# PHYS 124 LEC A01
# Final Examination
## Autumn 2007

**Name:** SOLUTIONS

**ID Number:** ______________________________

**Instructor:** Marc de Montigny
**Time:** Tuesday, December 18, 2007
9:00 AM – 12:00 PM
**Room:** Main Gym – Van Vliet Building
Rows 17, 19, 21, 23, 25

**Instructions:**

- Do not open this booklet until you are told to do so.
- This exam contains 11 pages.
- Items allowed: pens or pencils, calculator (programmable or graphic) without communication features. Personal digital assistants (PDAs) not allowed.
- Please turn off your cell phones. Remove your caps.
- Leave your ONECard on your desk while you are writing the exam.
- This is a closed-book exam. You may use the formula sheet provided earlier in class, subject to your own modifications. Specific rules were described in class. You may lose up to 10 marks (out of 50) if:
  1. solutions are included, of if
  2. the formula sheet is not returned with your exam.
- The exam contains 10 long problems. They are worth a total of 50 marks. Partial marks will be given. Show all work clearly and neatly. If you miss a result for a subsequent part of a question, then work algebraically.
- You may use the back of the pages for your own calculations. These will not be marked, unless you specify otherwise.
- Address all inquiries to a supervisor. Do not communicate with other students.
- If you become ill during the exam, contact a supervisor immediately. (Note that you may not claim extenuating circumstances and request your paper to be cancelled after writing and handing in your examination.)
- If you need to visit the washroom, bring your paper to a supervisor.
- You may not leave the exam until at least 30 minutes have elapsed.
- When the exam period is over, please stop writing immediately or you may lose marks. Do not discuss with anyone while you are handing it in. Students who finish early are asked to return their exam and to leave the exam room quietly. Examination rules apply until you have left the exam room.

**If anything is unclear, please ask!**

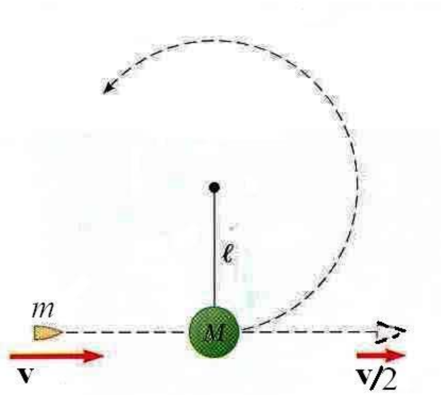
**P-1. Conservation of Momentum and Energy [4.0 marks]**

A bullet of mass $m$ and speed $v$ passes completely through a vertical pendulum bob of mass $M$, and emerges with a speed of $v/2$. The pendulum bob is suspended by a stiff rod of negligible mass and length $\ell$. Friction is negligible.

A. What is the speed of the pendulum bob immediately after the collision, in terms of $m$, $M$, and $v$? Use the conservation of momentum. **[1.0 mark]**

B. What is the kinetic energy of the pendulum bob immediately after the collision, in terms of $m$, $M$, and $v$? **[1.5 marks]**

C. What is the minimum value of $v$ such that the pendulum bob will barely swing through a complete vertical circle? Use the conservation of energy. Write your answer in terms of $m$, $M$, $g$ and $\ell$. "Barely" implies that the speed of the pendulum bob is zero at the top of the trajectory. **[1.5 marks]**




**SOLUTION**

A. $\sum p_i = \sum p_f$ ; $\quad mv = m\frac{v}{2} + MV \quad$ $\boxed{V = \frac{mv}{2M}}$

B. $K = \frac{1}{2}MV^2 = \frac{1}{2}M\left(\frac{mv}{2M}\right)^2 \quad$ $\boxed{K = \frac{m^2v^2}{8M}}$

C. $\Delta K + \Delta U = 0$; $\; -\frac{m^2v^2}{8M} + Mg(2\ell) = 0 \quad$ $\boxed{v = \frac{4M}{m}\sqrt{g\ell}}$

## P-2. Static Equilibrium [6.5 marks]

A uniform, horizontal 30.0-kg beam, 5.00-m long, is attached to a wall by a pin connection that allows the beam to rotate in a vertical plane. Its far end is supported by a cable that makes an angle of 53.0° with the horizontal. An 80.0-kg person stands 1.50 m from the wall. Hereafter you will find the tension in the cable and the force exerted on the beam by the wall, by following the steps below.

