PHYS 124 Section A01
Mid-Term Examination
Autumn 2007

Name :_____SOLUTIONS____________

ID Number :______________________

Instructor : Marc de Montigny
Time : Wednesday, October 24, 2007
9:00 – 9:50 AM
Room : Tory Lecture (Turtle) TL-B2

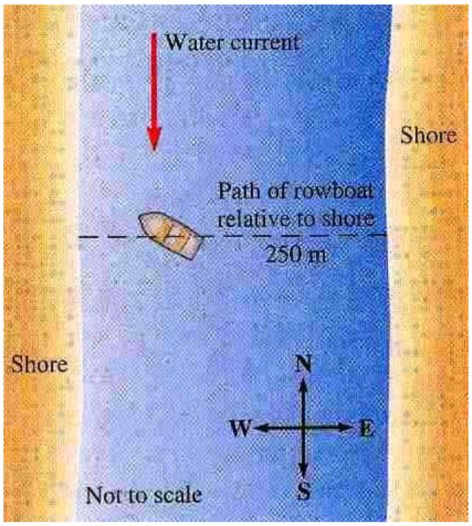
Instructions :

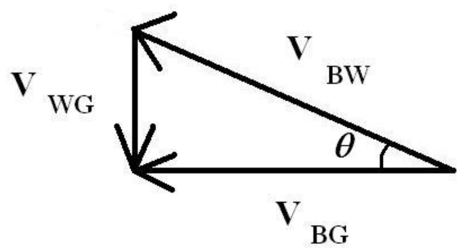
- This booklet contains four pages.
- Items allowed : pen or pencil, calculator (programmable or graphic allowed). Personal digital assistants not allowed.
- Please turn off your cell phones. Remove your caps.
- This is a closed-book exam. You may use the formula sheet provided earlier in class, subject to your own modifications. Specific rules were described in class. You may lose up to 5 marks (out of 20) if :
  1. solutions are included,
  2. the formula sheet is not returned with your exam, or if
  3. you have written formulas on the back of the sheet.
- The exam contains three problems. They are worth a total of 20 marks. Partial marks will be given. Show all work clearly and neatly. If you miss a result for a subsequent part of a question, then work algebraically.
- You may use the back of the pages for your own calculations. These will not be marked, unless you specify otherwise.
- When the exam period is over, please stop writing immediately, or you may lose marks. Do not discuss with anyone while you are turning it in. Examination rules apply until you have left the exam room.

P-1. Relative Velocity [6.0 marks]

Jack rows directly across a river from the east shore to a point on the west shore, as shown in the figure below. The width of the river is 250 m and the current flows from north to south at 0.610 m/s. The trip takes Jack 4 min 12 sec.

A. At what speed, with respect to still water, does Jack need to row? **[3.0 marks]**
B. In what direction does he have to head his rowboat in order to follow a course due west across the river? **[3.0 marks]**





SOLUTION

$\vec{v}_{BW} = \vec{v}_{BS} + \vec{v}_{SW} = \vec{v}_{BS} - \vec{v}_{WS}$

where $\vec{v}_{WS}$ is 0.610 m/s due south, or $\vec{v}_{WS} = (0, -0.610)$ m/s, and

$\vec{v}_{BS}$ has magnitude $v_{BS} = \dfrac{d}{t} = \dfrac{250\text{ m}}{252\text{ s}} = 0.9920634921$ m/s due west, or $\vec{v}_{BS} = (-0.9920634921, 0)$.

Thus we find $\vec{v}_{BW} = \vec{v}_{BS} - \vec{v}_{WS} = (-0.9920634921, 0) - (0, -0.610) = (-0.9920634921, 0.610)$ m/s. Next we use $v_{BW} = \sqrt{v_x^{\ 2} + v_y^{\ 2}}$, for the speed, and $\tan\theta = \dfrac{v_y}{v_x}$ for the direction, to find that

A. Speed is 1.16 m/s
B. Direction is 31.6° north of west

Of course, I will also accept for part B : 58.4° west of north, or 148° from the usual reference line. Also it is fine to use right triangles, Pythagorean theorem, to answer the questions.

