-2}[\sigma^{-2}_\zeta+(\sigma^2_n+\sigma^2_\theta)^{-1}]}. \tag{A10}$$

■

A.4 Solving for all coefficients

At the fixed point of Equation (19), $\psi(b)$ gives λ. Then, substituting Equations (A5) and (A9) for b_{pI} and b_{pU}, respectively, in (15) allows us to solve for a_d, which also fixes $a_n = a_d\lambda$ as well as b_{pI} and b_{pU}. Specifically, rewrite Equations (A5) and (A9) as

$$b_{pI} = -\frac{b_{nI}}{a_d}\frac{\sigma^{-2}_\zeta+(\sigma^2_n+\sigma^2_\theta)^{-1}}{\lambda\sigma^{-2}_\zeta} - \frac{1}{\gamma\sigma^2_{d|\mathbf{s}_I a}},$$

$$b_{pU} = -\frac{b_{nU}}{a_d}\frac{\sigma^{-2}_\zeta+(\sigma^2_n+\sigma^2_\theta)^{-1}}{\lambda\sigma^{-2}_\zeta} - \frac{1}{\gamma\sigma^2_{d|\mathbf{s}_U a}}.$$

Substituting into Equation (15) gives

$$a_d = \omega b_{sI}\left[\omega\left(\frac{b_{nI}}{a_d}\frac{\sigma_\zeta^{-2}+(\sigma_n^2+\sigma_\theta^2)^{-1}}{\lambda\sigma_\zeta^{-2}}+\frac{1}{\gamma\sigma^2_{d|\mathbf{s}_I a}}\right)+(1-\omega)\left(\frac{b_{nU}}{a_d}\frac{\sigma_\zeta^{-2}+(\sigma_n^2+\sigma_\theta^2)^{-1}}{\lambda\sigma_\zeta^{-2}}+\frac{1}{\gamma\sigma^2_{d|\mathbf{s}_U a}}\right)\right]^{-1}.$$

Solving for a_d yields

$$a_d = \gamma\left(\frac{\omega}{\sigma^2_{d|\mathbf{s}_I a}}+\frac{1-\omega}{\sigma^2_{d|\mathbf{s}_U a}}\right)^{-1}\left[\omega b_{sI}-[\omega b_{nI}+(1-\omega)b_{nU}]\frac{1+\sigma_\zeta^2(\sigma_n^2+\sigma_\theta^2)^{-1}}{\lambda}\right].$$

Finally, substituting Equations (A6) and (A10) for b_{0I} and b_{0U}, respectively, in (17) determines a_0 and, in turn, b_{0I} and b_{0U}. ■

A.5 Computation of volume

Market clearing requires that the mean of Δx_j is zero:

$$\int_j \Delta x_j = n - n = 0.$$

Equation (26) follows from the fact that, for any normally distributed variable $\widetilde{x}$ with a zero mean,

$$E[|\widetilde{x}|] = \sqrt{\frac{2}{\pi}var(\widetilde{x})}.$$

It suffices to explicitly compute Δx_j in Equation (25):

$$\begin{aligned}
\Delta x_j = x_j - n_j &= b_{sj}s_j + b_{nj}n_j + b_{pj}p + b_{0j} - n_j \\
&= b_{sj}(d+\sqrt{\rho}\varepsilon+\sqrt{1-\rho}\varepsilon_j) + (b_{nj}-1)(n+\zeta_j) \\
&+ b_{pj}[a_d(d+\sqrt{\rho}\varepsilon)+a_n n + a_0] + b_{0j} \\
&= (b_{sj}+b_{pj}a_d)(d+\sqrt{\rho}\varepsilon) + b_{sj}\sqrt{1-\rho}\varepsilon_j \\
&+ (b_{nj}-1+b_{pj}a_n)n + (b_{nj}-1)\zeta_j + b_{pj}a_0 + b_{0j},
\end{aligned}$$

and thus

$$\begin{aligned}
var(\Delta x_j) = &(b_{sj}+b_{pj}a_d)^2(\sigma_d^2+\rho\sigma_\varepsilon^2) + b_{sj}^2(1-\rho)\sigma^2_{\varepsilon,j} \\
&+ (b_{nj}-1+b_{pj}a_n)^2(\sigma_n^2+\sigma_\theta^2) + (b_{nj}-1)^2\sigma_\zeta^2.
\end{aligned}$$

Substituting this expression for $j = I, U$ into Equation (26) gives the formula for volume. ■

Appendix B Proofs

B.1 Proof of Lemma 1

To show the necessity of the existence condition, feed both sides of Equation (19) into mapping ψ to obtain $\psi(b) = \psi \circ \varphi(\psi(b))$, and write λ for $\psi(b)$. Conversely, substituting Equation (21) into mapping φ and replacing $\varphi(\lambda)$ by b produces Equation (19). Therefore, Equation (21) is a necessary and sufficient condition for (19). To show the condition for stability succinctly, define the 3 by 1 vectors $\frac{\partial\varphi}{\partial\lambda} \equiv \varphi_\lambda$ and $\frac{\partial\psi}{\partial b} \equiv \psi_b$. Post-multiplying φ_λ to the 3 by 3 matrix $\varphi_\lambda\psi_b'$ yields $\varphi_\lambda\psi_b' \cdot \varphi_\lambda = (\psi_b'\varphi_\lambda)\varphi_\lambda$, which shows that the Jacobian matrix $\varphi_\lambda\psi_b'$ has eigenvalue $\psi_b'\varphi_\lambda$ with the corresponding eigenvector φ_λ. The other two eigenvalues are 0, with the corresponding eigenvectors being any two linearly independent vectors in $\Re^3$ that are orthogonal to ψ_b. Therefore, the scalar $\psi_b'\varphi_\lambda = f'(\lambda)$ equals the only non-zero eigenvalue of matrix $\varphi_\lambda\psi_b'$, and hence $\max|eig\,(\varphi_\lambda\psi_b')| = |\psi_b'\varphi_\lambda|$. Rewriting this in the original notation gives $\max\left|eig\left(\frac{\partial\varphi}{\partial\lambda}\frac{\partial\psi}{\partial b'}\right)\right| = \left|\frac{\partial\psi}{\partial b'}\frac{\partial\varphi}{\partial\lambda}\right| = |f'(\lambda)|$. ■

