ARISTOTLE ON LOGICAL CONSEQUENCE

Compare two conceptions of validity: under an example of a modal conception, an argument is valid just in case it is impossible for the premises to be true and the conclusion false; under an example of a topic-neutral conception, an argument is valid just in case there are no arguments of the same logical form with true premises and a false conclusion. This taxonomy of positions suggests a project in the philosophy of logic: the reductive analysis of the modal conception of logical consequence to the topic-neutral conception. Such a project would dispel the alleged obscurity of the notion of necessity employed in the modal conception in favour of the clarity of an account of logical consequence given in terms of tractable notions of logical form, universal generalization and truth simpliciter. In a series of publications, John Etchemendy has characterized the model-theoretic definition of logical consequence as truth preservation in all models as intended to provide just such an analysis.

In this paper, I will argue that Aristotle intends to provide an account of a modal conception of logical consequence in topic-neutral terms and so is engaged in a project comparable to the one described above. That Aristotle would be engaged in this sort of project is controversial. Under the standard reading of the Prior Analytics, Aristotle does not and cannot provide an account of logical consequence. Rather, he must take the validity of the first figure syllogisms (such as the syllogism known by its medieval mnemonic ‘Barbara’: A belongs to all B; B belongs to all C; so A belongs to all C) as obvious and not needing justification; he then establishes the validity of the other
syllogisms by showing that they stand in a suitable relation to the first figure syllogisms. I will argue that Aristotle *does* attempt to provide an account of logical consequence—namely, by appeal to certain mereological theorems. For example, he defends the status of Barbara as a syllogism by appeal to the transitivity of mereological containment (if one thing is wholly contained in a second, and that second is wholly contained in a third, then the first is wholly contained in the third). There are, as I will discuss, reasons to doubt the success of this account. But the attempt is not implausible given certain theses Aristotle holds in semantics, mereology and the theory of relations.

It may be surprising to the reader that one would provide an account of logical consequence by appeal to mereological principles, and even more surprising that I would claim that such appeal is part of a project to provide an account of logical consequence in terms of notions of topic-neutrality. For surely mereology is a topic-specific theory. The source of the reader’s surprise is perhaps a certain view of topic-neutrality—a view which, I will argue, is not Aristotle’s. To get a historically accurate picture of Aristotle’s philosophy of logic, then, we need to distinguish between two construals of the topic-neutrality of logic and so two versions of the project of providing an account of a modal conception of logical consequence in terms of topic-neutrality. Briefly put, according to the first construal, a logical truth obtains solely in virtue of it’s form and so independently of the way the world is. Call this the view that logic is Formal. According to the second, a logical truth obtains in virtue of highly general features of the world. Call this the view that logic is General. Although the appeal to mereological principles would not provide an account of logical consequence in terms of notions of topic-neutrality, under the
Formal construal, nothing prevents the appeal to mereological principles from providing such an account, under the General construal.

Here is the plan for the paper. I will begin by discussing Aristotle’s own examples of modal and topic-neutral conceptions of logical consequence. Here I will distinguish between a broad syllogism, a modal consequence relation, and a narrow syllogism, an argument in one of the three figures of the syllogistic (§1). Then I will draw the distinction between Formal and General construals of topic-neutrality and argue that Aristotle holds that the syllogistic is General (§2). Next I will discuss the views in the philosophy of language and metaphysics which underlie Aristotle’s philosophy of logic. In particular, I will argue that Aristotle provides an account of narrow syllogisms by appeal to a fragment of a mereology. In light of these views, Aristotle would deny that the syllogistic is Formal (§3). Finally, I will note that Aristotle holds that any broad syllogism can be represented as a finite series of narrow syllogisms. So Aristotle intends to provide an account of a modal conception of logical consequence by appeal to mereological theorems. I will also assess the success of this account and draw a few cautious comparisons with contemporary philosophy of logic (§4).

Aristotle characterizes a syllogism as follows:

A syllogism is an argument in which, some things having been supposed, something other than what has been supposed results of necessity from their being so. I mean by “from their being so” resulting through them, and by “resulting through them,” needing no term from outside for the necessity to arise. (Prior Analytics, 24b18-22)
In what follows, I will treat consequence as a relation between a premiss-set and a conclusion. I will call the satisfaction of the characterization in 24b18-22 *syllogisity*. I also will follow terminology from Jonathan Barnes (1981) and distinguish between *broad* syllogisms and *narrow* syllogisms. A broad syllogism is any argument which satisfies the above characterization and so exhibits syllogisity. A narrow syllogism is any one of the moods of the three figures explicitly discussed in the *Prior Analytics*. In this section, I will discuss broad syllogisms. My aim here is to bring out both some of the similarities and some of the differences between the Aristotelian and model theoretic projects, and also to indicate the difficulties facing those of us who would give a more precise comparison. Syllogisity is a modal conception of logical consequence. But syllogisity is not classical validity and it is, furthermore, difficult to determine whether syllogisity is entailment.