A. Identify the forces acting on the beam. Draw the free-body diagram. **[0.5 mark]**

B. Write $\sum F_x = ma_x$ using the variables in this problem. **[1.0 mark]**

C. Write $\sum F_y = ma_y$ using the variables in this problem. **[1.0 mark]**

D. Write $\sum \tau = I\alpha$ using the variables in this problem. **[1.5 marks]**

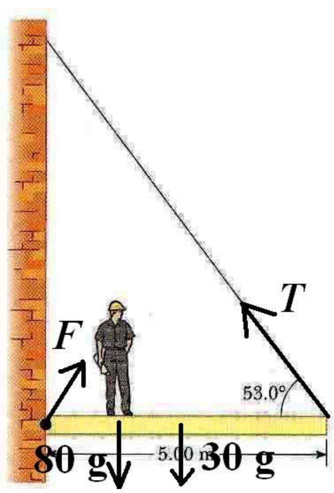
E. Compute the tension in the cable. **[1.0 mark]**

F. Compute the horizontal component of the force exerted on the beam by the wall at the pin connection. **[0.5 mark]**

G. Compute the vertical component of the force exerted on the beam by the wall at the pin connection. **[1.0 mark]**

**SOLUTION**

A.




Tension ***T***
Force by the wall ***F*** (components $F_H$, $F_V$)
Person's weight 80***g***
Beam's weight 30***g***

B. $F_H - T\cos 53 = 0$

C. $F_V + T\sin 53 - 80g - 30g = 0$

D. $(5)T\sin 53 - (1.5)80g - (2.5)30g = 0$

E. $T = 479$ N

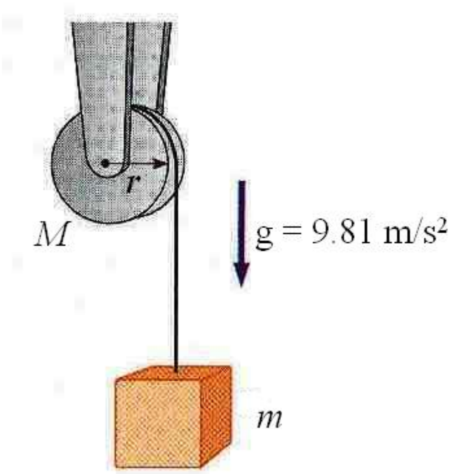
F. $F_H = 288$ N

G. $F_V = 697$ N

**P-3. Rotational Dynamics [4.5 marks]**

A. Use conservation of energy to determine the angular speed $\omega$ of the spool shown below, after the mass $m$ has fallen a distance $h$, starting from rest. The light string attached to this mass is wrapped around the spool and does not slip as it unwinds. Assume that the spool is a solid cylinder of radius $r$ and mass $M$. Write your answer in terms of $m$, $M$, $r$, $g$, and $h$. Use $I_{\text{cylinder}} = \frac{1}{2}Mr^2$. **[3.5 marks]**

B. Compute $\omega$ for $m = 5.0$ kg, $M = 0.50$ kg, $r = 50$ cm, and $h = 4.0$ m. **[1.0 mark]**




**SOLUTION**

A. $\Delta K + \Delta U = 0$

$\frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{1}{2}MR^2\right)\left(\frac{v}{R}\right)^2 - mgh = 0$ gives $v^2 = \frac{4mgh}{2m+M}$. With $v = \omega R$, we find the final answer: $\omega = \frac{2}{R}\sqrt{\frac{mgh}{2m+M}}$

