Sensitivity Analysis on Predictive Models of Annual Energy Production by Small Wind Turbines

by
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Thanks to Rob Beckers, Solacity Inc., I obtained a more complete Excel Sheet with the model commonly used to predict mean annual kWh production from small wind turbines. I have confirmed that this model gives the same result as the less-well-documented model from Bergey Inc., and hence I assume it is the identical model. See:
www.ualberta.ca/~mtyree/SWIEP/Docs/BobBeckersWindProductionModel.xls

This model attempts to predict average annual production in kWh from:

- The turbine's power curve
- The average annual wind speed, $\bar{v}$, m/s
- A Weibull function with a number of input constants that describe the site conditions at the turbine location.

In theory the power curve is given by

$$P = 0.5\rho AC_p v^3 \qquad (1)$$

where ρ = the density of air (kg/m^3) , A = an area perpendicular to the wind vector over which power is computed (m^2), C_p = the fraction of the power that might be captured by a wind or water turbine, v is the wind velocity (m/s) and power is in watts. Eq (1) without the C_p simply gives the power of either wind or water moving past area A. But C_p is not a constant for real-world turbines, hence the power curve has to be determined empirically. Power curves usually appear as tables of P versus v.

Ideally the average wind velocity ($\bar{v}$, m/s) should come from measurements on site and often 1- or 2-years of measurements are done at commercial sites. However, such measurements are usually quite expensive, so for small wind turbines the $\bar{v}$, m/s, is often estimated from published wind maps. Or from near by anemometers a lower heights with some corrections for height, wind shear and turbulence.

I am quite familiar with Weibull functions and use them often in my research. There are classes of Weibull functions with 1, 2, 3 or 4 parameters. The Weibull function predicts nothing from basic principles! It is simply a mathematical formula that is very good at fitting a wide range of empirical data sets, such as symmetric and asymmetric unimodal curves, sigmoid-shaped curves, and many other non-linear functions. In the wind industry the version of the Weibull function commonly used is:

$$\Gamma(v) = (0.89\frac{K}{\bar{v}})(0.89\frac{v}{\bar{v}})^{(K-1)}\exp(-[0.89\frac{v}{\bar{v}}]^K) \qquad (2)$$

where $\Gamma(v)$ = the probability that wind speed will be = v over a 1-year period, K = a Weibull constant, and $\bar{v}$ = the average annual velocity. Hence Eq. (2) is a three parameter Weibull function (counting 0.89) with one variable (v). Fig. 1 is an example of a typical Weibull curve for wind probability:

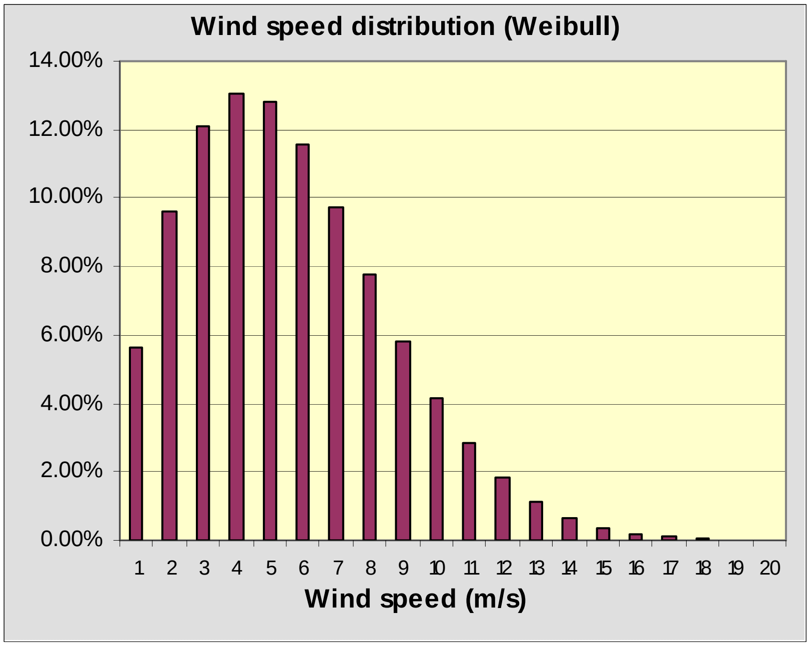

Fig. 1: Typical Weibull curve generated with K =1.892 & $\bar{v} = 5.464$

A typical power curve (measured empirically) is given in Fig. 2.