The locution ‘results of necessity’ in 24b18-22 resembles some modern characterizations of a modal conception of logical consequence and so suggests that Aristotle is targeting in this characterization a notion of validity which would be familiar to us. And indeed, some have held that Aristotle’s definition of a broad syllogism is a definition of a broad notion of deduction or valid argument. However, if Aristotle means to capture a notion of classical validity, then we must conclude that the syllogistic dramatically fails to achieve Aristotle’s intended goal. For much inferential reasoning is recalcitrant to syllogistic representation. Among non-syllogistic inferences are the most common derivation rules of classical propositional logic, including repetition, conjunction introduction and elimination, disjunction introduction and elimination, modus ponens and modus tollens. Moreover, the syllogistic is ill-suited to represent the
mathematical reasoning which is characteristic of Euclidean geometry. Aristotle was surely aware of many, if not all, of these argument forms. So, on this view, he is guilty of a rather obvious mistake: he intends to represent all validities but fails to notice that arguments, of which he is himself well aware, resist syllogistic representation.

However, there is no evidence that Aristotle intends to represent all validities. The characterization at 24b18-22 constrains broad syllogisms by several conditions. Broad syllogisms are non-circular: the conclusion must be “something other than what has been supposed.” And they are multi-premised: the premise set contains “some things.” Although 24b18-22 underdetermines what consequence relation syllogisity is, these constraints suffice to show that syllogisity is not classical validity. Classical validity is a consequence relation which is reflexive; syllogisity is irreflexive. According to reflexivity, we can infer any proposition from itself. Reflexivity is explicitly excluded from being a broad syllogism by the condition in 24b18-20 that the conclusion be a different proposition from any premise. And narrow syllogisms also fail to exhibit reflexivity. For example, an instance of repetition such as

All men are mortal
So all men are mortal

is not taken to be a narrow syllogism, although it is a classically valid argument.

Moreover, Aristotle holds that not all inferences are broad syllogisms, so it is unlikely that he intends to represent all validities with the syllogistic. Consider the conversion rules. Aristotle establishes the validity of the higher-order moods by showing that they stand in the relation of convertibility to one of the first-figure moods. Aristotle recognizes three conversion rules:
From ‘A belongs to no B’ infer ‘B belongs to no A’.
From ‘A belongs to some B’ infer ‘B belongs to some A’.
From ‘A belongs to all B’ infer ‘B belongs to some A’.

The conversion rules aren’t broad syllogisms: they fail to satisfy the requirement of a multiplicity of premises.

Syllogisity is arguably a consequence relation at least as strong as entailment. Entailment is a stronger consequence relation than classical validity. Contemporary relevance logicians find the relation of classical validity excessively permissive for, they claim, there are arguments which are valid according to the classical relation but which are intuitively not entailments. For example, classical logic allows explosions: according to the standard account of validity, under which an argument is valid just in case it is impossible for the premises to be true and the conclusion is false, an argument with contradictory premises is valid regardless of its conclusion. Relevance logicians claim that it is counterintuitive to hold that a conclusion follows from an unrelated contradiction. They propose to capture this intuitive notion of entailment by imposing a condition of relevance: in brief, any premise of the argument must be used in the derivation of the conclusion.

As many have noted, the syllogistic is a logical system which satisfies a constraint similar to relevance. Aristotle never canvasses narrow syllogisms corresponding to the so-called paradoxes of strict implication, such as the observations that everything strictly implies a tautology and a contradiction strictly implies everything. Yet to formulate such moods is well within the expressive power of the syllogistic’s object language. Indeed, narrow syllogisms are constrained by a condition more stringent than relevance: not only
must each term of the conclusion occur in just one premiss, each pair of adjacent premisses must share a term.

However, it is unclear whether syllogisity is entailment for it is difficult to determine whether a broad syllogism is monotonic, as entailments are, or counter-monotonic. Irvine and Woods (2004) read into Aristotle’s definition of a syllogism not only multiplicity of premisses and non-identity of the conclusion to any premiss but also a condition of counter-monotonicity: the addition of any premiss to a syllogism transforms it into a non-syllogism. However, Aristotle’s requirement in 24b18-22 that the conclusion needs no term from outside implies merely that the premisses are jointly sufficient, not that each premiss of a syllogism in necessary for the conclusion to follow. By contrast, Hitchcock (ms.) argues that there is evidence that Aristotle is committed to monotonicity:

in his discussion in the *Sophistical Refutations* of the fallacy of using *reductio ad absurdum* to refute a proposition that was not in fact used in generating the absurdity, Aristotle says, “Such arguments are not absolutely unsyllogistic, but are unsyllogistic in relation to the proposition.” (167b34-35) Thus Aristotle allows that a syllogism can have an irrelevant or redundant premiss, and *a fortiori* that the relation of resulting of necessity is at least not counter-monotonic.

However, in 167b34-35 Aristotle allows only that the premises of such an argument may form the premises of a deduction to some other conclusion. He is not committed to allowing irrelevant or redundant premises.

Aristotle’s concept of syllogisity is underdetermined by any of the passages we have considered in this section of the paper. However, it is clear that syllogisity is a modal consequence relation distinct from classical validity. This characterization, although incomplete, suffices for our present purposes.
Under the standard view, Aristotle does not and cannot offer an account of this modal conception of logical consequence. Rather, he must take the first figure narrow syllogisms to be obviously or self-evidently valid. So that in virtue of which the first figure syllogisms are valid—that is to say, that in which validity consists—is left unaddressed. He then shows that the narrow syllogisms of the second and third figure are valid, under the assumption that the first figure syllogisms are valid. Narrow syllogisms are two-premise arguments with categorical propositions as the premises and conclusion. The assertoric categorical propositions have the forms: B belongs to every A; B belongs to no A; B belongs to some A; and B does not belong to some A. The narrow syllogisms are classified into three figures, which have the following format. The premises contain the two terms of the conclusion respectively and a common or middle term: in the first figure, the middle term is in the predicate position of the first premise and in the subject position of the second premise; in the second and third figures, the middle is the predicate or the subject, respectively, of both premises. So, for example, one of the syllogisms of the first figure, called by its medieval mnemonic, ‘Barbara’, has this form:

(Barbara)  
A belongs to every B;  
B belongs to every C;  
so A belongs to every C.