B.2 Proof of Proposition 1

We will first show that $f(\lambda)$ converges to a finite negative number as $\lambda \to -\infty$, while the other limit as $\lambda \to -0$ is negative when $\rho > 0$ and diverges to positive infinity when $\rho = 0$ (Equations (A14) and (A15) below). To this end, explicitly write

$$\begin{aligned}
\sigma_{d|\mathbf{s}_I}^{-2}\{\sigma_{\varepsilon,j}^{-2}\} &= \sigma_d^{-2} + \frac{1}{\rho\sigma_\varepsilon^2 + (1-\rho)\{\sigma_{\varepsilon,j}^{-2} + (1-\rho)\lambda^{-2}[\sigma_\zeta^{-2} + (\sigma_n^2+\sigma_\theta^2)^{-1}]\}^{-1}} \\
&= \sigma_d^{-2} + \frac{\sigma_{\varepsilon,j}^{-2} + (1-\rho)\lambda^{-2}[\sigma_\zeta^{-2} + (\sigma_n^2+\sigma_\theta^2)^{-1}]}{1-\rho+\rho\sigma_\varepsilon^2\{\sigma_{\varepsilon,j}^{-2} + (1-\rho)\lambda^{-2}[\sigma_\zeta^{-2} + (\sigma_n^2+\sigma_\theta^2)^{-1}]\}} \\
&= \sigma_d^{-2} + \frac{1}{\rho\sigma_\varepsilon^2}\left[1 - \frac{1-\rho}{1-\rho+\rho\sigma_\varepsilon^2\{\sigma_{\varepsilon,j}^{-2} + (1-\rho)\lambda^{-2}[\sigma_\zeta^{-2} + (\sigma_n^2+\sigma_\theta^2)^{-1}]\}}\right] \qquad \text{(A11)} \\
\sigma_{d|\mathbf{s}_U}^{-2} &= \sigma_d^{-2} + \{\rho\sigma_\varepsilon^2 + \lambda^2/[\sigma_\zeta^{-2} + (\sigma_n^2+\sigma_\theta^2)^{-1}]\}^{-1} \quad (= \sigma_{d|\mathbf{s}_I}^{-2}\{0\} \text{ if } \rho < 1) \\
\sigma_{d|\mathbf{s}_j}^{-2}\sigma_{d|\mathbf{s}_j a}^2 &= g + (1-g)\frac{\sigma_{d|\mathbf{s}_j\theta}^2}{\sigma_{d|\mathbf{s}_j}^2}.
\end{aligned}$$

So, for the informed traders' strategy in the limit as $\lambda \to -\infty$, we have

$$\lim_{\lambda\to-\infty}\hat{\sigma}_{eI}^{-2}(\lambda) = \sigma_{\varepsilon,j}^{-2}, \quad \lim_{\lambda\to-\infty}\sigma_{d|\mathbf{s}_I}^{-2} = \sigma_d^{-2} + [\rho\sigma_\varepsilon^2 + (1-\rho)\sigma_{\varepsilon,j}^2]^{-1},$$

$$\lim_{\lambda\to-\infty}\sigma_{d|\mathbf{s}_I}^{-2}\sigma_{d|\mathbf{s}_I a}^2 = 1, \quad \lim_{\lambda\to-\infty} b_{sI} = \frac{1}{\gamma}\frac{\sigma_{\varepsilon,j}^{-2}}{1-\rho+\rho\sigma_\varepsilon^2\sigma_{\varepsilon,j}^{-2}} = \frac{1}{\gamma}\frac{1}{(1-\rho)\sigma_{\varepsilon,j}^2 + \rho\sigma_\varepsilon^2}.$$

In the other limit as as $\lambda \to -0$,

$$\lim_{\lambda \to -0} \hat{\sigma}_{eI}^{-2}(\lambda) = \begin{cases} \propto (1-\rho)\lambda^{-2}[\sigma_\zeta^{-2} + (\sigma_n^2 + \sigma_\theta^2)^{-1}] \text{ if } \rho < 1, \\ \sigma_{\varepsilon,j}^{-2} \text{ if } \rho = 1, \end{cases},$$

$$\lim_{\lambda \to -0} \sigma_{d|\mathbf{s}_I}^{-2} = \begin{cases} \propto \lambda^{-2}[\sigma_\zeta^{-2} + (\sigma_n^2 + \sigma_\theta^2)^{-1}] \text{ if } \rho = 0, \\ \sigma_d^{-2} + 1/\rho\sigma_\varepsilon^2 \text{ if } \rho > 0, \end{cases},$$

$$\lim_{\lambda \to -0} \sigma_{d|\mathbf{s}_I}^{-2}\sigma_{d|\mathbf{s}_I a}^2 = \begin{cases} g + (1-g)\frac{\sigma_\zeta^{-2} + (\sigma_n^2+\sigma_\theta^2)^{-1}}{\sigma_\zeta^{-2}+\sigma_n^{-2}} \text{ if } \rho = 0, \\ 1 \text{ if } \rho > 0, \end{cases},$$

$$\lim_{\lambda \to -0} b_{sI} = \begin{cases} \frac{1}{\gamma\sigma_{\varepsilon,j}^2}\left[g + (1-g)\frac{\sigma_\zeta^{-2} + (\sigma_n^2+\sigma_\theta^2)^{-1}}{\sigma_\zeta^{-2}+\sigma_n^{-2}}\right]^{-1} > 0 \text{ if } \rho = 0, \\ 0 \text{ if } 0 < \rho < 1 \\ \equiv 1/\gamma\sigma_\varepsilon^2 > 0 \text{ regardless of } \lambda \text{ if } \rho = 1, \end{cases} \tag{A12}$$

$$\lim_{\lambda \to -0} b_{nI} = \begin{cases} \infty \text{ if } \rho = 0, \\ 0 \text{ if } 0 < \rho < 1 \\ \equiv 0 \text{ regardless of } \lambda \text{ if } \rho = 1, \end{cases}$$

For the uninformed traders' strategy in the limit as $\lambda \to -\infty$, we have

$$\lim_{\lambda \to -\infty} \sigma_{d|\mathbf{s}_U}^{-2} = \sigma_d^{-2}, \quad \lim_{\lambda \to -\infty} \sigma_{d|\mathbf{s}_U}^{-2}\sigma_{d|\mathbf{s}_U a}^2 = 1, \quad \lim_{\lambda \to -\infty} b_{nU} = 0.$$

In the other limit,

$$\lim_{\lambda \to -0} \sigma_{d|\mathbf{s}_U}^{-2} \propto \lambda^{-2}[\sigma_\zeta^{-2} + (\sigma_n^2 + \sigma_\theta^2)^{-1}], \quad \lim_{\lambda \to -0} \sigma_{d|\mathbf{s}_U}^{-2}\sigma_{d|\mathbf{s}_U a}^2 = g + (1-g)\frac{\sigma_\zeta^{-2} + (\sigma_n^2+\sigma_\theta^2)^{-1}}{\sigma_\zeta^{-2}+\sigma_n^{-2}},$$

$$\lim_{\lambda \to -0} b_{nU} = \begin{cases} \infty \text{ if } \rho = 0, \\ 0 \text{ if } \rho > 0. \end{cases} \tag{A13}$$

Thus,

$$\lim_{\lambda \to -\infty} f(\lambda) = -\frac{\gamma}{\omega}[(1-\rho)\sigma_{\varepsilon,j}^2 + \rho\sigma_\varepsilon^2] < 0, \tag{A14}$$

$$\lim_{\lambda \to -0} f(\lambda) = \begin{cases} \frac{\omega \cdot \infty + (1-\omega)\infty - 1}{\text{finite positive}} = \infty \text{ regardless of } \omega \text{ if } \rho = 0, \\ \frac{\omega \cdot 0 + (1-\omega)0 - 1}{\omega \cdot 0} = -\infty \text{ if } 0 < \rho < 1, \\ \frac{\omega \cdot 0 + (1-\omega)0 - 1}{\omega \cdot 1/\gamma\sigma_\varepsilon^2} = \frac{-\gamma\sigma_\varepsilon^2}{\omega} < 0 \text{ if } \rho = 1. \end{cases} \tag{A15}$$

This implies that the graph of $f(\lambda)$ crosses that of λ at least once as long as $\rho > 0$, guaranteeing the existence.

