



***A Brief Guide to Engineering Management  
Financial Calculations  
in ENGM 401 Section B1 Winter 2009***

© MG Lipsett 2008  
last updated December 8, 2008

## **Introduction**

This document provides concise explanations of the financial calculations covered in lectures of ENGM 401, with brief context of the analysis scenario where each method applies. Chapter headings refer to the course text *Financial Management for Engineers (3rd Ed.)* by Dr. Peter Flynn.

## **Chapter 2: Introduction to Financial Statements**

Financial statements document the answers to three key questions about a business:

- How much did we make, and how much do we keep?
- How much do we have, and where did it come from?
- Where did our money go?

There are four types of financial statements:

- Income statements
- Balance sheets
- Cash flow statements
- Statement of retained earnings.

Financial statements are part of a chain that turns data into information analyzed in context for informed business decision making:

- Bookkeeping tracks all the transactions in a company
- Accounting assembles all the entries into different financial statements
- Analysis consists of interpreting Financial Statements.

Chronological **journal** entries are transferred to **ledgers**, which are the books of final entry, in the following categories:

- Assets
- Liabilities
- Equity (also called shareholders'/owners' equity or capital)
- Revenues
- Expenses

### ***Double-Entry Accounting***

There are two types of transaction entries, debits and credits.

#### **Debits:**

- An increase in an asset account
- A decrease in a liability account
- A decrease in an equity account
- A decrease in a revenue account
- An increase in an expense account

**Credits:**

- A decrease in an asset account
- An increase in a liability account
- An increase in an equity account
- An increase in a revenue account
- A decrease in an expense account

The process for doing double-entry bookkeeping is as follows:

- Figure out what the two halves of the transaction are
- Decide for each part of the transaction, whether it makes each account go up or down
- Check to make sure that one is a credit and the other is a debit.

**Chapter 3: The Income Statement & Contribution Margin****Key Assumptions**

In this course, unless otherwise stated, the Income Statement is considered to be a financial statement of earnings before interest and taxes, which is of particular interest to operations managers. Cost accounting allocates costs according to categories that relate to whether the cost is directly related to production or indirectly related overhead. How indirect costs are allocated to a product can give a clear (or distorted) picture of the overall cost of a product.

Activity-based accounting is an alternative method of accounting, which links each activity that the organization does to specific cost drivers (e.g., machining, transportation, procurement, engineering,...), and tries to minimize the amount of overhead charges. This method of accounting can be more accurate, but it requires care in budgeting and cost tracking. Activity-based accounting is not covered in this course.

Revenue is the money that comes into the company through sales of its goods or services.

Costs directly related to production are generally included in the category of Cost of Goods Sold (COGS).

Costs that are not directly related to production are generally listed under the category of Sales General & Administrative (SG&A). Depreciation is an element of the expenses that are included within SG&A. The depreciation is a non-cash expense that represents the loss in asset value during the reporting period. The other charges against income that are non-cash charges are:

- Allowances for future costs.
- Allowances for reclamation.

These would also go into cash from operations. We do not usually consider these allowances in most of our work.

Other Income includes one-time revenues and expenses such as writedowns. In some companies, the interest on the long-term debt is also included in Other Income.



Contribution margin represents the value that has been created above what is necessary to cover direct costs; that value can cover the indirect costs and, with Other Income, (hopefully) create profit.

### Calculations

***Net Revenue = Gross Revenue - (Bad debt, warranty, and allowances)***

***Contribution Margin = Net Revenue - COGS***

***Contribution Margin (%) = (Net Revenue - COGS) / Gross Revenue***

***Operating Income = Contribution Margin - SG&A (including Depreciation).***

(Operating Income is usually calculated before considering any taxes.)

***Net Income = Operating Income + Other Income.***

(Net Income is also called Profit or Earnings. If Net Income is negative it is a Loss.)

Quick checks in the Income Statement to assess whether there is enough cash being generated to run the operation are the Cash Flow from Operations and Net Cash Flow.

***Cash Flow from Operations = Operating Income + Depreciation.*** (typically)  
***= Contribution Margin - SG&A (excluding depreciation)***

***Net Cash Flow = Net Income + Depreciation + writedowns.*** (typically)

Net Cash Flow is also called Total Cash Flow. Here we assume that the only Other Income is writedowns.

Cash break-even is when operating income (with depreciation added back in) is zero. Book break-even is when operating income is zero without having to add depreciation back in. Cash break-even thus occurs at a lower operating income than book break-even.

### ***Production***

Production is the product of: the nominal rate of production, the duty (which is the fraction of the nominal production rate that is actually achieved, the utilization (the fraction of the time that production actually occurs), and the time interval:

***Production = Rate<sub>nominal</sub> x Duty x Utilization x time***

## **Chapter 3: Depreciation**

Depreciation is a *non-cash* expense that represents the need to replace assets eventually as they wear out. Different types of assets have different rates at which they lose value. This decline in value is not always linear, because some assets lose value faster earlier in their life, and some assets hold most of their value early but then lose value faster later on. Some assets have residual value at the end of their useful life, called salvage value.

### Key Assumptions

Unless otherwise stated, the fiscal year for a company is the calendar year.



## ENGM 401: Guide to Engineering Management Calculations

The useful life of the asset is estimated to be  $N$  years. The depreciation amount in the first year is adjusted if the asset is purchased in the middle of the year.

The Book Value for year  $t$  is  $BV_t$ , and it can not be less than the Salvage Value  $S$ .

### Calculations

#### *Straight-Line Method:*

The depreciation amount for each full year is the same amount: the original value of the asset  $B$  minus the salvage value  $S$  all divided by the number of years  $N$ :

$$d_t = \frac{B - S}{N}$$

The Book Value at the end of year  $t$  is the original book value  $B$  minus the sum of the accumulated depreciation amounts:

$$BV_t = B - \sum_{j=1}^t d_j$$

#### *Sum-of-Years-Digits Method:*

The depreciation amount is different for each year  $t$  (where  $t$  starts at 1 and goes to  $N$ ):

$$d_t = \frac{N - t + 1}{N(N+1)/2} \times (B - S)$$

The Book Value at the end of year  $t$  is the original book value  $B$  minus the sum of the accumulated depreciation amounts:

$$BV_t = B - \sum_{j=1}^t d_j$$

#### *Declining Balance Method:*

The depreciation amount in a given year depends on a depreciation rate  $D$  multiplied by the Book Value for the previous year (which in the first year is the original value  $B$ ).

$$d_t = D \times BV_{t-1}$$

The Book Value at the end of year  $t$  is the original book value  $B$  minus the sum of the accumulated depreciation amounts:

$$BV_t = B - \sum_{j=1}^t d_j$$

Alternatively, the Book Value at the end of year  $t$  can be calculated directly using:

$$BV_t = B \times (1 - D)^t$$

Note that this method does not explicitly use an estimate of the useful life of  $N$  years or salvage value  $S$ , but the rule still applies that  $BV_t$  can not be less than  $S$ .