In chapters A4-7 of the Prior Analytics, Aristotle considers various combinations for the three figures of syllogisms and shows which are valid and which invalid. The valid syllogisms of the first figure are taken to be obviously valid; the validity of the valid syllogisms of the second or third figures is established by showing that these syllogisms
stand in a certain relation to one of the syllogisms of the first figure—often, that of convertibility. That is to say, he takes such syllogisms as (one of the first figure syllogisms) Celarent:

\[
A \text{ belongs to no } B; \ B \text{ belongs to all } C; \ so \ A \text{ belongs to no } C
\]
as obviously valid. He then establishes the validity of such syllogisms as Cesare

\[
M \text{ belongs to no } N; \ M \text{ belongs to all } O; \ so \ N \text{ belongs to no } O
\]
by converting the first premise to

\[
N \text{ belongs to no } M
\]
by means of the conversion rule e-conversion and then using Celarent to infer the conclusion. Finally, the invalidity of the invalid combinations is typically established by counterinstance; I will discuss this method below.

I will argue that Aristotle does provide an account of logical consequence. Let me here parry a potential misunderstanding. I do not claim that Aristotle intends to provide an analysis of his modal conception of logical consequence in formalist terms. But Aristotle does, I will argue, aim to provide a systematic account of a modal conception of logical consequence—an account of what logical consequence consists in which relies not merely on the meaning of Greek words translated by’ syllogism’ or ‘results of necessity’, but which rather relies on substantive theses in semantics and metaphysics.

To get a historically accurate picture of Aristotle’s philosophy of logic, however, we first need to draw a distinction between two conceptions of topic-neutrality. This requires a bit of set-up. Aristotle is widely and rightly credited as the founder of logic, the formal study of consequence. That is to say, Aristotle founded a study of what it is for a conclusion to follow from premises; and the way in which Aristotle conducted this study
is topic-neutral, in—roughly—the following sense. To take a well-worn example, the
syllogism, ‘All Greeks are men; all men are mortal; so all Greeks are mortal’ is a valid
inference but it’s validity doesn’t depend on the meaning of the nonlogical words,
‘Greek’, ‘men’ or ‘mortal’. The inference would be licensed regardless of what these
words meant. The inference from ‘John is a bachelor’ to ‘John is unmarried’, on the other
hand, is also a permissible inference but it’s permissibility depends on the meanings of
the nonlogical words. If ‘bachelor’ meant Canadian, then the conclusion wouldn’t follow
from the premise.

But topic-neutrality, so characterized, can be read in one of two distinct ways.
Under one conception, logic is characterized by its indifference to all worldly facts or its
abstraction from any semantic content whatsoever. Under this conception, the above
syllogism is valid regardless of any worldly facts: whether Greeks are men, whether men
are mortal, and so on. This conception is often drawn on in contemporary
characterizations of logic; it underlies, for example, Ernest Nagel’s (1956: 66) claim that
logical laws are empty: they don’t tell us anything about the world. To give just one more
example: the conception underlies the view Quine (1970: 95) ascribes to Carnap: that “it
is language that makes logical truths true—purely language, and nothing to do with the
nature of the world.” Call this the Formal conception of topic-neutrality.

According to another conception of topic-neutrality, to claim that logic is topic-
neutral is not to characterize logic by its abstraction from all content whatsoever but
rather to characterize logic by its abstraction from the specific identities of things. Under
this conception, the syllogism is valid regardless of the specific identities of the referents
of ‘Greek’, ‘man’ and so on. Such a conception of logic, unlike the Formal conception, is
compatible with the claim that logical truths hold in virtue of highly general features of
the world. So call this the General conception of topic-neutrality. Such a conception
underlies Russell’s (1919: 169) well-known claim that “logic is concerned with the real
world just as truly as zoology, though with its more abstract and general features.” I am
thinking of worldly features in Quine’s (1970: 95) sense when he writes:

A logical truth, staying true as it does under all lexical substitutions, admittedly
depends upon none of those features of the world that are reflected in lexical
distinctions; but may it not depend on other features of the world, features that our
language reflects in its grammatical constructions rather than it’s lexicon?

There is an alternative sense of ‘worldly’, under which something is properly worldly
only if varies among the worlds of possible worlds semantics. Of course, the fact that
logic isn’t worldly in this sense is almost immediate. For the claim just is the
unobjectionable claim that logical truths are closed under permutation of the nonlogical
constants. In the sense of ‘worldly’ I am targeting, the grammatical constructions of a
language can represent worldly features not represented by its lexicon. For example, a
multi-place predicate logic represents, by its inclusion of n-ary relations, the general
worldly feature that objects can stand in complex relations to each other.

