PHYS 126 LEC B03
Final Examination
Winter 2008

Name: SOLUTIONS

ID Number: ______________________________

Instructor: Marc de Montigny
Time: Friday, April 18, 2008
2:00 – 5:00 PM
Room: Main Gym – Van Vliet Building
Rows 6, 8, 10

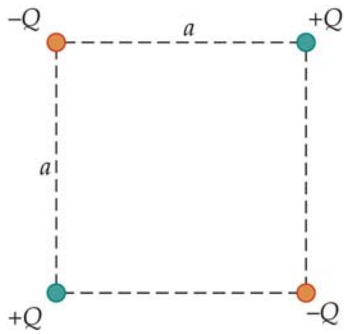
Instructions:

- Do not open this booklet until you are told to do so.
- This exam contains 11 pages.
- Items allowed: pen or pencil, calculator (programmable or graphic) without communication features. Personal digital assistants (PDAs) not allowed.
- Please turn off your cell phones.
- Leave your ONECard on your desk while you are writing the exam.
- This is a closed-book exam. You may use the formula sheet provided earlier in class, subject to your own modifications. Specific rules were described in class. You may lose up to 10 marks (out of 50) if:
    1. the formula sheet is not returned with your exam, or if
    2. solutions are included.
- The exam contains 10 problems. They are worth a total of 50 marks. Partial marks will be given. Show all work clearly and neatly. If you miss a result for a subsequent part of a question, then work algebraically.
- You may use the back of the pages for your own calculations. These will not be marked, unless you specify otherwise.
- Address all inquiries to a supervisor. Do not communicate with other students.
- If you become ill during the exam, contact a supervisor immediately. Note that you may not claim extenuating circumstances and request your paper to be cancelled after writing and handing in your examination.
- If you need to visit the washroom, bring your paper to a supervisor.
- You may not leave the exam until at least 30 minutes have elapsed.
- When the exam period is over, please stop writing immediately or you may lose marks. Do not discuss with anyone while you are handing it in. Students who finish early are asked to return their exam and to leave the exam room quietly. Examination rules apply until you have left the exam room.

If anything is unclear, please ask!

P-1. Electric Potential of Point Charges [4.0 marks]

One of the $-Q$ charges in the figure below is given an outward kick that sends it off with an initial speed v_0 while the other three charges are held at rest. If the moving charge has a mass m, what is its speed (in terms of Q, v_0, m, a, and k) when it is infinitely far from the other charges?



SOLUTION

$$K_i + U_i = K_f + U_f$$

$$\frac{1}{2}mv_0^{\ 2} + \left(-\frac{kQ^2}{a} - \frac{kQ^2}{a} + \frac{kQ^2}{\sqrt{2}\,a}\right) = \frac{1}{2}mv_f^{\ 2} + 0$$

$$v_0^{\ 2} + \frac{2kQ^2}{ma}\left(-2 + \frac{1}{\sqrt{2}}\right) = v_f^{\ 2}$$

Ans. $v_f = \sqrt{v_0^{\ 2} - \frac{2kQ^2}{ma}\left(2 - \frac{1}{\sqrt{2}}\right)}$ Of course, other forms are possible.

P-2. Capacitors and Dielectrics [5.0 marks]

A parallel-plate capacitor has plates of area 3.45×10^{-4} m^2.

A. If the capacitance is to be 1330 pF, what plate separation d is required, assuming that the space between the plates is filled with paper whose dielectric constant is equal to $\kappa = 3.7$? **[1.0 mark]**

B. If the capacitance is to be 1330 pF, what plate separation d is required, assuming that there is vacuum between the two plates? **[1.0 mark]**

C. For the capacitor with vacuum between its plates, what is the electric energy density u_E, in J/m^3, between the plates, if this space contains a uniform electric field of magnitude $E = 9.2 \times 10^5$ V/m? **[1.5 marks]**

D. What is the total electric energy U_E, in J, between the plates of the capacitor described in part C (i.e. with vacuum between the plates)? **[1.5 marks]**

SOLUTION

A. $C = \frac{\kappa \varepsilon_0 d}{d}$ gives $d = \frac{\kappa \varepsilon_0 d}{C} = 8.49 \times 10^{-6}$ m (Note: $1330 \text{ pF} = 1330 \times 10^{-12}$ F)