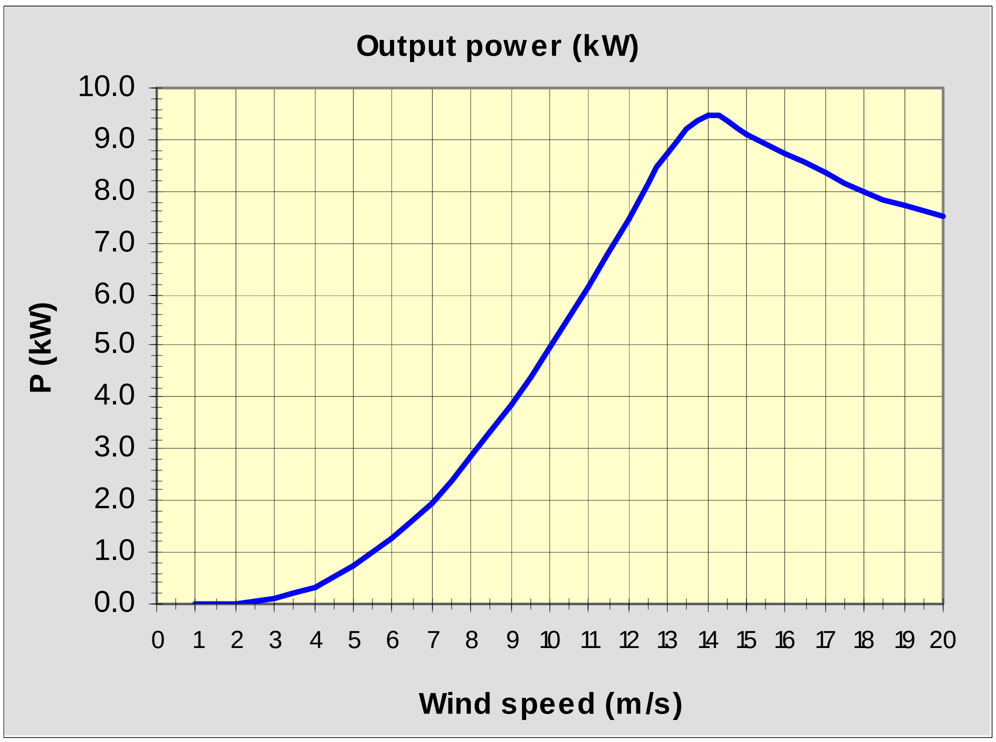

Fig. 2. A typical power curve for a Bergey 10 kW turbine.

The typical computation procedure is to compute annual power production in each wind speed class (bin) from:

$$\text{Power at bin}(v) = B(v) = P(v)\ \Gamma(v)\ (1\text{-}T_f)\ (1\text{-}D_f) \qquad (3)$$

where T_f = a turbulence factor with a typical range of (0.1 to 0.2) and D_f = an air density factor with a range that depends on elevation ranging from (0 to 0.3 for elevations from sea level to 10,000 ft). Both turbulence and elevation reduce energy production in a linear fashion as the equation predicts (data not shown).

The daily production in kWh is 24*$B(v)$ and the annual production is 365.25*24*$B(v)$, hence annual production is computed from the sum of all the bin values:

$$\text{kWh(annual)} = E_a = 8766(1\text{-}T_f)\ (1\text{-}D_f) \sum_{bin} P(v)\Gamma(v) \qquad (4)$$

Weibull Constant K:

The range of K values I have seen range from slightly under 2 to 4, where K = 2 is for inland sites, K = 3 is for costal sites and K = 4 is for island or ocean sites. The effect of increasing K is to make the $\Gamma(v)$ curve increasingly symmetrical as shown by the 3 Weibull distributions below all at $\bar{v} = 5.5$ m/s.

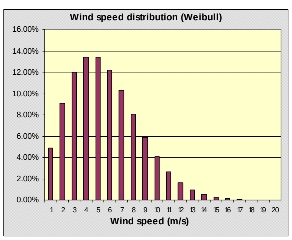

Fig. 3 Weibull K = 2

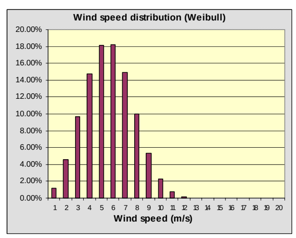

Fig. 4 Weibull K = 3

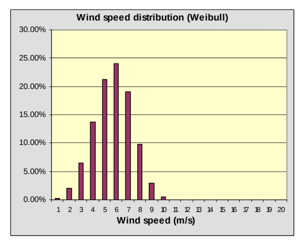

Fig. 5 Weibull K = 4

Empirically, the annual kWh production is a non-linear function of K but approximately linear function of 1/K.