To derive a sufficient condition for existence when $\rho = 0$, or indeed when $0 \leq \rho < 1$, note the fact that

$$\lambda + \frac{(1-\rho)\sigma_{\varepsilon,j}^2}{\sigma_\zeta^2 \lambda} \leq -2\sqrt{\frac{(1-\rho)\sigma_{\varepsilon,j}^2}{\sigma_\zeta^2}}, \tag{A16}$$

where the equality holds at λ given by

$$\lambda_{\#} \equiv -\sqrt{\frac{(1-\rho)\sigma_{\varepsilon,j}^2}{\sigma_\zeta^2}}.$$

Equation (24) in the proposition says that the value of f_{UI} is no more than the left hand side of Equation (A16) where the latter attains the maximum, or equivalently that $f(\lambda_{\#}) \leq \lambda_{\#}$. Since, again, $f(\lambda)$ converges to a finite negative number as $\lambda \to -\infty$ (see (A14)) and diverges to ∞ as $\lambda \to -0$ (see the first branch of (A15)), this implies that the graph of $f(\lambda)$ intersects that of λ at least once, and at least twice with a strict inequality, when condition (24) holds under $\rho = 0$. ■

B.3 Proof of Proposition 2

We show that larger ambiguity aversion makes investors' strategy less aggressive (a smaller norm of b) and their beliefs more pessimistic (larger absolute $f(\lambda)$). Observe that g affects the graph of $f(\lambda)$ in Equation (23) only through $b_{sI}(\lambda)$ and $b_{nU}(\lambda)$ in (A3) and (A8), respectively. These demand coefficients contain the reciprocal of the curly bracket in Equations (13) and (11),

$$\sigma_{d|\mathbf{s}_j}^2 \sigma_{d|\mathbf{s}_j a}^{-2} = \left\{ g + (1-g)\frac{\sigma_{d|\mathbf{s}_j\theta}^2}{\sigma_{d|\mathbf{s}_j}^2} \right\}^{-1}, \quad j = I, U.$$

Since $\sigma_{d|\mathbf{s}_j}^2 > \sigma_{d|\mathbf{s}_j\theta}^2$ (recall the discussion of Equation (11)), this variance ratio decreases in g and so do $b_{sI}(\lambda)$ and $b_{nU}(\lambda)$, holding everything else *including* λ constant.[12] This implies a less aggressive trading strategy.

Therefore, as g increases, the numerator of $f_{UI}(\lambda)$ in Equation (23) becomes negative and larger in magnitude, while its denominator becomes positive and smaller. Thus, $f_{UI}(\lambda)$ becomes negative and larger in magnitude for any given λ, consistent with a more "pessimistic" price belief. Since the first term in Equation (23) does not depend on g, the fixed point $\lambda = f(\lambda)$ must become negative and larger in magnitude where the graph of $y = f(\lambda)$ intersects that of

[12] Note the emphasis. Here we are holding the domain λ also constant to show that the whole graph of $f(\lambda)$ shifts down. It does *not* mean that equilibrium $b_{nU}(\lambda^*)$ and $b_{sI}(\lambda^*)$ also unanimously decrease, because a change in g also affects λ^* and the *ceteris paribus* condition is violated; observe this from Panels (A) to (C) of Figure 5, for example.

$y = \lambda$ from above (odd-numbered equilibria) and negative and smaller in magnitude where it does from below (even-numbered equilibria).

To show the last point more formally, implicit differentiation of Equation (23) gives

$$\frac{\partial \lambda}{\partial g} = -\frac{\partial[\lambda - f(\lambda)]}{\partial g}\left[\frac{\partial[\lambda - f(\lambda)]}{\partial \lambda}\right]^{-1} = \frac{\partial f(\lambda)/\partial g}{1 - f'(\lambda)},$$

where the prime indicates the derivative. By construction, $1 - f'(\lambda)$ is positive in odd-numbered equilibria and negative in even-numbered equilibria. Since $\partial f(\lambda)/\partial g < 0$ for any λ, it follows that $\partial\lambda/\partial g < 0$ in odd-numbered equilibria while $\partial\lambda/\partial g > 0$ in even-numbered equilibria. It is easy to see that all equilibria but the first will disappear as g rises. ■

B.4 Proof of Proposition 3

When $\rho = 0$, Equation (13) reduces to

$$\begin{aligned} b_{sI} &= \frac{1}{\gamma\sigma_{\varepsilon,j}^2}\frac{\sigma_{d|\mathbf{s}_I}^2}{\sigma_{d|\mathbf{s}_I a}^2} = \frac{1}{\gamma\sigma_{\varepsilon,j}^2}\left[g + (1-g)\frac{\sigma_d^{-2} + \sigma_{\varepsilon,j}^{-2} + \lambda^{-2}[\sigma_\zeta^{-2} + (\sigma_n^2 + \sigma_\theta^2)^{-1}]}{\sigma_d^{-2} + \sigma_{\varepsilon,j}^{-2} + \lambda^{-2}[\sigma_\zeta^{-2} + \sigma_n^{-2}]}\right]^{-1} \\ &= \frac{1}{\gamma\sigma_{\varepsilon,j}^2}\left[g + (1-g)\left\{1 - \frac{\sigma_n^{-2} - (\sigma_n^2 + \sigma_\theta^2)^{-1}}{\lambda^2(\sigma_d^{-2} + \sigma_{\varepsilon,j}^{-2}) + \sigma_\zeta^{-2} + \sigma_n^{-2}}\right\}\right]^{-1} \\ &= \frac{1}{\gamma\sigma_{\varepsilon,j}^2}\left[1 + (g-1)\frac{\sigma_n^{-2} - (\sigma_n^2 + \sigma_\theta^2)^{-1}}{\lambda^2(\sigma_d^{-2} + \sigma_{\varepsilon,j}^{-2}) + \sigma_\zeta^{-2} + \sigma_n^{-2}}\right]^{-1} > 0. \end{aligned} \tag{A17}$$

If further $\sigma_\zeta^{-2} = 0$, the first term in (23) is zero as well as $b_{nU} = 0$ (see (A8)). Then with (A17), we have

$$f(\lambda) = -\frac{1}{\omega b_{sI}} = -\frac{\gamma\sigma_{\varepsilon,j}^2}{\omega}\left[1 + (g-1)\frac{\sigma_n^{-2} - (\sigma_n^2 + \sigma_\theta^2)^{-1}}{\lambda^2(\sigma_d^{-2} + \sigma_{\varepsilon,j}^{-2}) + \sigma_\zeta^{-2} + \sigma_n^{-2}}\right]. \tag{A18}$$