**Double Declining Balance Method:**

This is a special case of the declining balance method, where a useful life **N** is defined, from which the depreciation rate is set to be  $D = 2/N$ . The depreciation amount in a given year **t** is:

$$d_t = D \times B \times (1 - D)^{t-1}$$

The Book Value at the end of year **t** can be directly calculated as:

$$BV_t = B \times (1 - 2/N)^t$$

**Units of Production Method:**

The depreciation amount in a given year **t** depends on the actual production **P<sub>t</sub>** and the total lifetime production **P<sub>lifetime</sub>** over the useful life of **N** years. The units of production will be in terms of the production method, such as tonnes of oilsand moved by a haul truck, number of widgets stamped out by a press, etc.

$$d_t = \frac{P_t}{P_{lifetime}} \times (B - S)$$

The Book Value at the end of year **t** is the original book value **B** minus the sum of the accumulated depreciation amounts:

$$BV_t = B - \sum_{j=1}^t d_j$$

**Assets Purchased Midway through a Year:**

When an asset is purchased in the middle of a year, depreciation is prorated in that year. The calculation method is as follows:

- 1) Calculate the nominal annual depreciation for the first year using the chosen method (straight-line, SOYD, etc).
- 2) Book 1/12 of the nominal first-year depreciation amount per month *on a straight-line basis* for the remainder of the year, booking the first depreciation at the end of the month in which the asset was purchased. For example, a purchase in April entails nine depreciation amounts for the remainder of the year (end of April, end of May, ..., end of December). Note that when an asset is purchased midyear, the nominal salvage value **S** is not actually achieved at the end of the **N**th year after the purchase.

**Depreciation for Tax Purposes:**

When calculating capital cost allowance for year **t** (**CCA<sub>t</sub>**) and the unclaimed capital allowance (**UCC<sub>t</sub>**), group assets by their taxation class and use the appropriate rate for each class. The sum of transactions for year **t** is **T<sub>t</sub>**; an asset transaction is positive for purchase of an asset & negative for a sale of an asset. For each class of asset, the calculation procedure is:

Find **T<sub>t</sub>**, which is the sum of the transactions for the current year **t**

If **T<sub>t</sub>** > 0, then **T<sub>t</sub><sup>\*</sup>** = 0.5 **T<sub>t</sub>**; otherwise **T<sub>t</sub><sup>\*</sup>** = **T<sub>t</sub>** (50 % rule)

Calculate **CCA<sub>t</sub>** = **R** \* (**UCC<sub>t-1</sub>** + **T<sub>t</sub><sup>\*</sup>**)

Calculate **UCC<sub>t</sub>** = **UCC<sub>t-1</sub>** + **T<sub>t</sub>** - **CCA<sub>t</sub>**



## Chapter 4: The Balance Sheet, Working Capital, and Leverage

### *Working Capital*

#### Key Assumptions:

A short-term lender may put a condition on a company requiring it to have a certain amount of Working Capital, which is calculated from figures on the balance sheet. A low Working Capital Ratio is an indication that the company may have trouble dealing with swings in short term expenditures.

#### Calculations:

***Working Capital = Current Assets - Current Liabilities.***

***Working Capital Ratio = Current Assets / Current Liabilities***

### *Leverage*

Most companies require financing. This is done either through equity (money from owners) or debt (money from others). Interest is the cost of debt. A company usually has a mix of both kinds of financing.

In good times, the earning power of the investment is higher than the cost of debt (i.e., interest on the Bank Debt). In that case, the extra value that has been created gets transferred to Equity, thus increasing the Return on Equity.

In bad times, the interest on the debt portion is higher than the earning power of the investment; and so the Return on Equity gets reduced to cover the shortfall. (A negative Return on Equity means that the owners are losing money.)

#### Key Assumptions

A number of methods can be used to calculate leverage. In this course, we only consider the leverage between Bank Debt (a type of long-term debt) and Equity. (In the course text, leverage is measured by the debt ratio, which is total liabilities divided by total assets.)

For leverage analysis, we assume Net Income to be the Operating Income minus the interest on the long-term debt (found on the Income Statement). (This is consistent with including interest on long-term debt as a negative entry in Other Income.)

- Bank Debt is found on the Balance Sheet, usually named Long-Term Debt.
- Total Assets and Total Equity are also found on the Balance Sheet.
- The bank interest rate is given information.

#### Calculations



## ENGM 401: Guide to Engineering Management Calculations

Three sets of calculations are made: leverage, net income, and return on equity.

***Leverage = Bank Debt / Total Assets (expressed in percent).***

***Net Income = Operating Income - (interest rate) x (Bank Debt)***

***Return on Equity = Net Income / Total Equity. (expressed in percent)***

(In the course text, leverage is measured by the debt ratio, which is total liabilities divided by total assets. This course takes a simplified approach.)

### Chapter 5: Cash Flow Statement

A business must have access to liquid funds (“cash”) to pay its day-to-day expenses. The statement of cash flow looks at the non-cash accounts to determine the overall flow of cash in and out of a company during a reporting period.

#### Key Assumptions

##### *Sources and Uses of Funds:*

Sources and Uses of funds are enumerated from two successive Balance Sheets and the Income Statement for the period between them.

- For Asset accounts, an increase is a use of funds; a decrease is a source of funds.
- For Liability accounts, an increase is a source of funds; a decrease is a use of funds.
- For Equity accounts, an increase is a source of funds; a decrease is a use of funds.

On the Statement of Cash Flow, a source of funds is entered as a positive number; and a use of funds is entered as a negative number. This is potentially confusing for asset accounts, where an increase is entered as a negative number.

The short-term credit line can be thought of as “negative cash.”

Depreciation is always considered to be a source of funds.

##### *Non-Cash Working Capital:*

Non-cash working capital is the sum of current assets except cash minus the sum of current liabilities except the short-term credit-line and the current portion of the long-term debt.