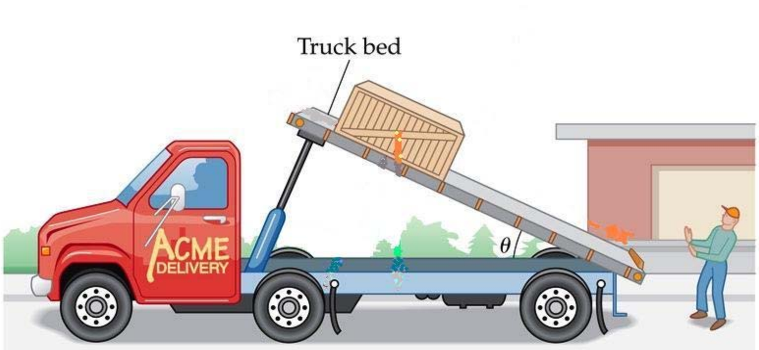
Comment: "With respect to still water" is a common expression which means "relative to water" in the sense of "relative to the reference frame in which water is still". After all, the question has already stated that water is flowing relative to the shore, so it would not make sense to ask a question for which water is at rest relative to ground...

P-2. Newton's Laws of Motion [6.5 marks]

A flatbed truck tilts its bed upward to dispose of a 95.0-kg crate. When the tilt angle exceeds 17.5°, the crate begins to slide. When the crate reaches the bottom of the flatbed, after sliding a distance of 2.75 m, its speed is 3.11 m/s. Find

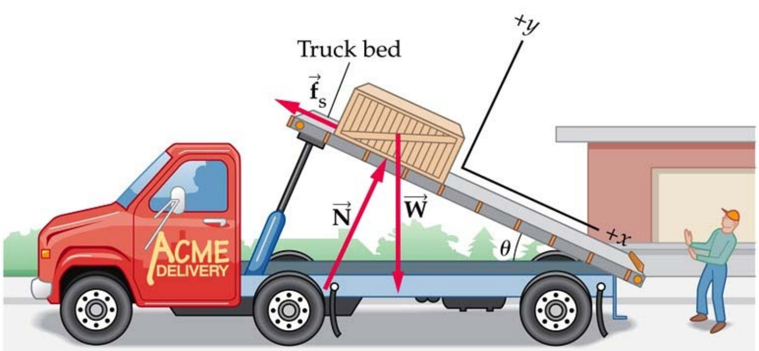
A. the coefficient of static friction, and **[3.0 marks]**
B. the coefficient of kinetic friction **[3.5 marks]**

between the crate and the flatbed. (Do *not* use conservation of energy.)



SOLUTION

Consider the following free-body diagram (shown for static friction, but also valid for kinetic friction).



A. $f_{s,\max} = \mu_s N$ since it begins to slide

$$\sum F_x : mg\sin\theta - f_s = 0$$

$$\sum F_y : N - mg\cos\theta = 0$$

These equations lead to $\mu_s = \dfrac{mg\sin\theta}{mg\cos\theta} = \tan(17.5) = 0.315$

B. $f_k = \mu_k N$ just after it began to slide down. So we take the same angle as in A.

$$\sum F_x : mg\sin\theta - f_s = ma$$

$$\sum F_y : N - mg\cos\theta = 0$$

Next we use the equation of kinematics at constant acceleration

$v = v_0^{\ 2} + 2a\Delta x$, so that $a = \dfrac{v^2}{2\Delta x}$, which we substitute into

$\mu_k = \dfrac{g\sin\theta - a}{g\cos\theta}$ where $\theta = 17.5$, to find μ_k = 0.127

Comments: The point of this question was to check that you know how to use Newton's Second Law. So, most marks were lost if the problem was solved using conservation of energy or the kinetic-energy theorem, even if the final answer was correct. Again, what matters to me is not so much the final answer as the solution itself.

After the crate begins to slide, it will keep sliding, without having to increase the angle. Moreover, since no other angle was given in the question, you were expected to use 17.5 for Part B as well. However, marks were not lost if slightly higher values (17.6 or 18) had been taken for Part B.