To summarize, I am contrasting two conceptions of topic-neutrality:

**Formal Conception**: topic-neutral truths obtain independently of the way the
world is;

**General Conception**: topic-neutral truths obtain independently of the particular
identities of things.

These conceptions are typically affiliated with other claims, such as claims about what it
is in virtue of which logical truths do obtain: the Formal conception is often affiliated
with the claim that logical truths obtain in virtue of language alone or in virtue of logical
form alone; the General conception may be affiliated with the claim that logical truths obtain in virtue of highly general features of the world. But, for our purposes, I will keep the terminology of Formal and General restricted to the above claims about that from which logical truths are independent.

Aristotle would both hold that logic is General and deny that logic is Formal. In the rest of this section, I will argue that Aristotle holds that logic is General. There’s good reason to think that Aristotle believes that an argument is valid only if every argument in the same form is valid. This claim is only tacit in the Prior Analytics but it plays two roles there, as John Corcoran (1974) noted. First, to establish validity of all arguments in the same form as a given argument, he establishes the validity of an arbitrary argument in the same form—that is to say, leaving its content words unspecified. As we’ve seen, he uses letters for the terms when stating syllogisms and when proving the higher-order syllogisms valid by conversion.

Second, Aristotle establishes the invalidity of a syllogistic form by a method of “contrasted instances,” as Ross (1949: 302) puts it. Consider the following explanation of this method:

If the first [i.e. the major term] belongs to every one of the middle and the middle belongs to none of the last [i.e. the minor term], there will not be a syllogism of the extremes; for nothing necessary results from their being so; for it is possible for the first to belong to every one of the last and possible for it to belong to none of the last, so that neither the particular nor the universal will become necessary; since nothing is necessary through these propositions, there will not be a syllogism. Terms for belonging to every animal-man-horse, for belonging to none animal-man-stone. (Prior Analytics I.4.26a2-9)

Here Aristotle shows that there is no deduction with the premises

A belongs to every B; and
B belongs to no C.

To show that nothing follows of necessity from these premises, Aristotle shows that different assignments of referents to the terms yields different propositions containing the extreme terms. For one assignment of referents to the terms

A: animal
B: man
C: horse

has the result that the alleged premises are true and a proposition where the extreme terms form a universal affirmation—namely, ‘animal belongs to all horses’—is also true.

But another assignment of referents to the terms

A: animal
B: man
C: stone

has the result that the alleged premises are true and a proposition where the extreme terms form a universal negation—namely, ‘animal belongs to no stone’—is also true.

The former situation shows that no universal negation follows of necessity; the latter situation shows that no universal affirmation follows of necessity. The considerations support the ascription to Aristotle of the General conception of logic. But they do not go so far as to support the ascription to Aristotle of the Formal conception. That is, although arguments in the same form are either all valid or all invalid, this does not show that the way the world is a matter of indifference to the question of an argument’s validity. And, especially in light of the fact that the Formal conception of logic is a currently controversial thesis, we need to proceed carefully. Aristotle nowhere expresses the Formal conception of logic. His methods do not require
it. And it is a substantial and currently controversial thesis. So we have as yet seen no reason to ascribe to Aristotle anything stronger than the General conception. To find whether Aristotle would hold or deny that logic is Formal, we need to dig deeper into Aristotle’s views on consequence. Recall, I have noted that, under the standard view, Aristotle fails to provide an account of logical consequence at all. But is this standard view the right one? There is textual evidence that Aristotle does justify the status of the first figure moods as syllogisms—by appeal to mereological principles. This justification is plausible, given certain views that Aristotle holds in metaphysics and the philosophy of language. I will begin by discussing these views.12

Aristotle provides an interpretation of categorical propositions early on in the *Prior Analytics* (24b26-8):

‘One thing is wholly in another’ means the same as ‘one thing is predicated universally of another’.13

Although Aristotle only provides a semantics for universal affirmations here, the extension to universal negations and particular propositions ought to be clear. Aristotle thus implies mereological truth conditions for all of the categorical propositions. Moreover, although these passages concern the semantics of only quantified propositions, it is Aristotle’s view that all propositions are one of universal affirmations, universal negations, particular affirmations or particular negations.14 So Aristotle holds that there are only four kinds of propositions. And so 24b26-30 suggests an extension to a general account of predication. So, just as ‘A belongs to every B’ is true iff B is mereologically
included in A, so too ‘A belongs to no B’ is true iff B is mereologically excluded from A. ‘A belongs to some B’ is true iff A and B mereologically overlap—that is to say, iff a part of B is a part of A. And ‘A does not belong to every B’ is true iff A is not mereologically included in B.

The part-whole talk suggests that Aristotle views predication at least somewhat in terms of the familiar set-theoretic interpretation of the syllogistic: under this interpretation, the terms range over sets; the categorical propositions express such set-theoretic notions as inclusion, exclusion, overlap and non-inclusion. Indeed, Aristotle’s discussion of predication in mereological terms has struck some as a confused conflation of mereology, the metaphysics of properties and set theory. The difficulty of interpretation here is partly that Aristotle is employing mereological notions which are foreign to us. Among various senses of ‘whole’, Aristotle distinguishes between what became known as quantitative wholes and integral wholes at *Metaphysics* 5.26 (1023b26-33):

> We call a whole … that which so contains the things it contains that they form a certain unity; and this in two senses—either as each part being one, or as a unity made up out of the parts. For what is universal and what is said wholly, since it is a certain whole, is universal in the sense that it contains many things by being predicated of each and by being all those and each of them one, as for instance man, horse, god are one because they are all living things. But the continuous and limited is also a whole, whenever there is a certain unity from the many.