##### ***Non-cash working capital***

***= (total current assets - cash)***

***- (total current liabilities - STCL - current portion of long-term debt).***

The change in non-cash working capital is easily tallied from the Statement of Cash Flow. If an asset account has increased during the period, then the change is a use of cash; and so the value of the change in that account would be a negative number. If a liability account has increased over the period, then the change is a source of cash; and so the value of the change in that account is a positive number.



## ENGM 401: Guide to Engineering Management Calculations

From a typical Statement of Cash Flow, the change in non-cash working capital is the sum of sources/uses of cash from these six accounts: receivables, inventory, prepaids, accounts payable, accrued expenses, taxes payable.

### Calculations

The Statement of Cash Flow has three sections, which sum to the change in net cash over the period.

#### ***Operating Activities (OA):***

Enter Net Income for the period from the Income Statement (not retained earnings).  
Add back in depreciation and any other non-cash charge (as sources of funds).  
Add (or subtract if appropriate) the changes in non-cash working capital.  
Total these entries.

#### ***Investing Activities (IA):***

Enter any additions or removal of asset acquisition costs (if new assets are purchased, then the asset account increases, which is a use of funds, so the value is negative).  
Enter the gain or loss on sale of shares or investments in other companies (Goodwill and intangibles). (Again, if new assets are purchased, then the value is negative.)  
Total these entries.

#### ***Financing Activities (FA):***

Enter any new equity injection (an increase in shares would be a source of equity funds) or any withdrawal (a use of equity funds).  
Add any new long term borrowing that has occurred (a source of funds) or subtract repayment (a use of funds).  
Subtract dividends paid (a dividend is a use of equity funds).  
Total these entries.

#### ***Change in Net Cash:***

Operating Activities plus Investing Activities plus Financing Activities equals the change in net cash during the period:

$$\mathbf{Change\ in\ Net\ Cash = OA + IA + FA.}$$

Change in Net Cash is also called the Funds Flow during the reporting period. Funds Flow is the Change in Cash Position from the start of the period to the end of the period, which equals the change in the sum of cash minus the amount of the short-term credit line.

A check can thus be made on the change in Net Cash calculation. Net cash at a point in time is cash minus the amount of the STCL (information found on the balance sheet). The balance sheet at the time when the reporting period starts tells you the net cash as of the start of the period: cash minus the amount of STCL. Similarly, net cash at the end of the period is found from the balance sheet at the time of the end of the period. That means that the Funds Flow





(the sum of OA + FA + IA) will equal the change (from the beginning to the end of the period) of the sum of cash minus the amount of the short-term credit line.

If the change in net cash is negative, the flow of funds is into the company, meaning that sources of cash had be available to the company during the period to support its operations (the cash account going down is a source of cash, just as an increase in the STCL is a source of cash). More cash is being used by the company than it produces. This is not a happy situation if it persists too long.

If the change in net cash is positive, then funds flow is outward. There are uses of cash and/or of the STCL coming out of the company during the period; this means that the company was able to support its activities without having to find more cash from outside.

## Chapter 6: Financial Ratios

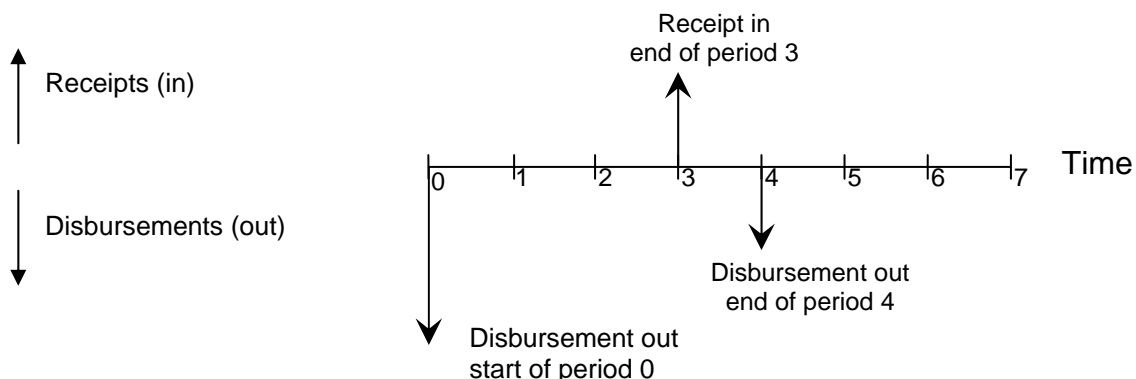
A list of financial ratios is given in the text. There is also a summary in the Glossary, for which there is a link on the course web site. A single ratio does not give a complete picture of the operational and financial situation of a company. Although most ratios have a standard definition, one should check what parameters are actually being used by a company to calculate its ratios.

## Chapter 7: The Time-Value of Money & Equivalence

A sum of money is not worth as much in the future as it is today. Interest is the cost of using money (until some time in the future) that could otherwise be used for something else today. Money has to earn interest to maintain its value. Equivalence allows the value of money at one point in time to be expressed at another point in time, for a given interest rate.

### *Cash Flow Diagram:*

A cash flow diagram is used to visualize inflows and outflows over a time period. Time is plotted along a horizontal (divided into equal periods). Inflows are cash the company receives (also called receipts); these are shown as positive arrows which point up. Outflows are cash the company pays out (also called disbursements); these are shown as negative arrows which point down. The amount shown on the diagram is in terms of value at the time at which the activity takes place.





### Key Assumptions

The original amount is called the Principal **P**.

Simple interest calculates interest on the Principal only.

Compound interest calculates interest on the Principal as well as the previous interest.

### Calculations

#### ***Simple Interest:***

The future value of a present sum (the Principal) at an interest rate **i** after **n** interest periods is:

$$F = P \times (1 + i \times n)$$

#### ***Compound Interest:***

The future value of a present sum (the Principal) at an interest rate **i** after **n** interest periods is:

$$F = P \times (1 + i)^n$$

This is consistent with our understanding that money has to earn interest to maintain its value into the future.