B. $\omega = 17.3$ rad/s

**P-4. Gravitational Energy [4.0 marks]**

A projectile launched vertically from the surface of the Earth ( $R_{\text{Earth}} = 6{,}370$ km; $M_{\text{Earth}} = 5{,}97 \times 10^{24}$ kg) rises to an altitude of 525 km, where it stops momentarily. Use conservation of energy in order to find the projectile's initial speed $v_0$. The distance is large enough to utilize $U_{\text{grav}} = -G\frac{m_1 m_2}{R}$, with $G = 6.67 \times 10^{-11}$ N·m²/kg², at all points of the trajectory.

**SOLUTION**

$$E_i = E_f$$

$$\frac{1}{2}mv_0{}^2 - \frac{GM_E m}{R_E} = -\frac{GM_E m}{R_E + h}$$

$$v_0 = \sqrt{GM_E\left(\frac{1}{R_E} - \frac{1}{R_E + h}\right)}$$

$v_0 = 3085$ m/s

**P-5. Simple Harmonic Oscillator [6.0 marks]**

A 0.60-kg mass is attached to a horizontal spring with a force constant of 24 N/m. Friction is negligible. The mass is released from rest a distance of 9.0 cm from the equilibrium position of the spring, and the system then performs oscillations.

A. What is the maximum speed of the mass during the oscillations? **[1.0 mark]**

B. How long does it take for the mass to go from a point of maximum speed to the next point where it is at rest? **[1.0 mark]**

C. What is the speed of the mass when it is halfway to the equilibrium position? **[2.0 marks]**

D. How far is the mass from the equilibrium position when its speed is half the maximum speed? **[2.0 marks]**

**SOLUTION**

A. $v_{MAX} = \omega A = \sqrt{\frac{k}{m}}\, A$ $\quad v_{MAX} = 0.569 \text{ m/s}$

B. $\frac{1}{4}T = \frac{1}{4}\frac{2\pi}{\omega}$ $\quad t = 0.248 \text{ s}$

C. $\frac{1}{2}mv^2 + \frac{1}{2}k\left(\frac{A}{2}\right)^2 = \frac{1}{2}kA^2$ leads to $v = \sqrt{\frac{3k}{4m}}\, A$ $\quad v = 0.493 \text{ m/s}$

D. $\frac{1}{2}m\left(\frac{v_{MAX}}{2}\right)^2 + \frac{1}{2}kx^2 = \frac{1}{2}mv_{MAX}{}^2$ leads to $x = \sqrt{\frac{3m}{4k}}\, v_{MAX}$ $\quad x = 7.79 \text{ cm}$

**P-6. Standing Waves on a String [4.5 marks]**

A steel guitar string with fixed ends has a tension $T$, length $L$, density $\rho$ (in kg/m$^3$), and radius $r$.

A. Find the fundamental frequency of the string in terms of $T$, $L$, $\rho$ and $r$. Consider the string as a cylinder of length $L$ and radius $r$. **[2.5 marks]**

B. Give the multiplicative factor by which the fundamental frequency of the string changes if the tension $T$ is increased by a factor of 3, the radius $r$ increased by a factor of 2, whereas the remaining quantities are unchanged. **[2.0 marks]**

**SOLUTION**

A. $f_1 = \frac{v}{2L}$, where $v = \sqrt{\frac{T}{\mu}}$ and $\mu = \frac{M}{L} = \frac{\rho\pi r^2 L}{L} = \rho\pi r^2$. $f_1 = \frac{1}{2Lr}\sqrt{\frac{T}{\rho\pi}}$

B. $f_1' = \frac{1}{2Lr'}\sqrt{\frac{T'}{\rho\pi}} = \frac{1}{2L(2r)}\sqrt{\frac{(3T)}{\rho\pi}} = \frac{\sqrt{3}}{2}\left(\frac{1}{2Lr}\sqrt{\frac{T}{\rho\pi}}\right)$ Factor $\frac{\sqrt{3}}{2}$

**P-7. Standing Waves in a Tube [4.5 marks]**

Consider a tube of a certain length. A certain harmonic produced inside the tube has a frequency of 450 Hz. The next higher harmonic has a frequency of 750 Hz. The speed of sound inside this tube is 343 m/s.