Its derivative is

$$\begin{aligned} f'(\lambda) &= \frac{\gamma\sigma_{\varepsilon,j}^2}{\omega}(g-1)\frac{[\sigma_n^{-2} - (\sigma_n^2 + \sigma_\theta^2)^{-1}] \cdot 2\lambda(\sigma_d^{-2} + \sigma_{\varepsilon,j}^{-2})}{[\lambda^2(\sigma_d^{-2} + \sigma_{\varepsilon,j}^{-2}) + \sigma_\zeta^{-2} + \sigma_n^{-2}]^2} < 0 \\ &= -\left[f(\lambda) + \frac{\gamma\sigma_{\varepsilon,j}^2}{\omega}\right]\frac{2\lambda(\sigma_d^{-2} + \sigma_{\varepsilon,j}^{-2})}{\lambda^2(\sigma_d^{-2} + \sigma_{\varepsilon,j}^{-2}) + \sigma_\zeta^{-2} + \sigma_n^{-2}} \\ &\to -2 \end{aligned}$$

at the fixed point, $f(\lambda) = \lambda$, as $g \to \infty$ and $\lambda \to -\infty$ in the first equilibrium by Proposition 2.
■

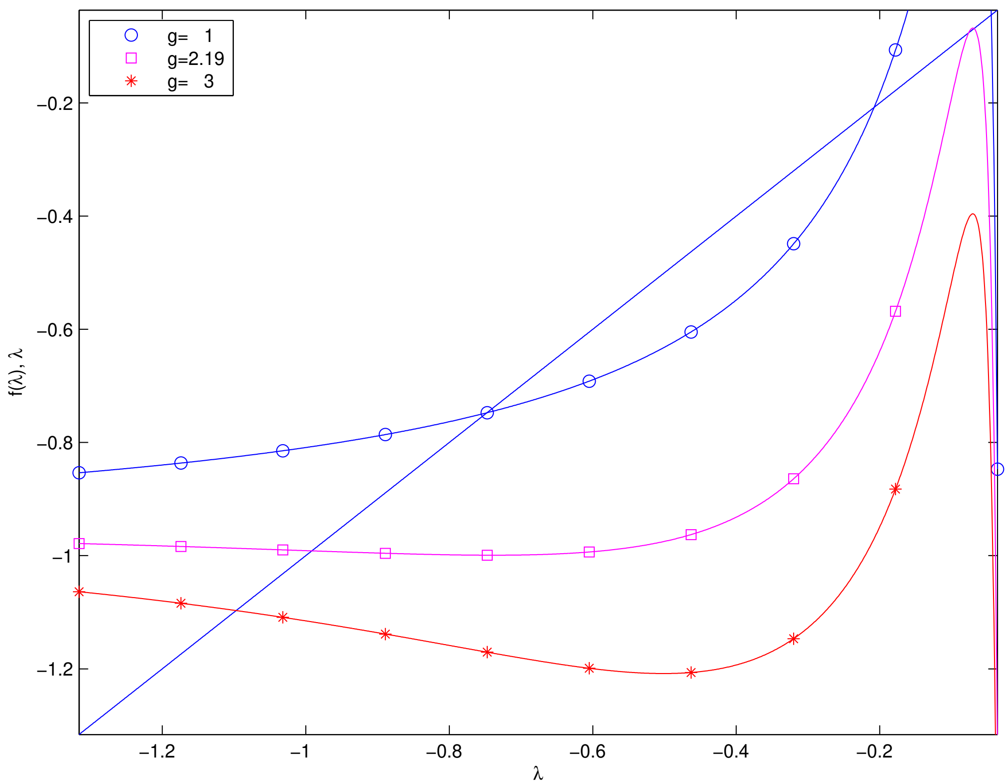

Figure 1: **Effect of ambiguity aversion**. An equilibrium is represented by an intersection of the graphs of λ (the 45° degree line) and $f(\lambda)$ for a given value of g shown in the legend. Parameter values: $\gamma = 1$, $\sigma_d^2 = 1$, $\rho = 0.01$, $\sigma_\varepsilon^2 = \sigma_{\varepsilon,j}^2 = \sigma_n^2 = \sigma_\theta^2 = 1$, $\sigma_\zeta^2 = 5$, $\omega = 1$.

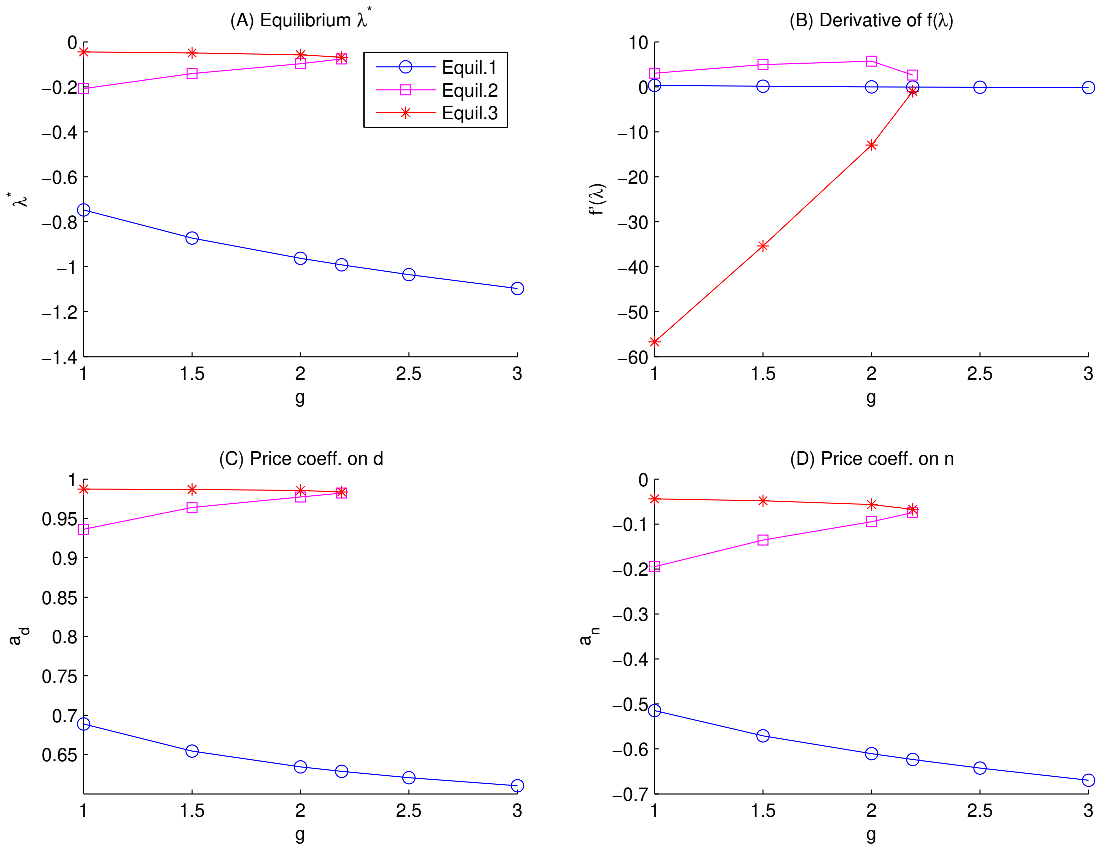

Figure 2: **Ambiguity aversion and price coefficients**. Parameter values: $\gamma = 1$, $\sigma_d^2 = 1$, $\rho = 0.01$, $\sigma_\varepsilon^2 = \sigma_{\varepsilon,j}^2 = \sigma_n^2 = \sigma_\theta^2 = 1$, $\sigma_\zeta^2 = 5$, $\omega = 1$.