This question is Problem 97, page 178 of Walker's text...

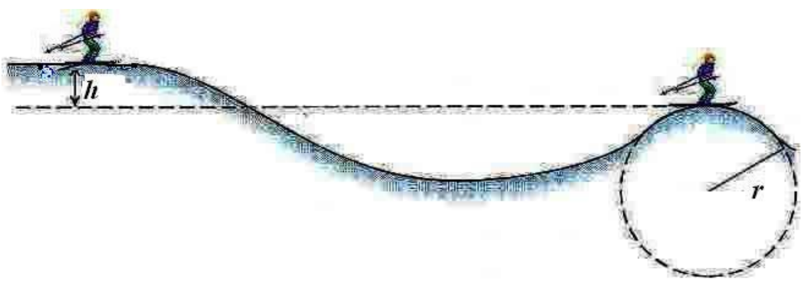
P-3. Conservation of Energy [7.5 marks]

A 75.0-kg skier starts from rest at the top of a hill. The skier coasts down the hill and up a second hill, as illustrated below. The crest of the second hill is circular, with a radius of $r = 36.0$ m. Neglect friction and air resistance.

A. What must be the height h of the first hill so that the skier just loses contact with the snow at the crest of the second hill? **[3.0 marks]**

B. What is the height h if the skier starts from the first hill with a speed $v_0 = 15.0$ m/s? **[2.0 marks]**

C. If, at the top of the second hill, the skier encounters a horizontal spring with a force constant of 10^5 N/m, by how much will the spring be compressed before the skier comes to rest? **[2.5 marks]**



SOLUTION

The main equations to use here are $E_i = E_f$ and $\sum F = \frac{mv^2}{r}$. The energy is a combination of $\frac{1}{2}mv^2$, mgh and $\frac{1}{2}kx^2$.

A. $\frac{1}{2}mv_i^2 + mgh_i = \frac{1}{2}mv_f^2 + mgh_f$ where $v_i = 0$, $h_i = h$ and $h_f = 0$. So the equation becomes $mgh = \frac{1}{2}mv_f^2$. At the peak of the second hill, Newton's Law reads $\sum F = N - mg = -\frac{mv^2}{r}$, so that with $N = 0$, we find $v_f^2 = rg$. Then we have $mgh = \frac{1}{2}mrg$ and $h = \frac{r}{2} = 18.0$ m.

B. Similar to Part A, with the substitution $v_i = v_0 = 15$ m/s. The equation is then $\frac{1}{2}mv_0^{\ 2} + mgh = \frac{1}{2}mv_f^{\ 2}$, where we have again $v_f^{\ 2} = rg$. As expected, we obtain a value of *h* smaller than in Part A: $h = \frac{1}{2}\left(r - \frac{v_0^{\ 2}}{g}\right)$ = 6.53 m.

C. The final energy is the potential energy, $\frac{1}{2}kx^2$, of the compressed spring. There is no kinetic energy and the gravitational potential energy is also taken to be zero at that point. There are three equivalent possibilities for the initial total energy:

(1) $mgh_A = (75)(9.81)(18.0) = 13243.5$ J

(2) $\frac{1}{2}mv_0^{\ 2} + mgh_b = \frac{1}{2}(75)(15)^2 + (75)(9.81)(6.53) = 13243.5$ J

(3) $\frac{1}{2}mv_f^{\ 2} = mrg = \frac{1}{2}(36)(9.81) = 13243.5$ J.

So, by taking any of these expressions equal to $\frac{1}{2}kx^2$, and solving for x gives, for instance, $x = \sqrt{\frac{mrg}{k}} = 0.515$ m, or 51.5 cm.

Comments: Of course, it is sufficient to consider only one of the three possibilities in Part C.

The meaning of case (3) is that the skier is moving at velocity v_f just before touching the spring. So then the total mechanical energy is kinetic energy.

Unlike Question 2, here you had to use conservation of energy, not Newton's Second Law. Some students have included the term $-kx$ (which is a *force*) in the total mechanical energy.