PHYS 126 Section B03
Mid-Term Examination
Winter 2008

Name: SOLUTIONS

ID Number:______________________________

Instructor: Marc de Montigny
Time: Wednesday, February 13, 2008
1:00 – 1:50 PM
Room: Central Academic Building (CAB) 243

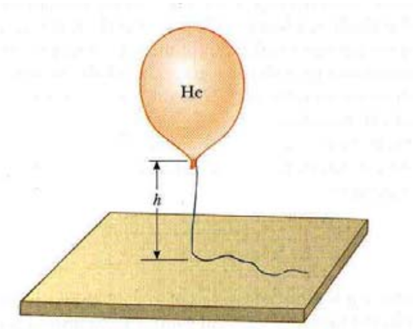
Instructions:

- This booklet contains five pages.
- Items allowed: pen or pencil, calculator (programmable or graphic allowed) without communication features. Personal digital assistants (PDAs) not allowed.
- Please turn off your cell phones. Remove your caps.
- The exam contains four problems. They are worth a total of 20 marks. Partial marks will be given. Show all work clearly and neatly. If you miss a result for a subsequent part of a question, then work algebraically.
- This is a closed-book exam. You may use the formula sheet provided earlier in class, subject to your own additions. Specific rules were described in class. You may lose up to 5 marks (out of 20) if :
    1. solutions are included,
    2. the formula sheet is not returned with your exam, or if
    3. you have written formulas on the back of the sheet.
- You may use the back of the pages for your own calculations. It will not be marked, unless you specify otherwise.
- When the exam period is over, *please stop writing immediately or you may lose marks*. Do not discuss with anyone while you are turning it in. Students who finish early are asked to return their exam and to leave the exam room quietly. Examination rules apply until you have left the exam room.

If anything is unclear, please ask!

P-1. Archimedes's Principle [4.5 marks]

A helium-filled balloon at atmospheric pressure is tied to a 2.0-m long, 0.05-kg string. The balloon is assumed spherical with a radius of 40 cm. When released, it lifts a length h of the string and then remains in equilibrium, as illustrated below. When deflated, the balloon itself has a mass of 0.25 kg. Note that only the part of the string above the floor contributes to the weight of the system in equilibrium. What is the value of h? Take $\rho_{\text{He}} = 0.179$ kg/m^3 and $\rho_{\text{Air}} = 1.29$ kg/m^3.



SOLUTION

1 force up: buoyant force F_b (due to air)

3 forces down: weights $W_{balloon}$, W_{He} (helium inside ballon), W_{string} (part of string above the ground)

Newton's Second Law : $F_b = W_{balloon} + W_{He} + W_{string}$

where $F_b = \rho_{air} \dfrac{4\pi}{3} r^3 g$

$W_{balloon} = m_{balloon} g$

$W_{He} = \rho_{He} \dfrac{4\pi}{3} r^3 g$

$W_{string} = \dfrac{h}{L} Mg$ (where h is the unknown of this problem)

We find

$$h = \frac{L}{M}\left[\frac{4\pi}{3} r^3 (\rho_{air} - \rho_{He}) - m_{balloon}\right] = \frac{2.0}{0.05}\left[\frac{4\pi}{3}(0.4)^3(1.29 - 0.179) - 0.25\right] = 1.91 \text{ m}$$

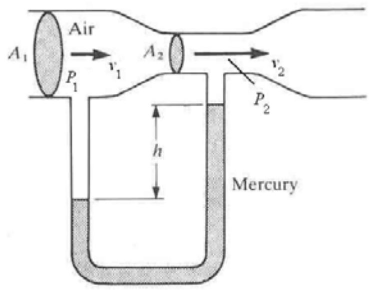
P-2. Fluids in Motion [6.5 marks]

Air (density: $\rho_{\text{Air}} = 1.29$ kg/m^3) flows from left to right through the horizontal main tube of the *Venturi meter* shown below. The U-tube of the meter contains mercury (density: $\rho_{\text{Hg}} = 13{,}600$ kg/m^3). The radii of the wide and narrow parts of the horizontal main tube are $r_1 = 1.0$ cm and $r_2 = 0.5$ cm, respectively. The speed of the air entering the meter is $v_1 = 15$ m/s.

A. Which pressure is greater, P_1 or P_2 ? **[1.0 mark]**

B. Find v_2 by using the continuity equation for the horizontal tube. **[2.0 marks]**

C. What is the mercury-level difference h between the two arms? **[3.5 marks]**



SOLUTION

A. From continuity eq, $v_2 > v_1$, then, from Bernouilli eq, $P_1 > P_2$.
An easier way to see it is that it causes the fluid to flow toward the right.

B. $$v_2 = \frac{A_1}{A_2} v_1 = \left(\frac{r_1}{r_2}\right)^2 v_1 = 60 \text{ m/s}$$

C. $P_1 + \frac{1}{2}\rho_{air} v_1^2 = P_2 + \frac{1}{2}\rho_{air} v_2^2$ For air in the horizontal tube. Air-pressure difference is in agreement with your answer to Part A.