The present value of a sum **n** periods in the future is calculated by using a negative value for the number of periods:

$$P = F \times (1 + i)^{-n}$$

#### ***Equivalent Annual Interest Rate:***

While the interest rate is quoted for an annual basis, in reality most loans have payment periods that are different from one year. The Effective Annual Interest Rate (EAIR) is the annual interest rate that takes into account any compounding that occurs over the year:

$$F = P \times \left(1 + \frac{i}{m}\right)^{m \times n}$$

where **F** = the future sum, **P** = the present sum, **i** = nominal annual interest rate, **m** = number of compounding periods per year, and **n** = number of years,

$$\therefore EAIR = \left(1 + \frac{i}{m}\right)^m - 1$$

To find the future value of a present sum with some number of compounding periods per year, calculate it as though the interest rate was the EAIR compounded annually:

$$F = P \times (1 + EAIR)^n$$

## **Chapter 7: Interest**



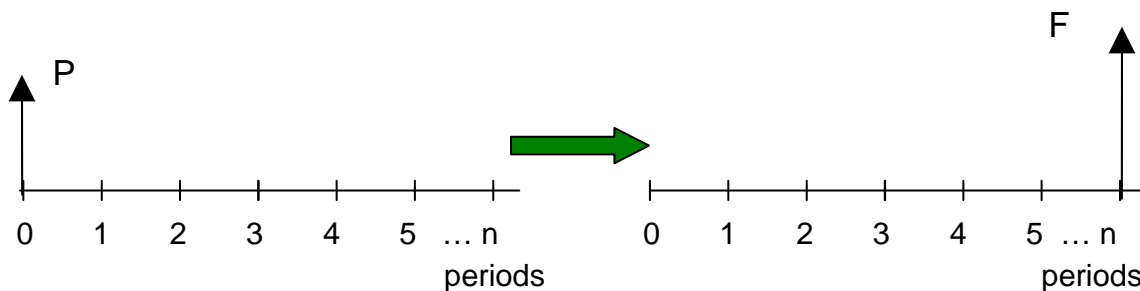
Key Assumptions

For each method, the interest rate  $i$  is assumed to be for the interest period. For example, if the period is one year, then the interest rate  $i$  is the annual interest rate; but if the interest period is one month, then the interest rate for the period  $i$  would be 1/12 of the annual interest rate. Interest rate, discount rate, and return rate all mean the same thing.

Calculations

Direct calculation methods and interest tables were developed to solve a number of different simple cash flow series. These cash flows are idealized cases (which makes them good for midterm questions that test understanding); but in practice most cash flow series are complicated and are easier to solve using spreadsheets.

**Single Payment Compound Amount:** The future amount  $F$  at the end of  $n$  periods after depositing a sum  $P$  at the start of the  $n$  periods, with interest rate  $i$ .

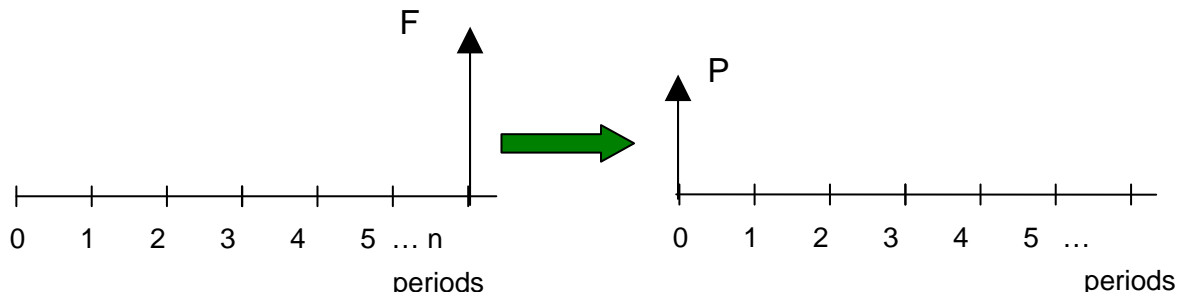


In interest tables, use  $F = P \times (F/P, i, n)$ . The formula for direct calculation is:

$$F = P \times (1 + i)^n$$

(This is the same as same as the compound interest calculation in the previous section for the future value of a present sum.)

**Single Payment Present-Worth Amount:** The present value  $P$  at the start of a set of periods (“end of year 0”) equivalent to future amount  $F$  at the end of  $n$  periods, with interest rate  $i$ .

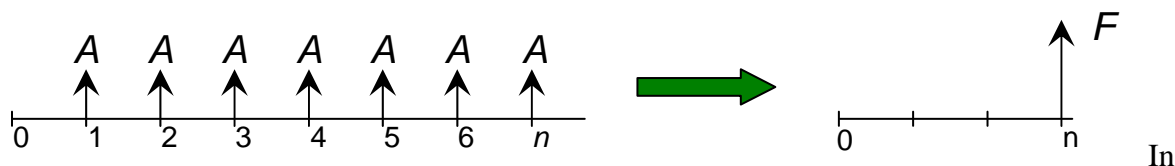


In interest tables, use  $P = F \times (P/F, i, n)$ . The formula for direct calculation is:

$$P = F \times (1 + i)^{-n}$$



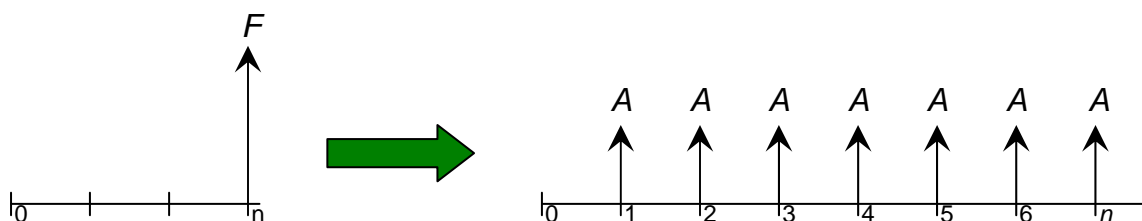
**Uniform Series Compound Amount:** The future amount  $F$  at the end of  $n$  periods after depositing a sum  $A$  at the end of each period with interest rate  $i$ .



In interest tables, use  $F = A \times (F/A, i, n)$ . The formula for direct calculation is:

$$F = A \times \left( \frac{(1+i)^n - 1}{i} \right)$$

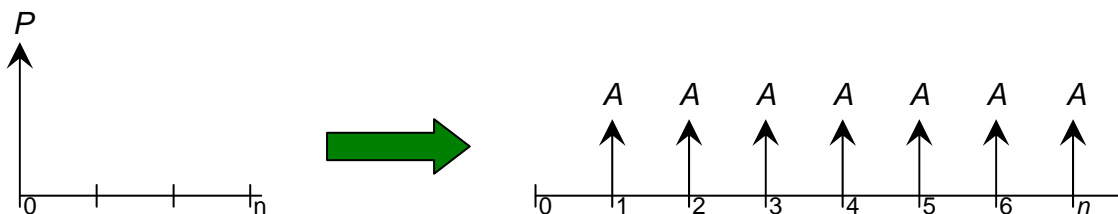
**Uniform Series Sinking Fund:** The amount  $A$  that would have to be deposited at the end of the period for  $n$  periods with interest rate  $i$  to yield the future amount  $F$ .