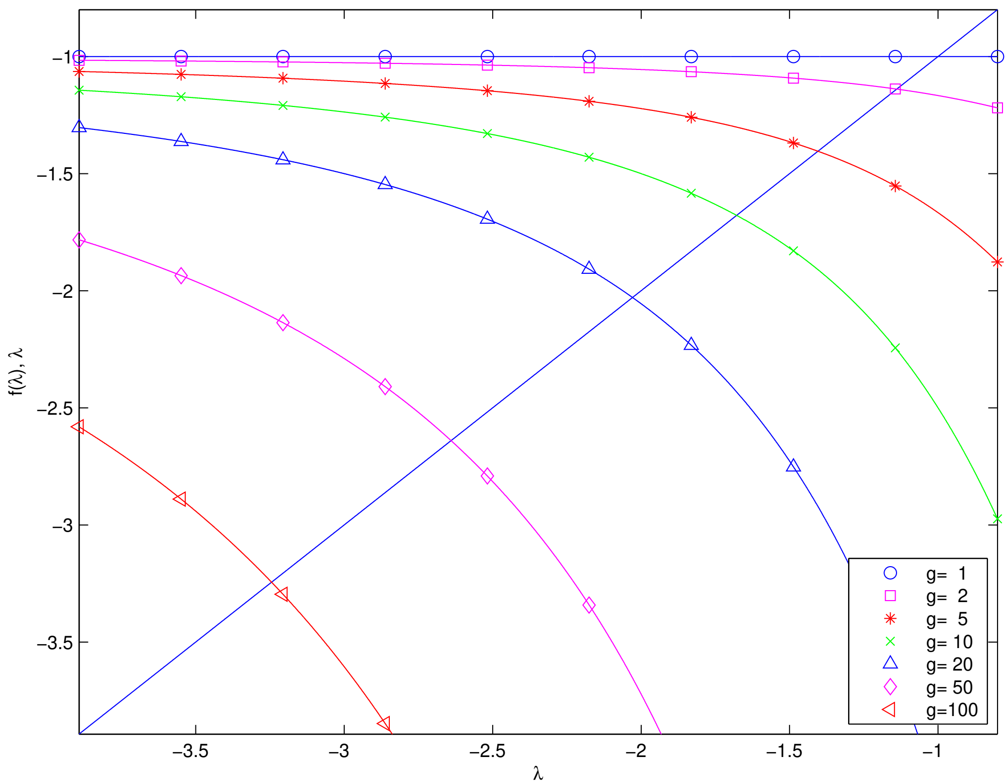

Figure 3: **Instability with large ambiguity aversion**. An equilibrium is represented by an intersection of the graphs of λ (the 45° degree line) and $f(\lambda)$ for a given value of g shown in the legend. The figure shows the case with no common noise in private signals ($\rho = 0$) and effectively uninformative endowment ($\sigma_\zeta^{-2} \approx 0$). The slope $f'(\lambda)$ at the intersection converges to -2 as g increases. Parameter values: $\gamma = 1$, $\sigma_d^2 = 1$, $\rho = 0$, $\sigma_\varepsilon^2 = \sigma_{\varepsilon,j}^2 = \sigma_n^2 = \sigma_\theta^2 = 1$, $\sigma_\zeta^2 = 10^{20}$, $\omega = 1$.

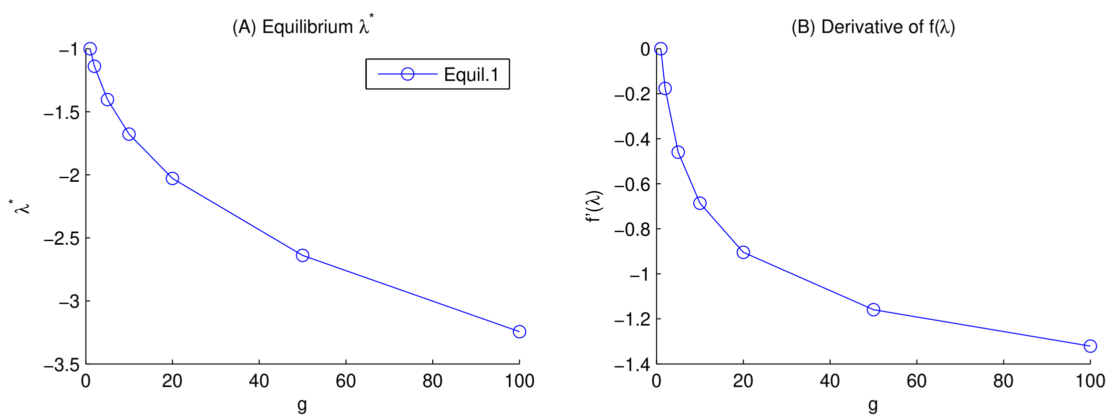

Figure 4: **The fixed-point λ^* and the slope $f'(\lambda^*)$.** The figure shows the equilibrium value of λ and the derivative of f under no common noise in private signals ($\rho = 0$) and effectively uninformative endowment ($\sigma_\zeta^{-2} \approx 0$). Parameter values: $\gamma = 1$, $\sigma_d^2 = 1$, $\rho = 0$, $\sigma_\varepsilon^2 = \sigma_{\varepsilon,j}^2 = \sigma_n^2 = \sigma_\theta^2 = 1$, $\sigma_\zeta^2 = 10^{20}$, $\omega = 1$.

