Derivation of Wind Power Equation: Playing with Newtonian Physics

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I have been puzzled for a while why power, *P* (watts), contained in wind should increase with the 3rd power of wind velocity, *v* (m/s), given that the kinetic energy of a moving mass increases with the 2nd power. The wind power equation is:

$$P = 0.5\rho A C_p v^3 \qquad (1)$$

where ρ = the density of air, *A* = an area perpendicular to the wind vector over which power is computed and C_p = the fraction of the power that might be captured by a wind or water turbine. Eq (1) without the C_p simply gives the power of the wind or water moving past area *A*. This equation applies to fluid movement also, e.g., I have seen under-sea water turbines on TV documentaries.

So here is my derivation:

The kinetic energy of a moving mass from high school physics is:

$$E = 0.5mv^2 \qquad (2)$$

Now consider that we are observing a lot of little masses, m, moving perpendicular to a plane of area *A*. The rate of energy 'movement' per unit time, t, in J/s = watts is by definition the power, hence,

$$P = \frac{dE}{dt} = 0.5\frac{dm}{dt}v^2 \qquad (3)$$

So Eq (3) gives the power of many moving masses traveling at a constant velocity, *v*, where the rate of mass movement is *dm/dt* kg/s.

So how much air or water mass might move across a plane of area *A* in time *t*? This will provide the last fact needed to derive Eq. (1).

Lets say that in time, *t*, the masses (water or air molecules) have moved a distance *L* while passing through the plane *A*. So the volume of air or water passing through the plane will be *A* times *L* and if you know the density of that volume, ρ (kg/m^3), you have: $m = \rho AL$ and hence the rate of mass movement is:

$$\frac{dm}{dt} = \rho A\frac{dL}{dt} = \rho Av \qquad (4)$$

given that *dL/dt* = velocity by definition. Hence if you substitute (4) into (3) you get $P_{MovingAir} = 0.5\rho Av^3$ and if you can capture a fraction C_p of that you have Eq. (1).

QED