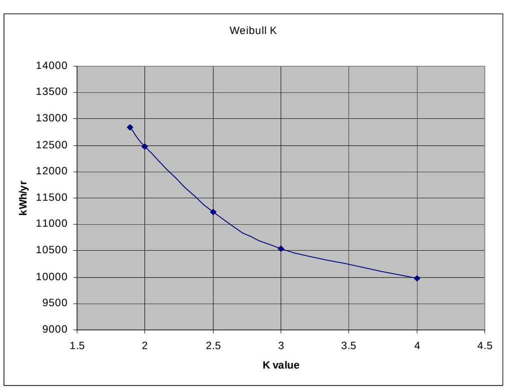

Fig. 6 Annual kWh vs K

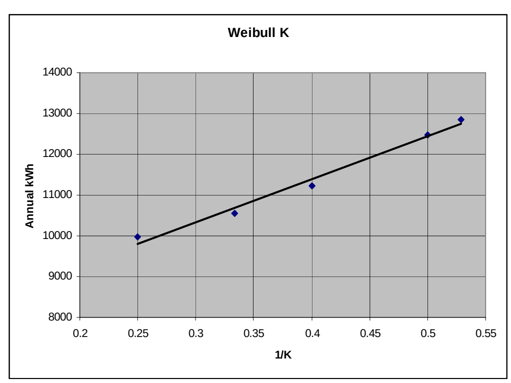

Fig. 7 Annual kWh vs 1/K

Impact of $\bar{v}$

As you might guess the predicted annual production at my site is strongly dependent on average annual wind velocity, $\bar{v}$, but what was unexpected was the VERY linear relationship given the number of non-linear factors entering into the analysis.

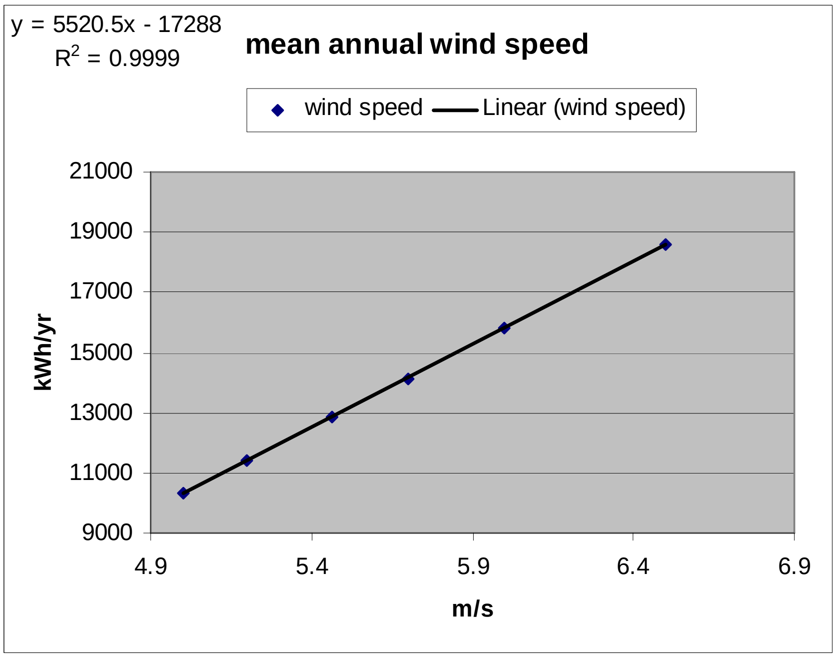

Fig. 8. Predicted kWh/yr versus average annual wind speed, $\bar{v}$, The value used in my original estimate was about 5.5 m/s

Another factor that can contribute to big differences in predicted annual production is the tower height. But once the tower height is selected the value is accurately known.

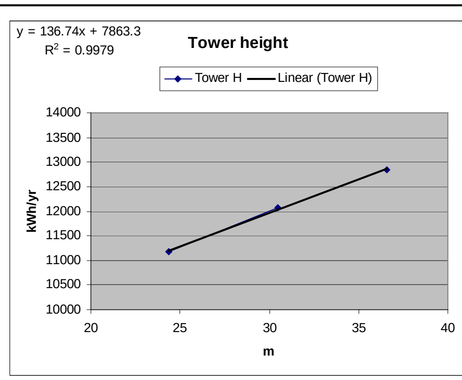

Fig. 9. Impact of tower height on predicted kWh/yr production.

Conclusion

I have a lot more to add for details and things to say, but I am running out of time and I have to leave for a two week trip. But I promised on two discussion groups to post something this weekend. So check back for a more detailed report in a week or two. You can tell if it is updated by the date/time at the top.

What I can say is that annual production will change from year to year simply because some years have more wind resource than others. In my case my annual wind resource was predicted at $\bar{v} = 5.5$ m/s for a annual production of almost 13,000 kWh. But what I got in the first 12 month period was 8,800. If the models are right this corresponds to a $\bar{v} = 4.7$ m/s. I did not have an anemometer installed in the first year but do have one now. So I will update this report as data becomes available.

Given the highly linear relationships between the predicted production and the input variables, there is a good possibility of finding out if current overestimates of production (21 out of 21 cases I have recorded) is due to one or more of the following:

- Annual variations in $\bar{v}$
- Incorrect evaluation of $\bar{v}$ by the installers
- A basic model that continuously over estimates the real world situation.

I hope to encourage people to send me data in the future. Sharing such data will benefit everyone: customers, installer, turbine builders and the ‘science’ of small wind energy.

PLEASE CONTRIBUTE.