Aristotle draws the contrast between quantitative and integral wholes by appealing to two distinct kinds of constitution relations. A quantitative whole is homoiomerous: the sum of animals, for example, is composed of parts each of which is itself an animal. An integral whole, by contrast, is heteromerous. A house, for example, is not a quantitative whole: it’s parts—the roof or the door, say—are not themselves houses; and not all of what can
be said of a house—that it’s final cause is to provide shelter, say—can be said of the parts of a house. So, for example, associated with the species humanity is a sum composed of individual humans. Any typical individual human has, of course, such parts as hands and feet. But these are integral parts of the individual, not quantitative parts. And so the hands and feet of the individual human are not themselves parts of the sum associated with the species. I do not expect that these comments will entirely dispel for the reader the foreignness of Aristotle’s mereological views. I cannot discuss in detail the relevant metaphysics. However, it suffices for my present purposes to bring out that Aristotle appeals to a notion that he characterizes as mereological, so to formulate the conditions under which ordinary predications express true thoughts. I will next attempt to establish that this relation is genuinely mereological.

According to our best available theories of parts and wholes, any legitimate part relation is a preorder—that is to say, a relation that is at least reflexive and transitive. So everything is a part of itself; and any part of a part of a thing is itself part of that thing. If we allow ‘Pxy’ to stand for ‘x is a part of y’, then we have the following axiom schemata:

(P1) Pxx (Reflexivity)
(P2) Pxy ∧ Pyz ⊃ Pxz (Transitivity)

These could be expressed as axioms were the variables bound by the appropriate quantifiers, but I will leave these omitted for ease of presentation. (P1)-(P2) characterizes a relation broader than any part relation. A reflexive and transitive relation need not be a part relation: for example, the less-than-or-equal-to relation is a preorder on the real numbers. For a system to be a mereology, we need to expand the axiom set. One common strategy for expansion is to introduce a supplementation principle. A commonly held intuition is that whenever an object has a proper part, it has more than one proper part.
That is to say, there is always a mereological difference between a whole and a proper part. Let us call this difference a *remainder*. The necessity of a remainder doesn’t follow from (P1)-(P2) alone. For example, consider a model of (P1)-(P2) with just two objects, one part-related to the other but not vice versa. To express the view that, when there is some proper part of a whole, there is always a distinct part of the same whole, it will be useful to define the notions of overlap and proper part. One mereological sum overlaps another just in case there is a shared part, i.e.

$$\text{Oxy} =_{df} \exists z (Pzx \land Pzy)$$

A proper part is a part which is non-identical with its whole, i.e.

$$\text{PPxy} =_{df} Pxy \land \neg (x =_{id} y)$$

Then the intuition that a proper part implies a remainder can be expressed by the axiom schema:

$$(P3) \text{ PPxy } \supset \exists z (\text{ PPzy } \land \neg \text{ Ozx}) \text{ (Weak Supplementation)}$$

Simons (1987), for example, holds that any system that can be truly called a mereology must conform to at least (P1)-(P3).

Since Aristotle characterizes the relation holding between a quantitative part and a whole as mereological, he is *prima facie* committed at least to the reflexivity, transitivity and weak supplementation of the relation. Moreover, as I will now argue, we have the textual evidence to establish that the quantitative part relation is reflexive, transitive and weakly supplementary. This supports Aristotle’s characterization of the quantitative part relation as mereological. I begin with reflexivity. Recall, Aristotle claims at 24b26-28 that one thing being wholly in another is equivalent to one thing being predicated universally of another. Aristotle draws a result:
And so we say ‘one thing is predicated universally of another’ whenever none of the subject can be taken of which the other cannot be said. (24b28-30)

It is easy to see that the implication, if none of the B’s can be taken of which A cannot be said, then A belongs to all B, is true under any substitution for the schematic letters whatsoever only if the relation of belonging to all is reflexive. Since Aristotle believes that the implication follows from the association of universal predication with the quantitative part relation, this passage gives us reason to hold that this relation is reflexive. I will discuss the transitivity of the quantitative part relation momentarily.

Aristotle claims that a universal term is predicated of many subjects at De Interpretatione 7 (17a39-b1):

I call a universal that which is by its nature predicated of many things, and individual that which is not; man, for instance, is a universal, Callias an individual.

So Aristotle is committed to Weak Supplementation. When there is some quantitative proper part of a whole, there is always a distinct part of the same whole. Since a universal is predicable of several subjects, when there is some quantitative proper part of a whole, there is typically a distinct part of the same whole. The characterization of the quantitative part relation as a weakly supplementary preorder is the weakest and least contentious ascription to Aristotle. Whatever else the quantitative part relation may be, it is a weakly supplementary preorder if it is a genuine mereological relation at all. Furthermore, these weak commitments suffice for our present purposes.

I will return to our consideration of Aristotle’s topic-neutral conception of logical consequence. Recall that Aristotle provides a mereological interpretation of the propositions used in the premises and conclusion: so, for example, he tells us that by ‘A belongs to all B’ he means that A is wholly in B. Furthermore, when he introduces the
first figure moods at Prior Analytics A4, he refers back to this interpretation and appeals to mereological principles. For example, Barbara holds in virtue of the transitivity of containment: if A is wholly in B and B is wholly in C then A is wholly in C. Barbara and Celarent are introduced this way.