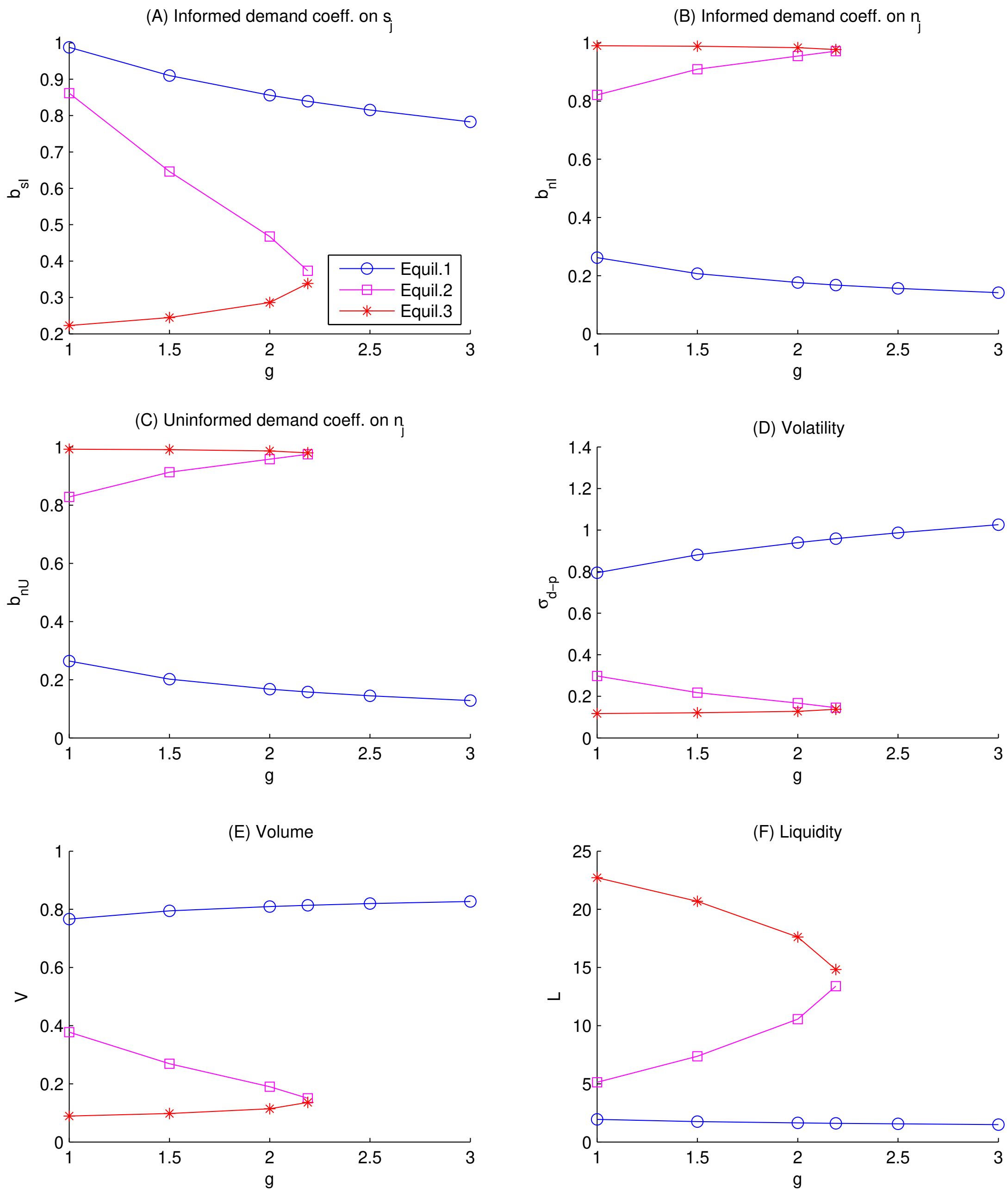

Figure 5: **Ambiguity aversion and equilibrium characteristics**. The figure plots the following quantities: (A) the informed investors' demand coefficient on private signal, b_{sI}; (B) the informed investors' demand coefficient on endowment, b_{nI}; (C) the uninformed investors' (shadow) demand coefficient on endowment, b_{nU}; (D) volatility, σ_{d-p}; (E) volume, V; (F) liquidity, L. Parameter values: $\gamma = 1$, $\sigma_d^2 = 1$, $\rho = 0.01$, $\sigma_\varepsilon^2 = \sigma_{\varepsilon,j}^2 = \sigma_n^2 = \sigma_\theta^2 = 1$, $\sigma_\zeta^2 = 5$, $\omega = 1$.

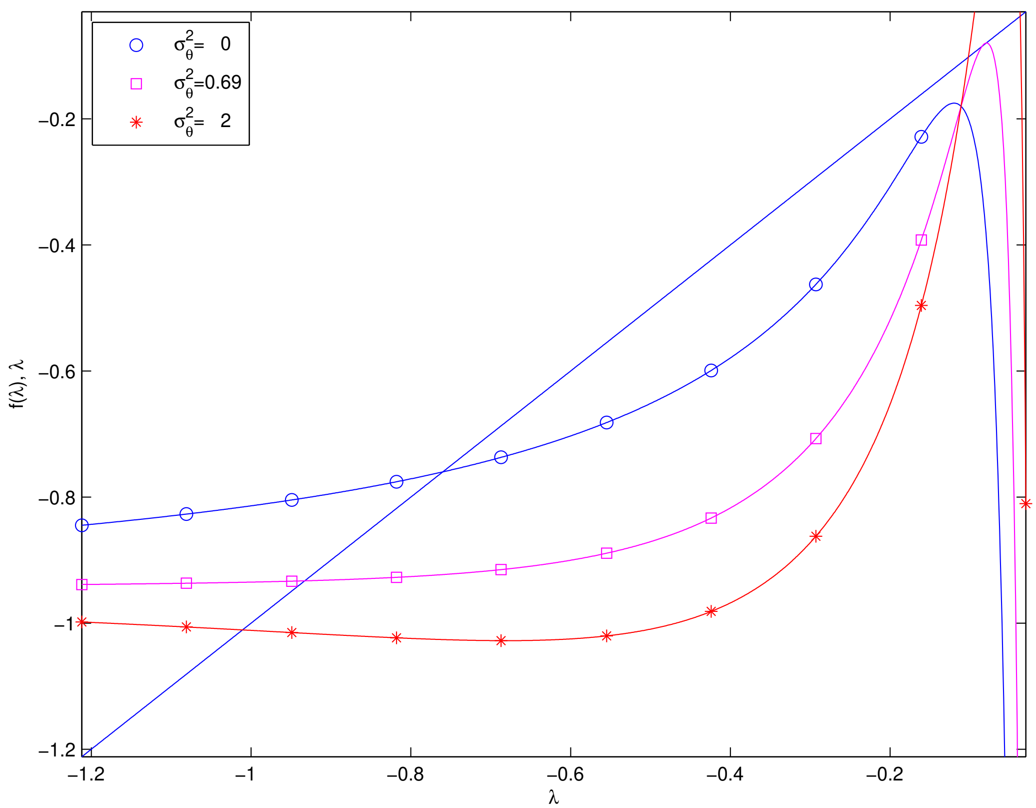

Figure 6: **Effect of supply ambiguity**. An equilibrium is represented by an intersection of the graphs of λ (the 45° degree line) and $f(\lambda)$ for a given value of σ_θ^2 shown in the legend. Parameter values: $\gamma = 1$, $g = 2$, $\sigma_d^2 = 1$, $\rho = 0.01$, $\sigma_\varepsilon^2 = \sigma_{\varepsilon,j}^2 = \sigma_n^2 =$, $\sigma_\zeta^2 = 5$, $\omega = 1$.