In interest tables, use  $A = F \times (A/F, i, n)$ . The formula for direct calculation is:

$$A = F \times \left( \frac{i}{(1+i)^n - 1} \right)$$

**Uniform Series Capital Recovery:** The size of payments  $A$  required at the end of each period to pay back a present sum  $P$  over  $n$  periods with interest rate  $i$ .

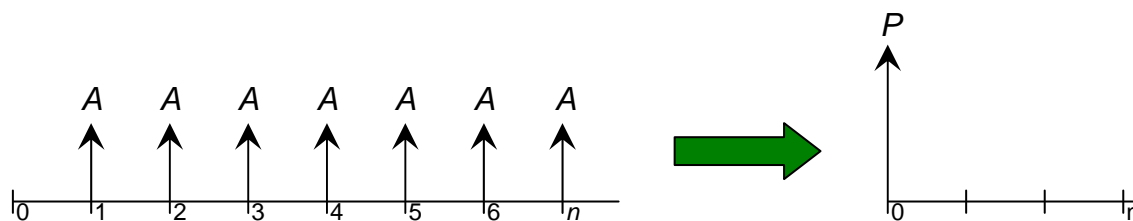


In interest tables, use  $A = P \times (A/P, i, n)$ . The formula for direct calculation is:

$$A = P \times \left( \frac{i \times (1+i)^n}{(1+i)^n - 1} \right)$$



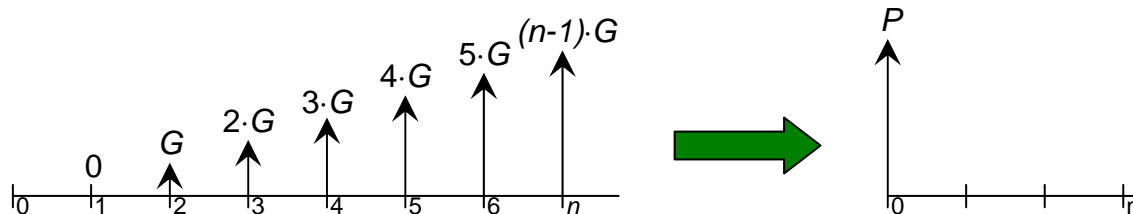
**Uniform Series Present Worth:** The present sum  $P$  that would be required to invest now to provide end-of-period payments of  $A$  for  $n$  periods with interest rate  $i$ .



In interest tables, use  $P = A \times (P/A, i, n)$ . The formula for direct calculation is:

$$P = A \times \left( \frac{(1+i)^n - 1}{i \times (1+i)^n} \right)$$

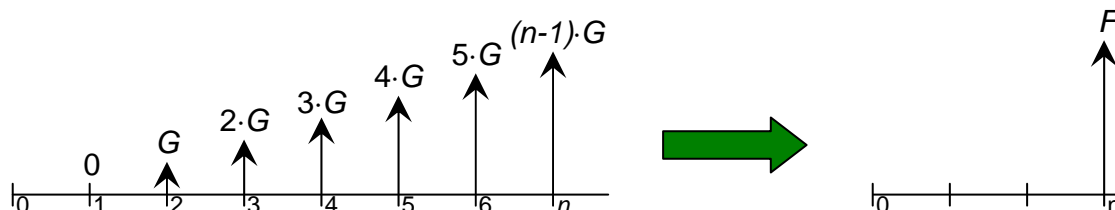
**Arithmetic Gradient Present Worth:** The present sum  $P$  from depositing a uniformly increasing series of sums,  $G$ ,  $2G$ , etc., for  $n-1$  periods with interest rate  $i$ . Note that the sequence starts at zero payment at the end of the first period, and the  $(n-1)^{\text{th}}$  payment is at the end of period  $n$ .



In interest tables, use  $P = G \times (P/G, i, n)$ . The formula for direct calculation is:

$$P = G \times \left( \frac{(1+i)^n - i \times n - 1}{i^2 \times (1+i)^n} \right)$$

**Arithmetic Gradient Future Worth:** The future worth  $F$  at the end of  $n$  periods, from depositing a uniformly increasing series of sums,  $G$  at the end of the second period,  $2G$  at the end of the third period, etc., to  $(n-1)G$  at the end of period  $n$ , with interest rate  $i$ . Note that the sequence starts at zero payment at the end of the first period, and the  $(n-1)^{\text{th}}$  payment is at the end of period  $n$ .

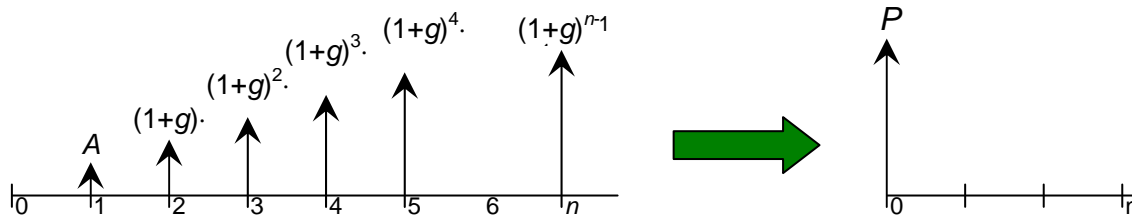


In interest tables, use  $F = G \times (F/G, i, n)$ . The formula for direct calculation is:

$$F = G \times \left( \frac{(1+i)^n - i \times n - 1}{i^2} \right)$$



**Geometric Series Present Worth:** The present sum  $P$  from depositing an increasing series of sums,  $A$  at the end of period 1,  $(1+g)A$  at the end of period 2, etc., up to a deposit of  $(1+g)^{n-1} \cdot A$  at the end of period  $n$  with interest rate  $i$ .



In interest tables, use  $P = A \times (P/A, g, i, n)$ . The formula for direct calculation is:

$$P = A \times \left( \frac{1 - (1+g)^n \times (1+i)^{-n}}{i - g} \right), \text{ if } i \neq g; \text{ otherwise, } P = \frac{A \times n}{1+i}, \text{ if } i = g.$$

### Combinations

These formulæ can be used in combination to represent a more complicated set of cash flows, in a linear combination of series.