(i) Whenever three terms so stand to each other that the last is wholly in the middle and the middle is either wholly in or wholly not in the first, it is necessary for there to be a perfect deduction of the extremes. (ii) I call ‘the middle’ that which both is itself in another and has another in it—this is also is middle in position—; the extremes are the terms which are [solely] in another [or solely] have another in them. (iii) If A is said of every B and B of every C, then it is necessary for A to be predicated of every C. (iv) For we have said earlier how to read ‘said of every’. (v) Similarly, if A is said of no B and B of every C, A will belong to no C. [APr 1.4, 25b32-26a2]

Barbara is stated in (iii). The status of Barbara as a perfect deduction is defended —‘for’ in (iv)—by appeal to the mereological interpretation of universal affirmative propositions: that is, I take the referent of ‘what has been said earlier’ in section (iv) to be 24b26-8 which, recall, states that ‘one thing is wholly in another’ is the same as ‘one thing is predicated universally of another’. The relation of mereological containment is transitive: if, for example, B is wholly in A and C is wholly in B, then C is wholly in A.

It’s clear that the passage is appealing to the transitivity of mereological containment to introduce Barbara and defend its status as a perfect syllogism. Section (v) of the above passage suggests that Celarent is defended in a like manner—that is to say, by appeal to the mereological principle that if one thing A is wholly excluded from another, B, and that thing, B, is wholly in a third thing, C, then the first thing, A, is wholly excluded from the third thing, C. Now, all valid assertoric syllogisms can be converted to either Barbara or Celarent. And Aristotle is well aware of this fact: he proves it at Prior Analytics 1.7.
So, in the presence of the conversion rules, the validity of all narrow syllogisms is hereditary on these two mereological principles.

Let me address a potential misunderstanding. One might take the referent of ‘what has been said earlier’ at 25b29-30 to be not 24b26-8, the claim that ‘one thing is wholly in another’ is the same as ‘one thing is predicated universally of another’, but 24b28-30, the claim that we say ‘one thing is predicated universally of another’ whenever none of the subject can be taken of which the other cannot be said. This is the textual source of the traditional dictum de omni on which, along with the corresponding dictum de nullo, Maier (1936) took the syllogistic to rest. However, I do not take this passage to be a definition of universal affirmation but a consequence of the mereological interpretation offered in 24b26-8. It is easy to see that 24b26-8 entails 24b28-30 but the latter does not entail the former. As such, although I hold that the validity of all narrow syllogisms is hereditary on two mereological principles, my interpretation is distinct from the traditional view of the syllogistic as resting on the dictum de omni et nullo.

My conjecture, then, is that mereological principles such as the transitivity of containment are Aristotle’s intended account of the narrow syllogism. The account, moreover, is plausible in the presence of certain assumptions in metaphysics and the philosophy of language to which Aristotle is clearly committed. These assumptions include metaphysical assumptions such as the claim that containment is transitive. Also, there are philosophy of language assumptions about what kinds of propositions can be expressed and what they mean, which I have discussed above.

I will address an objection. One might hold that Aristotle’s purpose in 25b32-26a2 is heuristic and the appeal to mereological theorems is a mere pedagogical or
illustrative aid. The objection might be fleshed out by considering an analogy with our use of Euler diagrams to teach introductory logic. Such diagrams provide a convenient decision procedure for testing certain validities: the intersection properties of circles are structurally isomorphic with the validity properties of certain arguments; and our visual and other cognitive abilities are such that we can apprehend the relevant spatial relations more easily than the abstract validities. However, it would be a neophytic error to conclude, upon being introduced to such diagrammatic representations of validities, that logic is about certain spatial relationships. So too, the objection might continue, Aristotle appeals to mereology without intending his readers to conclude that the syllogistic is about part-whole relationships. The transitivity of containment, for example, is structurally isomorphic to Barbara and is a principle we can quickly apprehend. But the transitivity is merely a useful representation of Barbara.²⁰

In response, I do not ascribe to Aristotle the views that mereological theorems are merely isomorphic to narrow syllogisms, that the transitivity of containment merely represents Barbara, that mereology is a convenient but potentially misleading decision procedure for testing the validity of syllogisms, or that mereological inclusion merely provides an easy but nonliteral way to appreciate the proposition expressed by a universal affirmation. On the contrary, as we have seen, there is good reason to take mereological relations such as inclusion to be the intended interpretation of the categorical propositions. The account of narrow syllogisms in mereological terms is not implausible in the presence of Aristotle’s commitments in semantics and metaphysics. In particular, given that Aristotle holds that categorical propositions express mereological relations, it is not so surprising that he holds also that inferences from a premiss set of categorical
propositions to a categorical proposition as conclusion are licensed by mereological theorems. Of course, Aristotle’s choice of which theorems to take as primitive, and so which syllogisms to take as first figure narrow syllogisms, is sensitive to issues of elegance, accessibility and perspicuity. The syllogistic rests in part on the two mereological theorems licensing Barbara and Celarent; arguably, the correctness of each is easily grasped. But to grant such considerations in the structure of the syllogistic is not to take the mereological terminology as a mere heuristic. If this is correct, then the objection lapses.