B. $d = \frac{\varepsilon_0 d}{d} = 2.30 \times 10^{-6}$ m

C. $u = \frac{1}{2}\varepsilon_0 E^2 = 3.75$ J/m^3

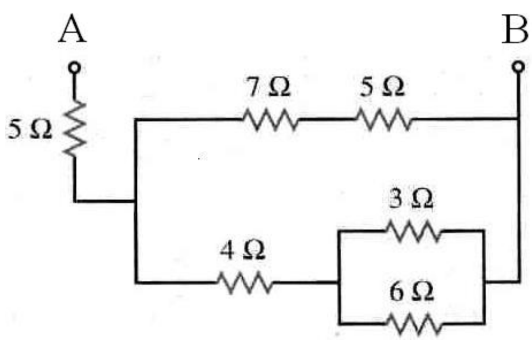
D. $U_E = u(\text{volume}) = u(Ad) = 2.97 \times 10^{-9}$ J

P-3. Combination of Resistors [4.0 marks]

A. What is the equivalent resistance between the points A and B for the combination of resistors shown below? **[2.0 marks]**

B. If a 12-V battery is connected between the points A and B, how much current flows through the 5-Ω resistor that is located just below the point A? **[1.0 mark]**

C. Calculate the power, in watts, delivered by the battery? **[1.0 mark]**



SOLUTION

A. 3 Ω and 6 Ω in parallel gives $R_{3,6} = 2\ \Omega$, which is in series with 4 Ω, to give 6 Ω for the lower leg. The upper leg has $R_{7,5} = 12\ \Omega$, so that the total equivalent resistance is $R_{eq} = 5 + \left(6^{-1} + 12^{-1}\right)^{-1} = 5 + 4 = 9\ \Omega$. The operations above may be summarized into a single formula:

$$5 + \left\{\left[\left(3^{-1} + 6^{-1}\right)^{-1} + 4\right]^{-1} + \left[7 + 5\right]^{-1}\right\}^{-1} = 9\ \Omega$$

B. Since $I_{4\,\Omega} = I_{Req}$, we find $I = \dfrac{V}{R_{eq}} = \dfrac{12}{9} = \dfrac{4}{3}\ \text{A} = 1.33\ \text{A}$

C. $P = VI = (12)\left(\dfrac{4}{3}\right) = 16\ \text{W}$ (Note: One could also use $P = \dfrac{V^2}{R_{eq}}$ or $P = R_{eq} I^2$)

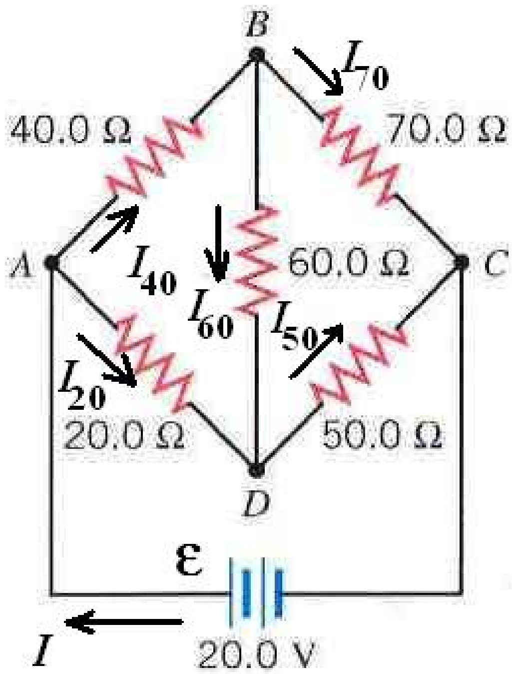
P-4. Kirchhoff's Rules [5.5 marks]

The figure below shows a 20-V battery connected to a combination of resistors known as a *Wheatstone bridge*. In the questions below, I ask you to write down equations which follow from Kirchhoff's rules, without solving these equations.