The formulæ can be used to convert from one type of series to another. One formula is used in an intermediate calculation of the equivalent  $P$  (or  $F$ , if you prefer), and then that value of  $P$  (or  $F$ ) is then substituted into another equation that yields the equivalent set of payments for a different type of series, or for an equivalent sum at a different point in time. Some examples include:

$$(F/P, i, N) = i(F/A, i, N) + 1$$

$$(P/F, i, N) = 1 - (P/A, i, N)i$$

$$(A/F, i, N) = (A/P, i, N) - i$$

$$(A/P, i, N) = i / [1 - (P/F, i, N)]$$

$$(F/G, i, N) = (P/G, i, N)(F/P, i, N)$$

$$(A/G, i, N) = (P/G, i, N)(A/P, i, N)$$

$$(F/A, g, i, N) = (P/A, g, i, N)(F/P, i, N)$$

## Chapter 7: Present Worth Analysis

Future values are a set of monetary values expressed in terms of the time at which each of the transaction occurs. Each amount is expressed in the value of the currency at that particular time, referred to as future dollars or more generally as “as-spent” dollars.

Present values (PV) convert the set of future values to the equivalent set of present sums in terms of a single value of currency at a single point in time, referred to as “today’s dollars.” This conversion is done using  $(P/F, i, n)$ , where  $i$  is the discount rate (or interest rate or return rate).

In Present Worth analysis, the focus is on the future. Any values from a time before the present time are ignored. These are called sunk costs.



The sum of the PV values over the course of the overall time determines the net present value (NPV) of financing plus the returns on the investment over and above the interest rate  $i$ . A company usually has a discount rate that is the minimum that is acceptable as a “hurdle rate”: the Minimum Acceptable Rate of Return (MARR).

When MARR is used as the interest rate, then the NPV needs to be greater than zero to be accepted. (That’s why it is called a hurdle rate: the NPV has to be more than zero to clear the bar at MARR.) The Weighted Average Cost of Capital (WACC) is another hurdle rate that a company may use, which represents the cost of its financing and its overall level of investment risk. If the investment does not provide a return at that rate or better, then the company’s performance slips.

### Present Worth Analysis Using A Spreadsheet

Present Worth Analysis is most easily done using a spreadsheet program. The formulæ shown in the section above are useful for specific situations; but in reality most cash flows are not as well behaved as the idealized series.

#### Calculation

To set up the spreadsheet, each future value  $F$  is listed with time at which occurs, expressed as a number of time intervals  $n$  (usually years). The interest rate  $i$  is entered into a cell on its own. Usually the interest period is one year, but not necessarily.

The future values are entered in column B starting “today” in row 4, with the value one period later in row 5, etc.

	A	B
1	Present value	
2	i =	3%
3	n	FV
4	0	\$0.00
5	1	\$100.00
6	2	\$100.00
7	3	\$100.00

Recall that the present value  $P$  of a sum that is  $n$  periods in the future (the future value  $F$ ) is calculated by using a negative value for the number of compounding periods:

$$P = F \times (1 + i)^{-n}$$

This formula is used to calculate the present value of each of the individual future values. In the example spreadsheet, the expression that is typed into cell \$C\$5 to calculate the present value  $P$  of \$100 at the end of period 1:  $P = 100 \times (1 + 0.03)^{-1}$  is =B5\*(1+\$B\$2)^(-A5). An absolute cell reference is used for the interest rate in cell \$B\$2.

The value in cell \$C\$6 calculates the present value at the end of year 2, which in this case is  $P = 100 \times (1 + 0.03)^{-2}$ , the formula in the cell is =B6\*(1+\$B\$2)^(-A6). The value in cell



## ENGM 401: Guide to Engineering Management Calculations

\$C\$7 to calculate the present value at the end of year 3, or  $P = 100 \times (1 + 0.03)^{-3}$ , is  $=B7*(1+B$2)^(-A7)$ .

	A	B	C
1	Present value		
2	i =	3%	
3	n	FV	PV
4	0	\$0.00	\$0.00
5	1	\$100.00	\$97.09
6	2	\$100.00	\$94.26
7	3	\$100.00	\$91.51

The NPV is simply the sum of the present values. In Excel, use the SUM( ) function. In this example the formula in cell \$C\$8 is  $=SUM(C4:C7)$

	A	B	C
1	Present value		
2	i =	3%	
3	n	FV	PV
4	0	\$0.00	\$0.00
5	1	\$100.00	\$97.09
6	2	\$100.00	\$94.26
7	3	\$100.00	\$91.51
8		NPV	\$282.86

In this particular example, the NPV of the series happens to be the same as the Uniform Series Present Worth in the case of  $P = 100 \times (P/A, 3\%, 3)$ .

Here is a tip for setting up a spreadsheet for NPV. Use an absolute reference for the interest rate. Remember that the convention for an absolute reference to a cell is to use dollar signs in front of the column (letter) and/or the row (number), \$B\$2 in this example. In this way, the present value is calculated by pointing directly to the cell that has the number that you use repeatedly, in this case the interest rate. A common error can occur when setting up the spreadsheet by copying cells into another location that end up referring to the wrong cell. Check your calculation formulæ in the cells to make sure you are using the correct parameters from other cells.

### Chapter 7: Rate of Return

The internal rate of return (IRR) is the interest rate for which the sum of present values totals to zero. This is the rate of return at which the benefits of the investment are the same as its costs. If the IRR is greater than MARR, then it is a worthwhile investment.

If IRR is higher than MARR, then later future sums have more value at the MARR hurdle rate than at the IRR rate, which means that the investment will deliver value beyond what is demanded at the MARR. If IRR is below MARR, then later future sums have less value at the MARR hurdle rate than at IRR rate, and so the investment will not deliver value required at MARR.





### Calculation:

Tabulate a time series set of future values on a spreadsheet and calculate their individual present values using a first guess at the IRR interest rate. Calculate the Net Present Value of the series of Present Values. Then, use the Excel solver function to make this IRR Present Value total equal to zero by solving for the IRR interest rate, which is chosen by selecting the cell that contains the interest rate.

It is good practice to create an additional column in a spreadsheet for each present value case (e.g. MARR and IRR). You should question an IRR in the bazillions of percent; a common error is not putting the correct signs on costs or benefits.

### ***Solving for $i$ of a Cash Flow Series by Interpolation Using Interest Tables***

We can find an approximate solution for the interest rate for a cash flow series which we can express as a formula, provided that  $i$  is the only unknown, by finding the value of the formula factor, and then interpolating in the interest table to find the approximate value of  $i$ .