I have argued that Aristotle provides an account of narrow syllogisms by appeal to a fragment of a mereology. In this section, I will note that Aristotle holds that any broad syllogism can be represented as a finite series of narrow syllogisms. So Aristotle intends to provide an account of a modal conception of logical consequence by appeal to mereological theorems. I will also tentatively assess the success of this account and draw a few cautious comparisons with contemporary philosophy of logic.

Aristotle asserts in APr 1.23 that any broad syllogism can be represented as a finite series of narrow syllogisms:

It is clear from what has been said that the syllogisms in these figures are made perfect by means of the universal syllogisms in the first figure and are reduced to them. That every syllogism without qualification can be so treated, will be clear presently, when it has been proved that every syllogism proceeds though one of these figures. (40b17-22)

The referent of ‘what has been said’ is APr 1.7, where Aristotle shows that each of the narrow syllogisms is reducible to Barbara and Celarent. Aristotle argues for the
conclusion that any broad syllogism—‘every syllogism without qualification’—is reducible to Barbara and Celarent in the rest of APr 1.23. Despite implying here that every syllogism proceeds through one of the narrow syllogism, Aristotle’s argument rests on the claim that any broad syllogism can be represented as a series of narrow syllogisms. He argues that this series is finite in APr 1.19-22.\(^{22}\)

The claim that any broad syllogism can be represented as a series of narrow syllogisms resembles a contemporary completeness theorem. Contemporary logics typically employ two notions of consequence: a proof theoretic notion of derivability and a semantic notion of truth preservation in all models. The notions might be fruitfully thought of as extensions of the methods of demonstrating validity introduced in typical first-year logic courses: the step-wise use of intuitively valid derivation rules to transform the premises of an argument into its conclusion, and the use of truth tables to demonstrate that there are no interpretations of the nonlogical constants where the premises are all true and the conclusion false. The metalogical results of soundness and completeness ensure that the two notions are extensionally equivalent. If a conclusion \(S\) is derivable from a set of premises \(K\), then soundness ensures that there are no models where all of \(K\) is true but \(S\) is false; if, on the other hand, there are no models where all of \(K\) is true but \(S\) is false, then completeness ensures that \(S\) is derivable from \(K\). The completeness theorem then shows that there is no undergeneration of the derivation system: there are no validities expressible in the object language which cannot be demonstrated to be valid by the derivation rules.

There are reasons to resist pressing the analogy too far.\(^{23}\) But Aristotle’s claim resembles modern completeness theorems in this sense: he appears to hold that all the
valid inferences which are expressible in an object language consisting of just categorical propositions, and meeting the constraints of a broad syllogism, are derivable within the syllogistic. There are, however, difficulties in assessing this claim. Aristotle defends his thesis that all broad syllogisms proceed through a series of narrow syllogisms by appeal to two lemmas. First, Aristotle holds that

It is necessary that every demonstration and every syllogism should prove either that something belongs or that it does not, and this either universally or in part. (40b23-5)

Aristotle does not say why he holds that every broad syllogism must conclude with a categorical proposition. But, as we have seen, he seems to hold that any genuine proposition is categorical. So here is a first difficulty that faces us: it is difficult to assess Aristotle’s claim that all broad syllogisms proceed through the narrow syllogisms without a detailed study of Aristotle’s philosophy of language.

It is clear then that Aristotle’s claim that all broad syllogisms can be represented as a finite series of narrow syllogisms is made under certain restrictions. The expressive power of the object language is restricted: the conclusion of any narrow syllogism must be a categorical proposition. As we have seen, the characterization of a broad syllogism also places certain constraints on narrow syllogisms: for example, syllogisms must be non-circular and multi-premised. However, as we have also seen, the narrow syllogisms are held to constraints not placed on them by the characterization of a broad syllogism: for example, adjacent premises must share a term. Some have understandably held that Aristotle fails to defend this claim. Smith (1989: 140), for example, asserts that the requirement that adjacent premises have a common term is left unproven. Aristotle does purport to defend the requirement. He writes:
For a syllogism, without qualification, is from premises; a syllogism in relation to this term is from premises in relation to this term; and a syllogism of this term in relation to that is through premises of this term in relation to that. And it is impossible to take a premise in relation to B without either predicking or rejecting anything of it, or again to get a syllogism of A in relation to B without taking any common term, but <only> predicating or rejecting certain things separately of each of them. (41a4-11)

The writing here is crabbed. But charitably, we might take the move to be an appeal to a theorem in the theory of relations. Broad syllogisms must conclude in a categorical proposition, and these express a relation between the terms. If this is correct, then Aristotle is relying on substantive but admittedly, by our lights, implausible claims about relations. In particular, he seems to hold that either a relation is primitive and indemonstrable or else obtains in virtue of each relata being related to a third relata. Here then is a second difficulty that faces us. And so it is difficult to assess Aristotle’s argument without a detailed study of both his semantics and his idiosyncratic theory of relations. However, it is the intention, to provide an account of syllogisity, and not the success of this project, which is my primary concern in this paper.

I will begin to bring the paper to a conclusion. Aristotle’s appeal to mereological principles such as the transitivity of containment are prima facie evidence of a philosophical interest in providing an account of logical consequence. Since deductions concern interrelations among propositions and propositions express mereological relationships, deductions are justified in terms of mereological principles. Such an account is not an analysis of logical consequence, for the account does not rely solely on the meaning of words like that translated as ‘syllogism’; rather, the account relies on substantive assumptions in the philosophy of language and metaphysics.