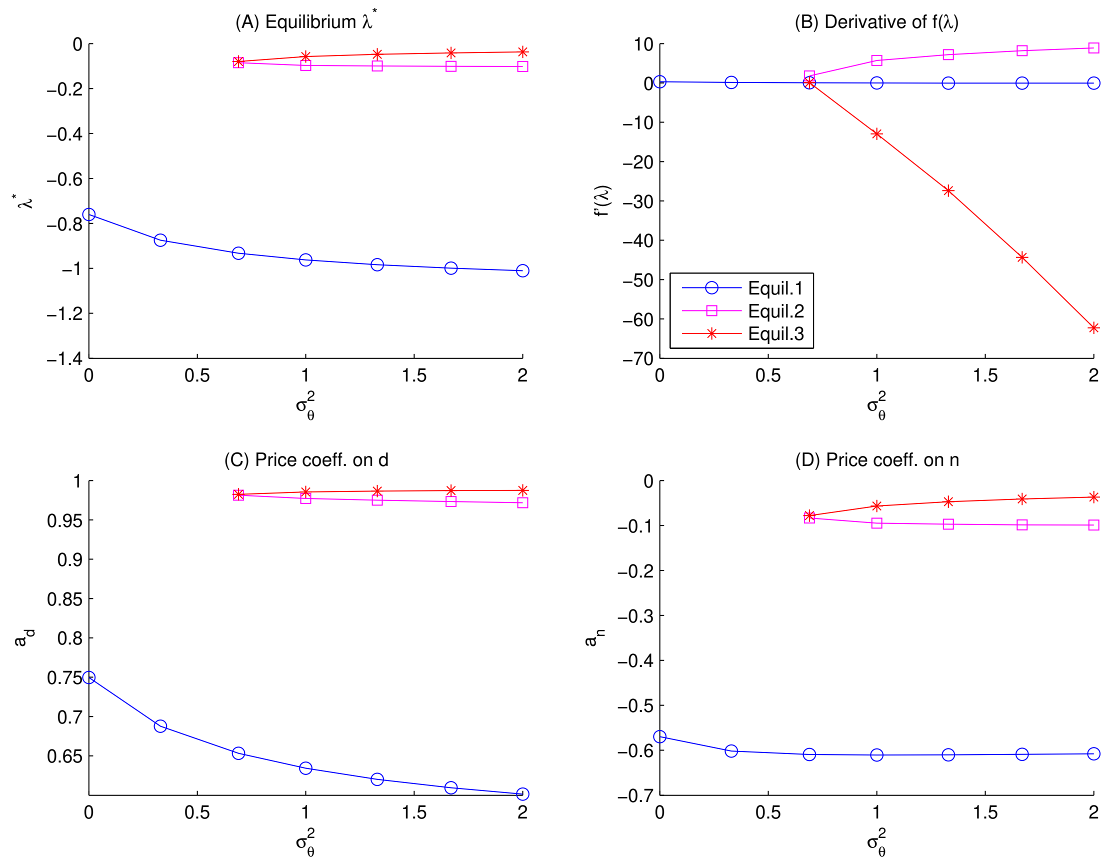

Figure 7: **Supply ambiguity and price coefficients**. Parameter values: $\gamma = 1$, $g = 2$, $\sigma_d^2 = 1$, $\rho = 0.01$, $\sigma_\varepsilon^2 = \sigma_{\varepsilon,j}^2 = \sigma_n^2 = 1$, $\sigma_\zeta^2 = 5$, $\omega = 1$.

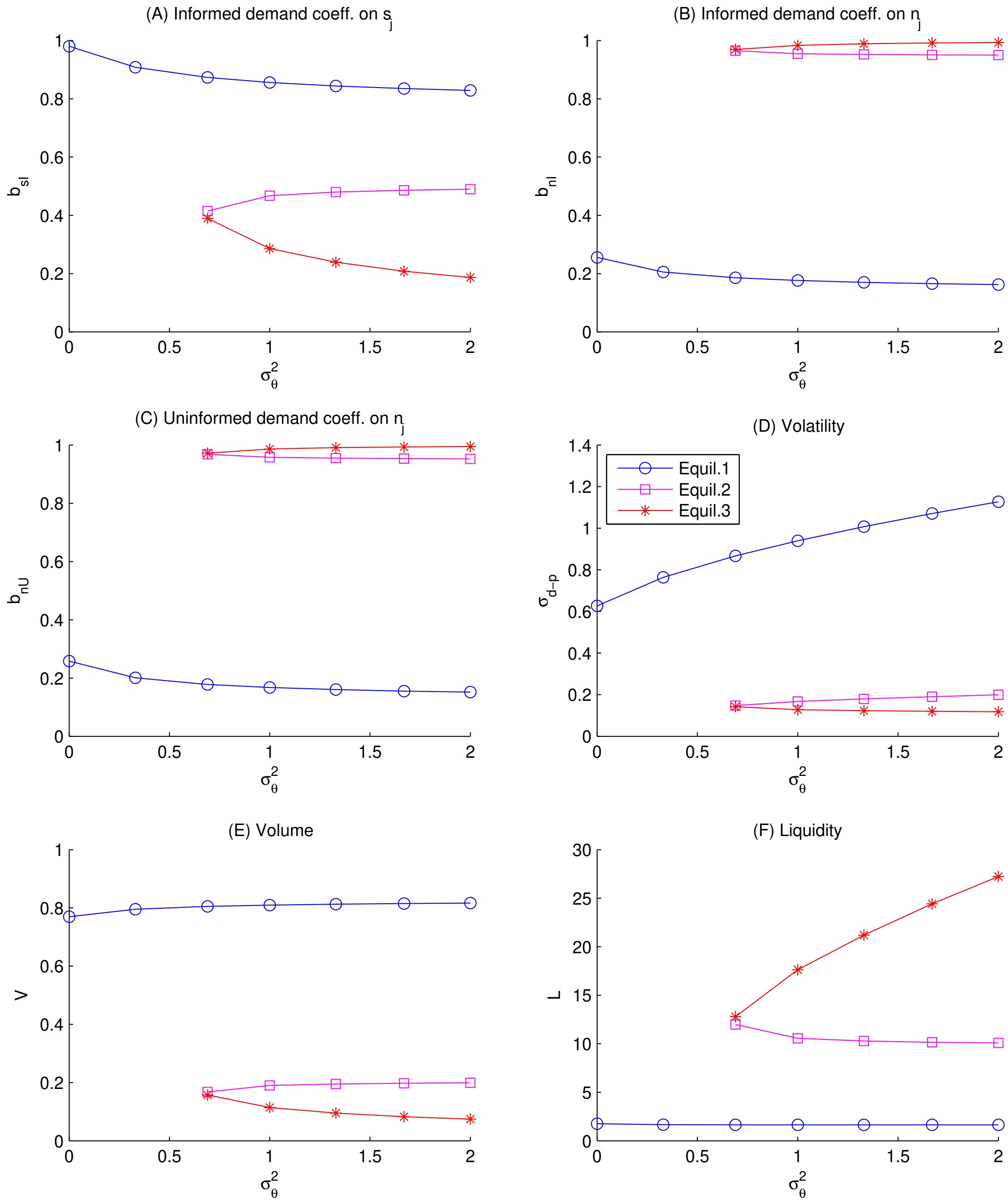

Figure 8: **Supply ambiguity and equilibrium characteristics**. The figure plots the following quantities: (A) the informed investors' demand coefficient on private signal, b_{sI}; (B) the informed investors' demand coefficient on endowment, b_{nI}; (C) the uninformed investors' (shadow) demand coefficient on endowment, b_{nU}; (D) volatility, σ_{d-p}; (E) volume, V; (F) liquidity, L. Parameter values: $\gamma = 1$, $g = 2$, $\sigma_d^2 = 1$, $\rho = 0.01$, $\sigma_\varepsilon^2 = \sigma_{\varepsilon,j}^2 = \sigma_n^2 = 1$, $\sigma_\zeta^2 = 5$, $\omega = 1$.