### ***Incremental IRR***

Incremental IRR (IIRR) is used for an investment that is being considered as an alternative to another investment. The investment with the lower capital cost is considered to be the base case. The cash flow of the higher-capital-cost alternative (in future dollars) is subtracted from the cash flow of the base investment (also in future dollars) to give the incremental cash flow. The IIRR is then found using an iterative solution (such as Solver in Excel) for the series of present values of the incremental cash flow that gives an NPV of \$0. If the IIRR is great than MARR, then the higher capital cost alternative should be selected, because it adds enough extra value to be worthwhile. In fact, if the IIRR is greater than the IRR of the base investment, then the incremental investment adds more value to the investment on a relative basis than the base investment does.

## **Chapter 7: Other Analysis Techniques**

### ***Benefit-Cost Analysis***

The Benefit-Cost ratio compares the Present Worth of the benefits of an investment  $PW_{benefits}$ , and the Present Worth of the costs  $PW_{costs}$

$$BCR = PW_{benefits} / PW_{costs}$$

If  $BCR < 1$ , then the benefits are less than the costs, and so the investment is not worthwhile.

### ***Incremental Benefit-Cost Analysis***

Incremental Benefit-Cost analysis compares increasing costly alternatives.



## ENGM 401: Guide to Engineering Management Calculations

- For each option, calculate the PW of its benefits the PW of its costs, and its Benefit to cost ratio. Reject any option that has a BCR < 1.
- Arrange the remaining options in ascending order of costs.
- Then calculate the incremental BCR between cases, starting with the lowest two cost options. As analysis proceeds, reject any option that gives an incremental BCRs that is less than 1, and use the next higher cost option to calculate the incremental BCR.
- Once all the valid incremental BCRs have been calculated, choose the option with highest of the acceptable incremental BCRs. Note that the choice will not necessarily have the highest BCR.

### ***Payback***

Payback happens when an investment has paid for itself. The point at which the cumulative sum of values becomes positive is when payback occurs.

For engineering projects, the payback time is usually counted as the time from start-up to the point at which sum of the series of time values goes positive, rather than from the beginning of the project itself. Payback usually happens in the middle of a period, but payback period is usually expressed as an integer: the end of that period minus beginning of start-up. For example, if an engineering project started at the beginning of year 1, started up sometime in year 4, and the sum of values went positive sometime in year 11, then the payback time is  $11 - 4 = 7$  years. Some people interpolate within a year to get more precision, but it's not usually worth the extra effort, given the inherent uncertainties in such analyses.

Simple payback is found from the series of future values.

Discounted payback is found from the series of present values (the discounted values of the future sums).

### ***Break-Even Analysis***

Break-even analysis considers the effect of a parameter in an investment option on its equivalence to another investment option. The key is to express one investment option in terms of the variable to be solved and the equivalent value of the other option.

If the cash flow is simple, then it is often easiest to solve for a parameter directly using an interest formula and solving by interpolation from the interest tables. Of course, a spreadsheet can be used with the Solver function to find the value of the parameter.

### ***Solving for $n$ of a Cash Flow Series by Interpolation Using Interest Tables***

Break-even analysis is often concerned with the time period associated with different investment options. Interpolation can also be used to find the value of  $n$ . In Lecture 27, there is a break-even analysis problem that determines how many years a corrosion-resistant part have to last to have the equivalent uniform annual cost of the untreated part. The untreated part costs \$350 and lasts 6 years, so the uniform annual costs is  $\$350 \times (A|P, 10\%, 6) = \$80.36$ . The corrosion-resistant part equivalent uniform annual cost (Option B) must be at



least this low, so we have to find the number of years that will give the same amount, i.e., \$500 (A|P,10%, n) = \$80.36. So now we know that (A|P,10%, n) = \$80.36 / \$500 = 0.1607, and we look in the Uniform Series Capital Recovery Table (on slide 13 of Lecture 21) and find that in the column for (A|P,10%,n) the value 0.1627 for n = 10 and 0.1540 for n=11. That means that the solution lies somewhere between 10 and 11 years. By interpolation, we can solve n = 10 + (0.1627 - 0.1607)/(0.1607 - 0.1540) = 10.23 years.

(A|P, 10%, n) = \$80.36 / \$500 = 0.1607

so we solve for n iteratively using the uniform series capital recovery formula:  $A = P \times \frac{i \times (1+i)^n}{(1+i)^n - 1}$

or (more simply) use an interpolation between values in the interest table:

$n = 10 + \frac{0.1627 - 0.1607}{0.1627 - 0.1540} = 10.23 \text{ years}$

### What Is Included in the Cash Flow Series for an Investment Project

In evaluating an investment, only future costs are considered. The cash flow series should only include the incremental costs to do the project, and none of the costs associated with running the current business. For example, allocation of costs for head office overheads does not enter into a cash flow analysis, since no incremental money is spent. (The head office would run whether the project went ahead or not.) Providing an accounting reserve for future expenditures is a common accounting practice (recall that allowances are used to blend out expenditures); but cash flow forecasts use actual cash flows that are expected to occur, not allowances. Some projects may do preliminary project development work as part of regular business activities. These sunk costs would not be attributed to the project.

## Chapter 7: Inflation

Inflation is a quantitative measure of how much the purchasing power of money decreases as time moves forward. (Remember that a dollar tomorrow will buy less than it does today.) Inflation is a negative compound interest, i.e., 3% inflation is like a -3% interest rate.

In reality, the inflation rate varies continually over time and in different regions. In this course, the inflation rate is assumed to be fixed unless stated otherwise.



### Calculation:

#### ***Real Interest Rate***

The real interest rate for a period  $i_r$  is approximately equal to the nominal inflation rate  $i_n$  minus the inflation rate  $f$ :

$$i_r = i_n - f$$

The more accurate formula for real interest rate is

$$i_r = \frac{1 + i_n}{1 + f} - 1$$

If the interest rate (or discount rate) needs to include inflation, then the real interest rate should be used.

#### ***Net Present Worth including Inflation***

A calculation of NPV is typically done using a nominal MARR. If inflation  $f$  is a factor, then NPV should be calculated using inflation-corrected future values. This is done as follows:

- Adjust the future values in each period for inflation  $f$  (the future values will go up, reflecting the equivalence associated with the declining value of money).
- Then calculate the PVs using these inflation-corrected future values and the nominal MARR. In the case of taxable income, do the inflation calculation first to get the inflation-corrected FV of the cash flows before doing the tax calculation.