$P_1 = P_2 + \rho_{Hg} gh$ For mercury in U-tube. The pressure difference is the same as in the equation above.

Substituting the second equation into the first equation, we find
$P_2 + \rho_{Hg} gh + \frac{1}{2}\rho_{air} v_1^2 = P_2 + \frac{1}{2}\rho_{air} v_2^2$, from which we obtain the height

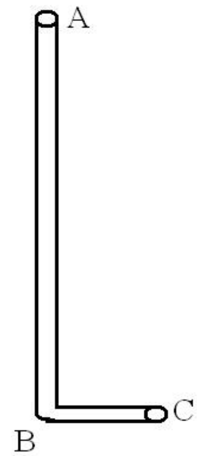
$$h = \frac{\rho_{air}}{2\rho_{Hg}}\left(v_2^2 - v_1^2\right) = \frac{1.29}{2(136000)}\left(60^2 - 15^2\right) = 1.63 \text{ cm}$$

P-3. Viscosity [4.0 marks]

A patient receives a blood transfusion through a horizontal needle of inner radius equal to 0.20 mm, and length 2.0 cm. The bottle supplying the blood is 50 cm above the patient's arm. The density of blood is $\rho_{\text{Blood}} = 1050$ kg/m^3 and its coefficient of viscosity is $\eta_{\text{Blood}} = 0.0027$ N·s/m^2. Assume that the absolute pressure of the blood in the patient's arm (where the blood exits the needle) is equal to the absolute pressure of the blood in the bottle, 50 cm above the needle.

A. What is the volume flow rate through the needle? **[3.0 marks]**
B. What is the mass flow rate through the needle? **[1.0 mark]**

SOLUTION



A. Poiseuille's relation for blood in needle $\frac{\Delta V}{\Delta t} = \frac{\Delta P \pi R^4}{8\eta_{Blood} L}$

First, we have $P_B > P_C$ (see figure), even though the two points are at the same level. It is due to viscosity and the fluid is flowing. Second, the last sentence in the question means that $P_A = P_C$. Therefore, we find $\Delta P = P_B - P_C = P_B - P_A = \rho_{Blood} gh$, so that the equation above becomes

$$\frac{\Delta V}{\Delta t} = \frac{\rho_{Blood} gh\pi R^4}{8\eta_{Blood} L} = \frac{(1050)(9.81)(0.5)\pi(2\times10^{-4})^4}{8(0.0027)(0.02)} = 5.99\times10^{-8} \text{ m}^3/\text{s}$$

B. $$\frac{\Delta m}{\Delta t} = \rho_{Blood}\frac{\Delta V}{\Delta t} = (1050)(5.99\times10^{-8}) = 6.29\times10^{-5} \text{ kg/s}$$

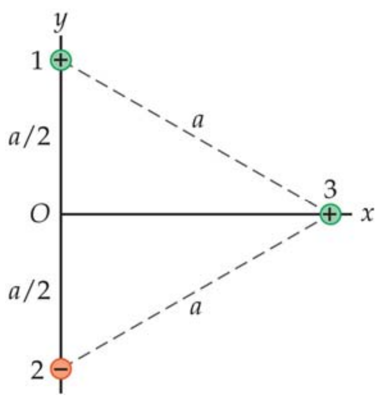
P-4. Electrostatic [5.0 marks]

Three charges are located at the vertices of an equilateral triangle of side equal to a = 50 cm, as shown in the figure below. Charges 1 and 3 are equal to +5.0 μC each, and charge 2 is equal to -6 μC.

A. What are the vector components of the electric field at the location of charge 2, due to charges 1 and 3? **[3.0 marks]**

B. What are the vector components of the net electric force acting on charge 2, due to charges 1 and 3? **[1.0 mark]**

C. What is the acceleration (give its vector components) of charge 2 if the mass of this particle is equal to 0.040 kg? **[1.0 mark]**



SOLUTION

A. $\vec{E}_1 = \left(0, -k\dfrac{|q_1|}{a^2}\right)$, $\vec{E}_3 = \left(-k\dfrac{|q_3|}{a^2}\cos 30°, -k\dfrac{|q_3|}{a^2}\sin 30°\right)$ so that

$$\vec{E}_1 + \vec{E}_3 = \left(-k\frac{|q_3|}{a^2}\cos 30°, -k\frac{|q_1|}{a^2} - k\frac{|q_3|}{a^2}\sin 30°\right) = \left(-1.56\times10^5, -2.70\times10^5\right) \text{ V/m}$$

B. $\vec{F} = q_2\left(\vec{E}_1 + \vec{E}_3\right) = \left(-6\times10^{-6}\right)\left(-1.56\times10^5, -2.70\times10^5\right) = (0.934, 1.62)$ N

C. $\vec{a} = \dfrac{\vec{F}}{m} = (23.4, 40.5)$ m/s^2