A. Write down the Kirchhoff's rule for currents at junction A. **[0.5 mark]**
B. Write down the Kirchhoff's rule for currents at junction B. **[0.5 mark]**
C. Write down the Kirchhoff's rule for currents at junction C. **[0.5 mark]**
D. Write down the Kirchhoff's rule for potentials around ABDA-loop. **[1.0 mark]**
E. Write down the Kirchhoff's rule for potentials around ABCDA-loop. **[1.0 mark]**

F. Write down the Kirchhoff's rule for potentials around ADCƐA-loop. **[1.0 mark]**

G. Write down the Kirchhoff's rule for potentials around ABCƐA-loop. **[1.0 mark]**



SOLUTION

A $I - I_{20} - I_{40} = 0$

B. $I_{40} - I_{60} - I_{70} = 0$

C. $I - I_{50} - I_{70} = 0$

D. $40I_{40} + 60I_{60} - 20I_{20} = 0$

E. $40I_{40} + 70I_{70} - 50I_{50} - 20I_{20} = 0$

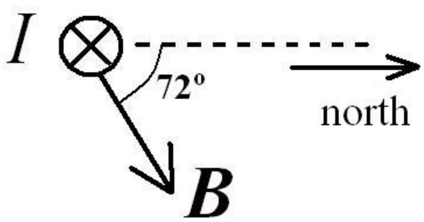
F. $20 - 20I_{20} - 50I_{50} = 0$

G. $20 - 40I_{40} - 70I_{70} = 0$

P-5. Magnetic Force on a Current [3.5 marks]

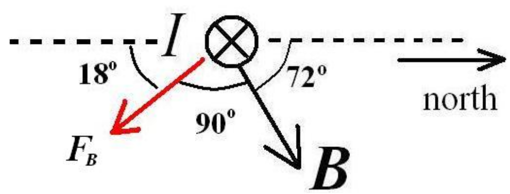
A straight wire is aligned east-west in a region where the Earth's magnetic field has a magnitude of 4.80×10^{-4} T and points to the north, 72° below the horizontal. The wire carries a current I toward the west. The magnetic force on the wire per unit length of wire has a magnitude of 0.020 N/m.

A. What is the direction of the magnetic force on the wire? **[1.0 mark]**
B. What is the magnitude of the current I ? **[2.5 marks]**



SOLUTION

A. The magnetic force points to the south, 18° below the horizontal, as shown on the right.



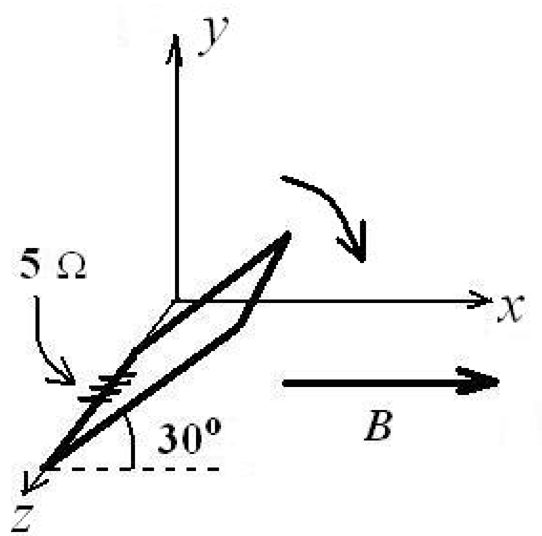
B. From $F = ILB\sin\theta$ we find $I = \dfrac{F/L}{B\sin\theta} = \dfrac{0.02}{\left(4.8 \times 10^{-4}\right)(\sin 90)} = 41.7\ \text{A}$

P-6. Electromagnetic Induction [4.5 marks]

A square loop with 75-cm sides has one side lying on the z axis, and its plane makes a 30º-angle above the x-z plane, as shown in the figure below. The loop contains 75 turns and a 5-Ω resistor lying on the z axis. A uniform magnetic field of magnitude 0.32 T points in the direction of the positive x axis. If the loop falls toward the x-z plane by rotating about the z axis, so that the angle goes from 30º to 10º in 0.6 seconds,

A. does the induced current flow in the resistor toward $+z$ or $-z$? **[1.0 mark]**

B. What is the magnitude of the induced current I_{induced}. **[3.5 marks]**



SOLUTION

A. The angle changes from $\theta_i = 90 - 60 = 30$ to $\theta_f = 90 - 10 = 80$. During this time, the flux Φ_B decreases, so that $\vec{B}_{\text{ind}}$ is in the same direction as the initial $\vec{B}$, an the right-hand rule shows that I_{ind} points toward $-z$.