An alternative approach is to use the nominal future values and a MARR that is corrected for inflation:

- Use the future values as quoted, without any inflation correction, and then use the real  $MARR_{real} = (1 + MARR)/(1 + f) - 1$ .

**Caution:** The alternative method only works if there is no intermediate calculation to do, such as taxes, and so it is prone to error. For this reason, the first method is preferred.

## **Chapter 7: After-Tax Cash Flow**

### Key Assumptions

After-tax cash flow is discussed for revenue from normal operations, i.e., earnings before interest and taxes is considered to be the same as operating income.

The tax implications of Other Income depend on the specific situation (including how interest is earned and booked as Other Income), and so its tax is handled on a case-by case basis and not discussed further here.



## ENGM 401: Guide to Engineering Management Calculations

In some cases, depreciation may have been booked as a non-cash expense in Operating Income. In that case, the taxable cash flow from operations will be the sum of operating income plus depreciation.

### Calculation:

For each year, the after-tax income involves a set of calculations:

- 1) Take the Operating Income and adjust for any expenses that are not eligible for tax deductions (e.g. some kinds of meal expenses). Interest on loans is a taxable expense.
- 2) Add depreciation back into the Operating Income (if necessary) to give the taxable cash flow from operations. (If inflation is a factor, then Operating Income is corrected for inflation before doing the taxation calculations in subsequent steps.)
- 3) Calculate the Capital Cost Allowance (CCA) for each class of capital costs (using the Canada Revenue Agency guidelines). (See Depreciation for Tax Purposes in the section on depreciation in this guide for details on how to do that calculation.)
- 4) Deduct the CCA (up to the limit of net income from **all** sources) to get the taxable income. Taxes for an incremental cash flow may actually be negative, if enough other sources of income exist in the company from which to deduct taxes. (See the section below if there is disposal of an asset in the year being considered.)
- 5) Calculate income tax payable based on taxable income and the applicable percentage tax rates for the business.
- 6) Deduct the income tax from taxable income to get net income after tax for the year of interest. This is the after-tax cash flow for that year.

This set of steps is done for each period (generally annually) over set of intervals in time to give a series of after-tax cash flows, which is a set of future sums. For a series of after-tax cash flows, the NPV is calculated using the same procedures as for any other time-series of future sums.

### *Accounting for Salvage Value*

Because CCA is calculated using declining balance (i.e., it carries on forever), disposal of assets is not straightforward. A disposal tax effect (DTE) is included to account for the fact that salvage value is either a loss or a gain in the year  $t$  in which it occurs. DTE in year  $t$  is equivalent to the difference between the salvage value and the UCC<sub>t</sub> book value multiplied by the tax rate  $TR$ :

$$DTE = (BookValue - SalvageValue) \times TR$$

The After Tax Cash Flow in that year includes the Salvage Value and the DTE:

$$ATCF = ATCF_{Nominal} + Salvage\ Value + DTE$$



Three cases are of interest:

- 1) If an asset is sold for less than its UCC value, then the DTE is positive.
- 2) If an asset is sold for greater than its UCC value, then the DTE is negative (salvage value is larger than book value), meaning the tax effect is an increase in taxes (i.e., ATCF is reduced by DTE).
- 3) If an asset is sold for greater than its purchase price (Basis Value), then the DTE is negative, and equal to the sum of two tax effects:

$$\begin{aligned} \text{Tax On Recapture} &= (\text{Book Value} - \text{Basis Value}) \times \text{TR} \\ \text{Capital Gains Tax} &= (\text{Basis Value} - \text{Salvage Value}) \times \text{TR} / 2 \\ \text{DTE} &= \text{Tax On Recapture} + \text{Capital Gains Tax} \end{aligned}$$

### ***Loan Principal Repayment:***

When a loan is taken out, there is an agreed schedule for payments. Mortgage payments are generally done on a level payment basis: each payment is the same; and that payment will be the sum of the interest on the remaining principal amount for the period, plus some repayment of the principal. Because part of the principal gets paid off, the interest amount goes down over time and more of the payment goes toward paying down the remaining principal amount.

The interest part of a payment in a particular period is the principal for that period times the interest rate. The mortgage payment minus the interest is applied to repayment of the principal, and so the principal for the next period is the current principal minus the repayment.

Because interest is a taxable expense, the cost of interest is subtracted from income before debt service (that's the interest portion only, not the principal repayment part). That pre-tax net income then has the taxes subtracted (at the tax rate) to yield the after-tax net income. The cash flow after tax is typically the after-tax net income with depreciation added back in (a simplification of the real world).

Principal repayment is done using after-tax dollars, and so

$$\text{Cash flow after tax and principal repayment} = \text{Cash flow after tax} - \text{principal repayment.}$$

## **Chapter 7: Uncertainty**

### Key Assumptions

Most of our calculations have been done with the assumption that the parameters are known. In reality, there is often a lot of uncertainty in our estimates, and no one really knows what interest rates will be a couple of years in the future. To deal with such uncertainty, analysts use probability distributions or other methods to describe the uncertainty. A simple way to describe uncertainty is to state the most likely value, and a number on either side that represents the “worst-case” value and the “best-case” value. Ideally these would bracket the risk associated with the uncertainty.



## Calculations

For each parameter, choose a value for the most likely case, the worst case, and the best case.

### ***Best-Case/Worst-Case Scenarios***

Do the analysis first using the most likely values for the parameters. This gives the realistic result.

Repeat the analysis, assigning all the uncertain parameters their best-case values. This gives the optimistic result.

Repeat the analysis, assigning all the uncertain parameters their worst-case values. This gives the pessimistic result.

### ***Sensitivity Analysis***

Do the analysis first using the most likely values for the parameters. This gives the realistic result.

Repeat the analysis, but varying only one of the uncertain parameters. The set of results shows the sensitivity of the answer to variability of that particular parameter. Computing the variation of a particular parameter that would cause a decision to change yields the sensitivity of the decision to that parameter.

### ***Monte Carlo Analysis***

A more sophisticated uncertainty analysis uses not just three values for the range of a parameter, but a broader range in a probabilistic distribution. (The distribution can be a continuous function, or it can be a discrete set of values.) Monte Carlo analysis does the same analysis over and over, picking random variables from all of the probably distributions of the parameters. The result is a probabilistic representation of the result of the analysis.