Research Towards Improved Models for Annual Production of Energy of Small Wind Turbines. Part II How Air Density is Influenced by Temperature, Pressure and Elevation

An Opinion Paper by Mel Tyree

Introduction

There is increasing evidence that the WindCad models (industry standard models) for estimation of Annual Production of Energy (Y_p) of small wind turbines consistently overestimates actual measured production (Y_m). The purpose of this series of opinion papers is to examine each element of theory and data involved in the estimation of Y_p in order to determine the cause(s) of the overestimation. Two previous SWIEP papers address this issue. The first is a sensitivity analysis of the WindCad model (R#22) and the second examines one aspect of C_v = Turbulence Intensity on Y_p (R#30). See also (R#32) for an introduction on how models are formulated and validated. All reports can be downloaded from:
http://www.ualberta.ca/~mtyree/SWIEP/Publications.html Part II examines the influence of air density.

Wind turbine power production, P_w, is a function of air density, ρ, because

$$P_w = 0.5\rho A v^3 C_p \qquad (1)$$

Where, P_w = power in W, A = sweep area of blade in m^2, v = wind speed in m/s, and C_p = coefficient of performance. Standard atmosphere at sea level is sometimes defined as having a pressure of 101.3 kPa at 20 ^{o}C and a ρ = 1.225 kg/m^3.

So how much does normal (maximum) variation in atmospheric conditions influence ρ? The ideal gas law states that $PV = nRT$, where P = pressure, V = volume containing n moles of air, T = Kelvin temperature and R = the gas constant. If $\overline{M}_{air}$ = the average molecular weight of air then by definition

$$\rho = \frac{n}{V}\overline{M}_{air} = \frac{P}{T}\frac{\overline{M}_{air}}{R} \qquad (2)$$

R is a universal constant and $\overline{M}_{air}$ is approximately constant in the first 5000 m above sea level, hence ρ (air density) should be proportional to P (atmospheric pressure) and inversely proportional to T (absolute Kelvin temperature), where ^{o}K = 273.16 + ^{o}C.

Temperature ranges

Temperature ranges experienced by a turbine might be -40 ^{o}C to + 40 ^{o}C (= 233 to 313 ^{o}K). So the maximum range in air densities that might affect production due to temperature range is give by

$$\left(\frac{\rho_{cold}}{\rho_{hot}}\right) = \frac{313}{233} = 1.34 \qquad (3)$$

Hence if all else is the same then a turbine will produce 1.34 x the power (34% more) in really cold air than in really hot air.

Atmospheric pressure ranges

Atmospheric pressure ranges experienced by turbines will be influenced most by elevation and then secondarily by pressure changes due to weather fronts. Air pressure declines exponentially with elevation above sea level, but from 0 to 3 km (about 0 to 10,000 ft) this change is nearly linear and about 9.2% per 1 km if the air temperature changes -6.5 C per km change in elevation. See Barometric formulas in (also shorter version in the Appendix of this report):
http://en.wikipedia.org/wiki/Atmospheric_pressure#Altitude_atmospheric_pressure_variation)
Hence the approximate difference in turbine production between sea level and 3000 m is about

$$\left(\frac{\rho_{0km}}{\rho_{3km}}\right) = \frac{1}{0.724} = 1.38 \qquad (4)$$

Note that this is not far from the range from temperature effects (Eq. 3).

As weather fronts pass through the air pressure can change at sea level from a fairly high value of 105 kPa to a fairly low value of 98 kPa so this corresponds to

$$\left(\frac{\rho_{High\,Pressure}}{\rho_{Low\,Pressure}}\right) = \frac{105}{98} = 1.07 \qquad (5)$$

Interaction between temperature and elevation

People commonly notice that air seems to get cooler as you travel up a mountain. This is due to the DALR = Dry Adiobatic Lapse Rate, i.e., the tendency of dry air to become colder as it rises. This cooling rate is -6.5 ^{o}C per km rise in elevation for dry air. In contrast, wet air does not cool nearly as much as dry air as it rises. This is because as clouds form (moisture condenses) some heat is added back into the air due to the latent heat of vaporization of water. Heat is required to vaporize water and that is why evaporating sweat feels cool. In the reverse process, when water condenses into clouds/fog the air is warmed.

How the WindCad model handles the effect of elevation on air density?

By inspection of the WindCad model, it can be seen that ρ is modified 9.18% per km rise in elevation (see Excel cell H14 in R#21). Hence it seems to use the barometric formula that applies to DALR. If you assume constant air temperature the value should be about 9.31%.

How the WindCad model handles the effect of mean annual temperature.

Answer: It seems not to do it at all. There is no explicit allowance for temperature in the WindCad model. If you did include a temperature affect (e.g., a mean annual temperature), you might want to bias the value a little towards the low side since it is windier and colder in winter than in summer. I don’t expect that ignoring mean annual temperature will affect the annual predicted energy production more than a few percent because mean annual temperature ranges from 275 to 298 ^{o}K (Alaska to Hawaii = the coldest and warmest States in terms of mean annual temperature). So the effect of mean annual temperature on air density and hence P_w would be = 298/275 = 1.08 or 8% from the oldest to the warmest states.

What does NREL do about T in their standard test procedure (R#26)? They correctly allow for temperature effects on density and correct both wind speeds and density for sea-level conditions. However, the WindCad models do not include the average temperature at which power curves were measured hence it is currently not possible to correct for T for any possible differences between T at which the power curve was measured and average annual T at which Y_p is predicted. This correction might help some but can’t explain all the difference in $Y_m/Y_p = 0.24$ according to the Cadmus Report (R#17 & R#25). We have to explain away an error of 76%.

More work needs to be done to determine how much Y_m/Y_p is influenced by temperature corrections. At this point such corrections might make Y_m/Y_p closer to 1 (desirable) or farther from 1 (undesirable).

Please send comments and corrections to mtyree@ales.ualberta.ca so that I can improve this report in revised versions.

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APPENDIX Barometric formula

Eq. (1) applies to temperature laps ≠ 0

$$\frac{P}{P_b} = \left[\frac{T_b}{T_b + L_b(h - h_b)}\right]^{\frac{g_b M}{R L_b}}$$

Eq. (2) applies to temperature laps = 0

$$\frac{P}{P_b} = \exp\left(-\frac{g_b M(h - h_b)}{R T_b}\right)$$

P = pressure (Pa or any unit)
T_b = standard temperature = 288.15 K for h=0 to 11,000 m
L_b = temperature laps = -6.5E-3 K/m for h = 0 to 11,000 m
h = height above sea level, h_b – 0 for h < 11,000 m)
R = gas constant 8.13432E3 N-m/(kmol K)
g_o = standard gravity acceleration 8.80665 m/s^2
M = molar mass of Earths air = 28.9644