B.

$$I_{\text{ind}} = \frac{|\varepsilon_{\text{ind}}|}{R} = \frac{1}{R}\left|N\frac{\Delta\Phi_B}{\Delta t}\right| = \frac{1}{R}\left|NB\ell^2\frac{\cos\theta_f - \cos\theta_i}{\Delta t}\right| = \frac{1}{5}\left|(75)(0.32)(0.75)^2\frac{\cos 80 - \cos 60}{0.6}\right|$$

$I_{\text{ind}} = 1.47\ \text{A}$

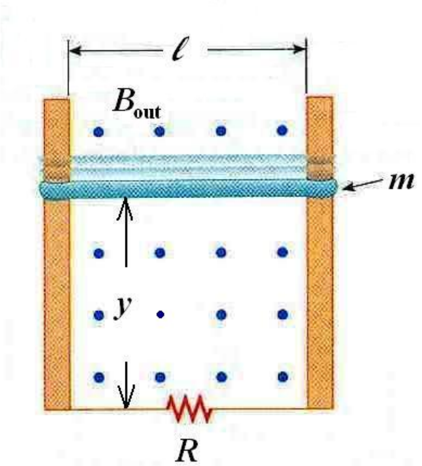
P-7. Electromagnetic Induction [6.0 marks]

A horizontal wire is free to slide on the vertical rails of a conducting frame, as shown in the figure below. The wire has mass m and length ℓ, and the resistance of the circuit is R. The weight of the wire tends to drag it downward. An external uniform magnetic field $\vec{B}_{\text{out}}$ is directed perpendicularly to the frame and out of the page.

A. What is the magnitude of the induced current in the wire when the speed of the wire is equal to v? Express your answer in terms of ℓ, B_{out}, v, and R. **[2.0 marks]**

B. What is the direction of the induced current in the resistor R? **[1.0 mark]**

C. The horizontal wire attains a terminal speed v_t when the magnetic force exerted by $\vec{B}_{\text{out}}$ on the induced current (obtained in part A) becomes equal to the weight of the wire. Compute v_t in terms of m, R, $\vec{B}_{\text{out}}$, ℓ, and g. **[3.0 marks]**



SOLUTION

A. $I_{\text{ind}} = \dfrac{\varepsilon}{R} = \dfrac{1}{R} B_{\text{out}} \ell \dfrac{\Delta y}{\Delta t} = \dfrac{B_{\text{out}} \ell v}{R}$

B. As the rod is falling, Φ_B decreases, so $\vec{B}_{\text{ind}}$ is out of the page, and

I_{ind} is counterclockwise and points to the right in R.

C. $F_g = F_B$, where $F_g = mg$ and $F_B = I_{\text{ind}} \ell B_{\text{out}}$. Using the answer in Part A to replace I_{ind} by $\dfrac{B_{\text{out}} \ell v_t}{R}$, we obtain

$$v_t = \frac{mgR}{B_{\text{out}}^2 \ell^2}$$

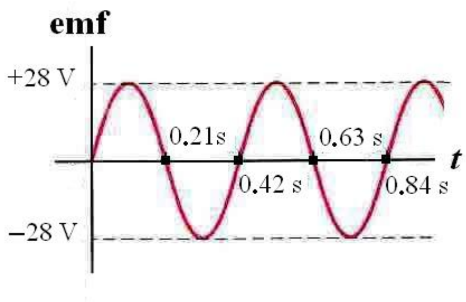
P-8. Electric Generators [3.5 marks]

The drawing shows a plot of the output emf of a generator as a function of time. The coil of this device has a cross-sectional area per turn of 0.020 m^2 and contains 150 turns. The coil of wire can be rotated in a magnetic field.