A. Is the tube open only at one end, or is it open at both ends? **[1.0 mark]**

B. What is the integer $n$ that describes the harmonic whose frequency $f_n$ is equal to 450 Hz? **[1.0 mark]**

C. What is the length of the tube? **[1.0 mark]**

D. Draw the standing wave inside the tube for the 750-Hz harmonic. **[1.5 marks]**

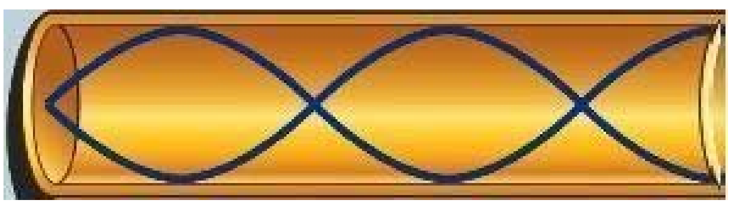
**SOLUTION**

A. $\frac{750}{450} = \frac{5}{3}$, so that $f_3 = 450$ Hz, $f_5 = 750$ Hz. Since n increases by 2, the tube is open at one end only.

B. $f_3 = 450$ Hz means that $n = 3$

C. $f_1 = \frac{f_3}{3} = \frac{450}{3} = 150$ Hz $\qquad L = \frac{v}{4 f_1} = 57.2$ cm

D. $f_5 = 750$ Hz means that the drawing contains 5 quarter-waves



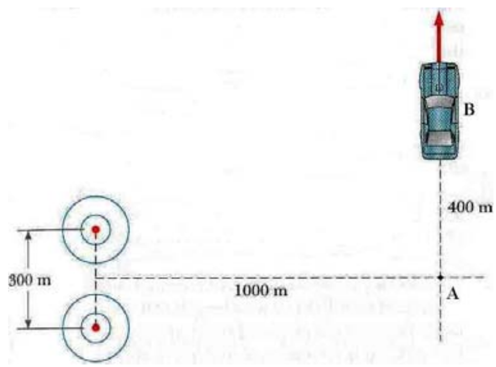
## P-8. Interference [4.5 marks]

Two radio antennas separated by 300 m transmit signals that are 180° out of phase and of the same wavelength. A radio in a car traveling due north receives the signals.

A. When the car is at point A, is the interference constructive or destructive? Explain briefly. **[1.0 mark]**

B. If the next point (car moving forward) of destructive interference occurs when the car is at point B, what is the wavelength of the signals? **[2.0 marks]**

C. If the wavelength of the signals is modified so that, instead, the point B denotes the next location of constructive interference, what is the new wavelength? **[1.5 marks]**




**SOLUTION**

A. Destructive because $\Delta d = 0$ and the sources are out of phase.

B. Destructive interference occurs when $\Delta d = 0, \lambda, 2\lambda, \ldots$

$$\Delta d = d_2 - d_1 = \sqrt{1000^2 - 550^2} - \sqrt{1000^2 - 250^2} \cong 110.4948\ldots$$

The largest possible wavelength is $\lambda = \Delta d = 110\ \text{m}$. (Because the question did not specify the largest wavelength, other possible answers are 110/integer.)

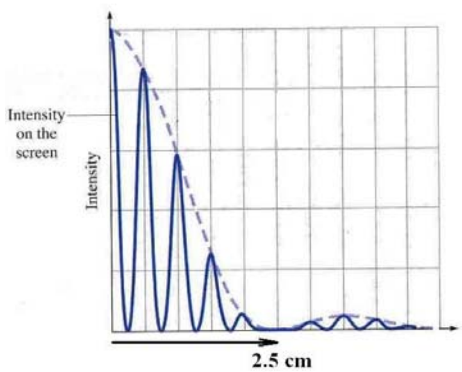
C. Next point of constructive interference is at $\Delta d = \lambda / 2$, so that $\lambda = 2\Delta d$, which gives $\lambda = 221\ \text{m}$. (*Remark:* I also accepted $\lambda = 73.7\ \text{m}$, since some students interpreted "next location" not as being after Point A, but after the point found in part B.)