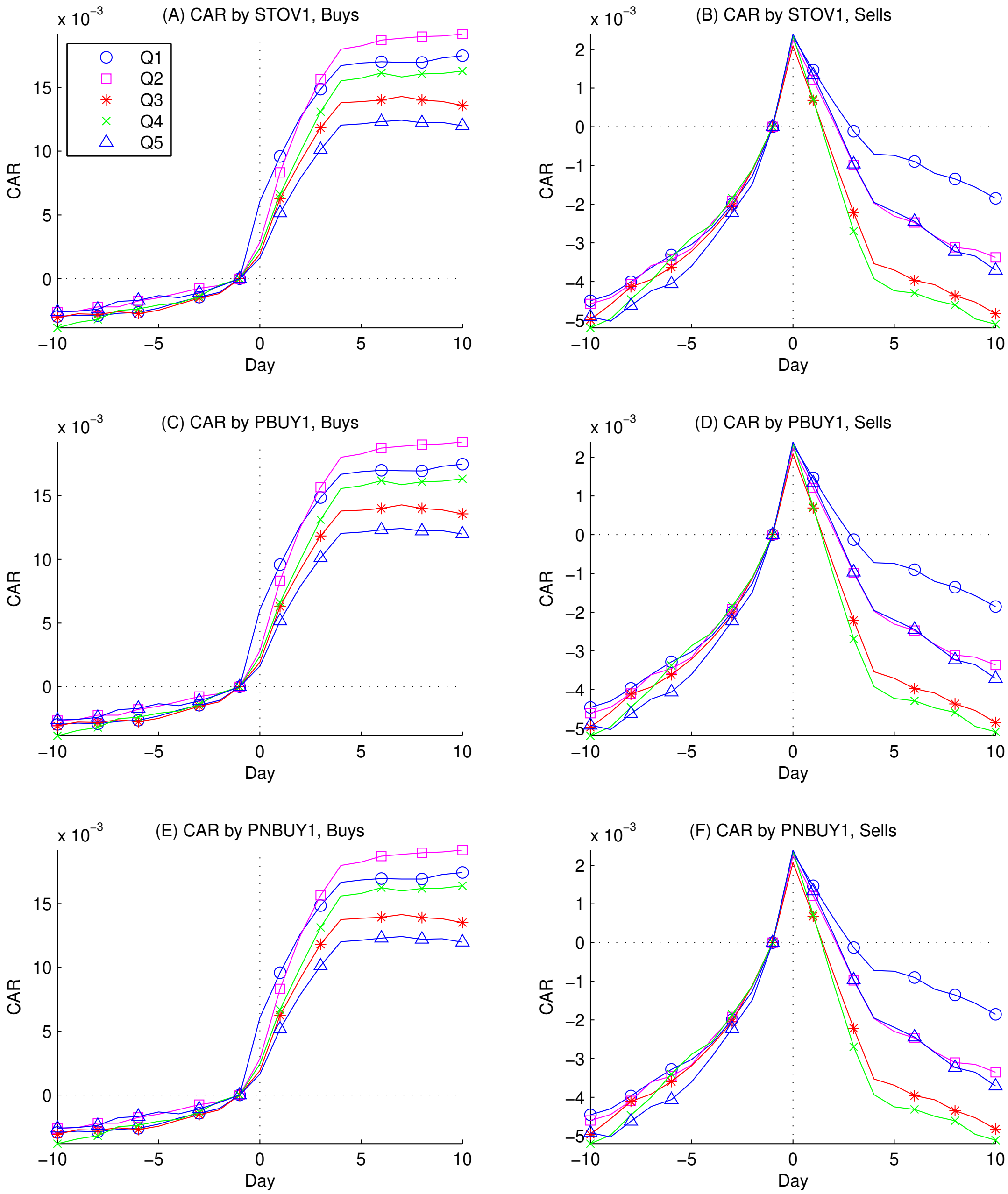

Figure 9: **Cumulative abnormal returns (CAR) around insider trades**. The sample contains insider trades from Thomson Financial (Table 1, Forms 3-5) over the period 1985 through 2012, matched to prices and returns in CRSP, and quotes and trades in TAQ, by the stocks traded. Denote the day of insider trading as Day 0. On Day -1, separately for insider buys and sells, we form equally weighted portfolios by sorting stocks into quintiles based on one of the following characteristics of TAQ trades in the smallest trade-size quintile: (A)(B) signed share turnover ($STOV1$), (C)(D) proportion of buys ($PBUY1$), (E)(F) proportion of the number of buys ($PNBUY1$). CAR is then the alpha from the regression of a portfolio return on the CRSP equally weighted market return, cumulated over days and normalized to 0 on Day -1.

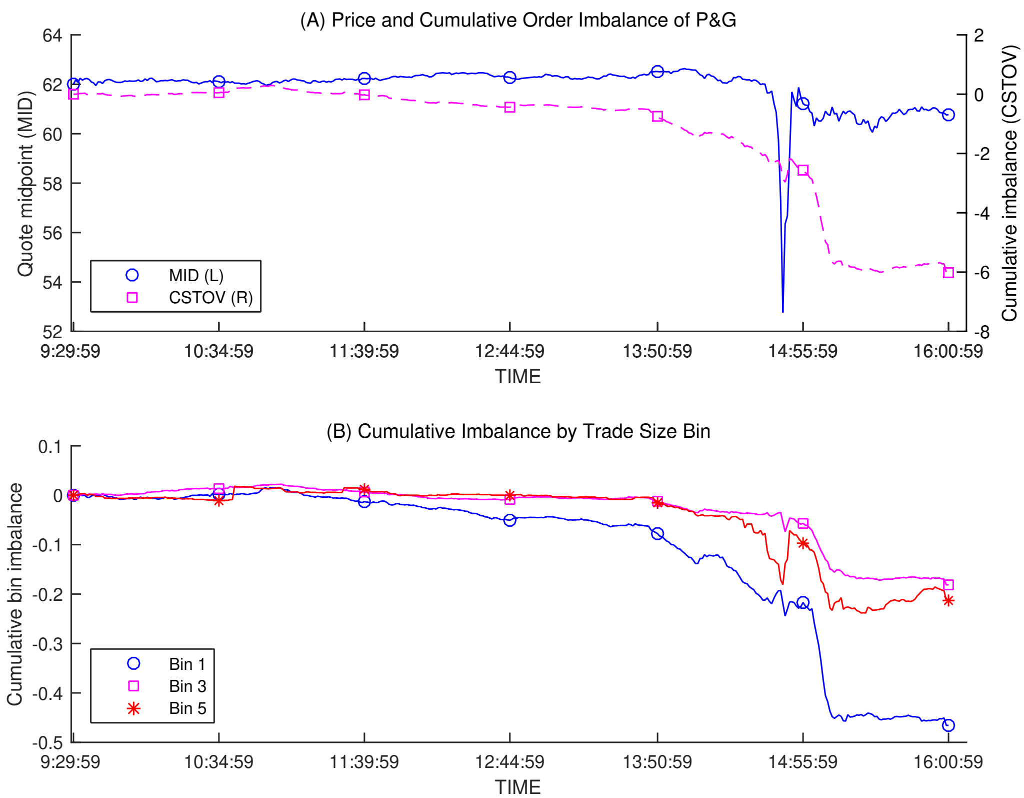

Figure 10: **Quote midpoint and cumulative order imbalance of Procter & Gamble on May 6, 2010**. Panel A shows the quote midpoint (MID, left axis) and cumulative order imbalance ($CSTOV$, right axis) of Procter & Gamble on May 6, 2010 at one minute intervals. Panel B shows the cumulative order imbalance in selected trade size bins, 1 (smallest), 3 (medium), and 5 (largest).