A. What is the frequency f of the generator, in hertz? **[1.0 mark]**

B. What is the angular speed ω, in rad/s? **[0.5 mark]**

C. What is the magnitude of the magnetic field? **[2.0 marks]**



SOLUTION

A. $f = \frac{1}{T} = \frac{1}{0.42} = 2.38\ \text{Hz}$

B. $\omega = 2\pi f = 15.0\ \text{rad/s}$

C. $\varepsilon = \underbrace{NBA\omega}_{\varepsilon\,\max} \sin \omega t$ so that $B = \frac{\varepsilon_{\max}}{NA\omega} = 0.624\ \text{T}$

P-9. AC Circuits [7.0 marks]

An RLC circuit has a 150-Ω resistor in series with a 1.50-μF capacitor, a 35.7-mH inductor, and a 500-Hz AC generator which supplies a peak emf of $V_{\max} = 30$ V.

A. What is the impedance Z of this circuit? **[2.0 marks]**

B. What is the phase ϕ? **[1.0 mark]**

C. What is the amplitude V_R of the voltage across the resistor, given by $V_R = RI_{\max}$, where $I_{\max} = \dfrac{V_{\max}}{Z}$ is the amplitude of the current provided by the source? **[1.0 mark]**

D. Same question for the amplitude V_C across the capacitor. **[1.0 mark]**

E. Same question for the amplitude V_L across the inductor. **[1.0 mark]**

F. Is the relation $V_R + V_C + V_L = V_{\max}$ satisfied? Explain briefly. **[1.0 mark]**

SOLUTION

A. $Z = \sqrt{R^2 + (X_L - X_C)^2}$ where $X_L = \omega L = 2\pi f L \cong 112\ \Omega$ and $X_C = \dfrac{1}{\omega C} = \dfrac{1}{2\pi f L} \cong 212\ \Omega$. We find $Z = 180\ \Omega$.

B. $\tan\phi = \dfrac{X_L - X_C}{R}$ gives $\phi = -33.7°$.

C. $V_R = RI_{\max} = R\dfrac{V_{\max}}{Z} = 25.0$ V

D. $V_C = X_C I_{\max} = X_C \dfrac{V_{\max}}{Z} = 35.3$ V

E. $V_L = X_L I_{\max} = X_L \dfrac{V_{\max}}{Z} = 18.7$ V

F. $V_R + V_C + V_L = 78.9$ V which is different from $V_{\max} = 30$ V. The relation is not satisfied because the potential differences V_R, V_C, V_L and V_{total} are not in phase. Only their instantaneous values are equal, not their maximum values. In terms of the phasor diagram, the vectors are not parallel.

P-10. Radioactivity and Dosage [7.0 marks]

A patient ingests a radioactive pharmaceutical containing Phosphorus ${}^{32}_{15}\mathrm{P}$, which emits β rays with an RBE of 1.60. The half-life of ${}^{32}_{15}\mathrm{P}$ is 14.28 days, and the initial activity of the medication is 1.21×10^6 decays per second.

A. Knowing the initial activity, what is the initial number of nuclei? **[1.5 marks]**
B. How many nuclei remain after 10 days? **[2.0 marks]**
C. If each emitted electron has an energy of 650 keV, what is the total amount of energy absorbed by the patient's body in 10 days? Hint: the number of emitted electrons is equal to the number of nuclei that have decayed over these 10 days. **[1.5 marks]**
D. Find the absorbed dosage, in rem, assuming that the radiation is absorbed by 95 grams of tissue. **[2.0 marks]**

SOLUTION

A. $N_0 = \dfrac{R_0}{\lambda} = \dfrac{R_0 T_{1/2}}{\ln 2} = 2.15 \times 10^{12}\ \text{nuclei}$

Note: $T_{1/2} = 14.28\,\text{d} \times 24\,\dfrac{\text{hr}}{\text{d}} \times 3600\,\dfrac{\text{s}}{\text{hr}}$

B. $N = N_0 e^{-t \ln 2 / T_{1/2}} = 1.33 \times 10^{12}\ \text{nuclei}$

C. $E = (N_0 - N) E_{1e} = 5.38 \times 10^{17}\ \text{eV} = 0.0861\ \text{J}$

D. $(\text{dose in rem}) = (\text{dose in rad})(\text{RBE}) = \left(\dfrac{E}{m}\right)\left(\dfrac{1\,\text{rad}}{0.01\,\text{J/kg}}\right)(\text{RBE}) = 145\ \text{rem}$