**P-9. Young Interference and Diffraction [5.5 marks]**

In a real double-slit interference experiment, the bright fringes are equally spaced but not equal in intensity, due to the combination of interference and diffraction. The figure shown below displays the intensity for a Young experiment where the spacing between the two slits is $d$ and the width of each slit is $W$. Assume the first minimum of diffraction to be located 2.5 cm from the central bright fringe, on a flat screen which is 3.0 m away from the slits when light of wavelength 450 nm is used.

A. Find the numerical value of the width $W$. Assume $\sin\theta \cong \tan\theta$. **[2.5 marks]**

B. From the information shown in the figure, deduce the numerical value of the spacing $d$ between the slits. **[3.0 marks]**




**SOLUTION**

A. With $W\sin\theta = m\lambda$ and $\sin\theta \cong \tan\theta = \frac{y}{L}$, we find $W = \frac{m\lambda L}{y}$, so that

$W = 5.40\times10^{-5}$ m

B. From $W\sin\theta = m_d\lambda$ and $d\sin\theta = m_i\lambda$, we find $d = \frac{m_i}{m_d}W$. The figure shows that $m_d = 1$ corresponds to $m_i = 5$, so that $d = 5W = 2.70\times10^{-4}$ m

Another solution consisted in $d\sin\theta = m\lambda$ and $\sin\theta \cong \frac{y}{L}$, giving also $d = \frac{m\lambda L}{y} = 2.70\times10^{-4}$ m.

**P-10. Blackbody Radiation and Photoelectric Effect [6.0 marks]**

In this question, we investigate the electromagnetic radiation emitted from the heating element of a stove as the source of light in a photoelectric experiment.

A. Use Wien's Law ( $f = (5.88 \times 10^{10})T(°K)$ ) to find the frequency of the most intense radiation emitted by a heating element at a temperature of 191 °C. **[1.0 mark]**

B. If radiation with the frequency found in part A is incident on a surface with a work function of 2.17 eV, what is the maximum kinetic energy of electrons ejected from this surface? Interpret briefly your result. ($1\,\mathrm{eV} = 1.60 \times 10^{-19}\,\mathrm{J}$) **[2.5 marks]**

C. Find the cut-off frequency of the surface in part B. **[1.0 mark]**

D. If $6.50 \times 10^{14}$-Hz radiation is incident on the surface of part B, find the maximum kinetic energy of electrons ejected from this surface. **[1.5 marks]**

**SOLUTION**

A. $f_{peak} = (5.88 \times 10^{10})T(°K) = (5.88 \times 10^{10})[T(°C) + 273.15]$ gives

$f_{peak} = 2.73 \times 10^{13}$ Hz

B. $K_{\max} = hf - W_0 = (6.63 \times 10^{-34})(2.73 \times 10^{13})\left(\dfrac{1\mathrm{eV}}{1.60 \times 10^{-19}\ \mathrm{J}}\right) - 2.17$ so that

$K_{\max} = -2.06$ eV or $-3.29 \times 10^{-19}$ J No photoelectrons are ejected.

C. $f_0 = \dfrac{W_0}{h} = 5.24 \times 10^{14}$ Hz

D. $K_{\max} = hf - W_0 = (6.63 \times 10^{-34})(6.50 \times 10^{14})\left(\dfrac{1\mathrm{eV}}{1.60 \times 10^{-19}\ \mathrm{J}}\right) - 2.17$ so that

$K_{\max} = 0.523$ eV or $8.38 \times 10^{-20}$ J