The points of similarity with the model theoretic definition of logical consequence, as truth preservation in all models, are striking. Recently, there’s an
emerging consensus that the model theoretic account of logical consequence is also wrongly characterized as an analysis. Etchemendy (1990 and 2008) has argued that model theory cannot provide an analysis of logical consequence. However, although model theory is not an analysis of logical consequence, it may nonetheless be an account. The model theoretic definition of logical consequence does not rely solely on the meaning of ‘logical consequence’ but rather relies on substantive claims in metaphysics and philosophy of language. Etchemendy notes that the model theoretic definition is extensionally adequate only because of the strength of the underlying set theory and the weakness of the object language. One such assumption of the former kind is that the universe is infinitely large, an assumption built into the definition by the set theoretic axiom of infinity. Consider a sentence which is true in every finite model but false in some infinite model. For example, consider the sentence asserting that a transitive, irreflexive relation has a minimal element:

\[ (1) \forall x \forall y \forall z (Rxy \land Ryz \supset Rxz) \land \neg \forall x Rxx \supset \exists x \forall y \neg Ryx. \]

Etchemendy (1990: 111-22) notes that a finitist can consistently assert both

(2) There are only finitely many objects, and
(3) Sentence (1) isn’t a logical truth.

(2) and (3) are consistent but, conjoined with the model theoretic definition of logical truth, yield a contradiction. Just as the model theoretic account of logical consequence depends on extralogical assumptions in set theory, so too the Aristotelian account of syllogisity depends on extralogical assumptions in mereology.

Etchemendy (2008: 272) also notes that the model theoretic definitions avoid overgeneration of logical truths in part by the choice of logical constants. The object language of propositional logic takes as its sole logical constants the classical truth functional connectives. This is an expressively weak language. If we expand the
expressive power of the object language, we can express sequences that are truth preserving in all models but which are not logically valid. To take an example from Etchemendy (2008: 272), if we add the standard quantifiers and identity, we can express the extra-logical fact that there are more than three billion objects; any argument with this conclusion is truth preserving but intuitively is not logically valid. So too the success of the Aristotelian appeal to mereological theorems in an account of logical consequence depends in part on the severe limitations in expressive power of the object language of the syllogistic. Indeed, the plausibility of appealing to such theorems as the transitivity of mereological containment depends in part on the restriction to categorical propositions and on the semantic profile Aristotle gives to them.

There are also notable differences between Aristotle’s project and the model theoretic project. As we have seen, Aristotle’s modal notion of logical consequence is not our notion of validity. His topic-neutral conception does not employ notions of universal generalization but rather relies on highly general features of the world. And finally, Aristotle’s account of logical consequence is plausible only in the presence of views in the philosophy of language and metaphysics which strike us as quite foreign.

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1 Aristotle’s other two definitions of a broad syllogism are similar: “a syllogism is an argument in which, some things having been supposed, something other than that which has been supposed results of necessity (ex anankēs sumbainei) through (dia) the posits” (Topics I.1.100a25-27; “the syllogism is from some things having been posited so as to say of necessity something other than the posits through (dia) the posits” (Sophistical Refutations 164b27-165a2). However, in this paper I will discuss only the syllogism in the Prior Analytics.

2 The view that Aristotle intends the notion of a broad syllogism as the notion of valid inference is explicit in much of the recent secondary literature and is implicit in the recent tendency towards translating the Greek syllogismos with the English ‘deduction’. For example, see Rose (1968: 10-11, nb 27): “Aristotle’s ‘definition of ‘syllogism’ (at 24b18-20) seems so broad as to include any valid inference…. This definition seems inconsistent with Aristotle’s restriction of ‘syllogism’ to categorical syllogisms in the first, second, and third figures.” Smith (2007) describes the definition of a broad syllogism as “a general definition of ‘valid argument’,” although he later notes that the definition “is not a precise match for the modern definition of validity.”

3 Some have attempted to adapt the syllogistic so to represent such inferences. See, for example, Sommers (1982) and Englebretsen (1981) who, following Leibniz, attempt to
include singular terms by assigning them ‘wild’ quantity. Whatever the success of such projects, they cannot be taken as clear vindications of Aristotle’s intentions.

4 For the difficulty of representing mathematical reasoning solely with syllogisms, see Mueller (1974). Mendell (1998) persuasively argues that the ‘belongs to’ terminology which Aristotle uses to characterize categorical propositions, is sufficiently malleable to allow for mathematical theorems to be expressed in the object language of the syllogistic. However, Mendell concedes that geometric constructions resist syllogistic representation.

5 For the characterization of classical validity see, for example, Scott (1971). For the observation that syllogistic is not reflexive, see Irvine and Woods (2004). Reflexivity is a necessary but insufficient condition for a consequence relation to be classical validity. For example, classical validity is also transitive and systems targetting classical validity typically have cut as a primitive inference rule. Syllogisity is not a transitive relation, although it adheres to an inference schema akin to cut. If \{A, B\} entails C and \{C, D\} entails E then \{A, D\} entails E.

6 See, for example, Priest (1998: 411), Smith (1989).

7 See, for example, Jonathan Lear (1980: 3), who writes that Aristotle “simply states that it is evident that the first figure syllogisms are perfect. No argument is given for their validity. For if the syllogisms are perfect, no argument need be given.” Compare Lynn Rose (1968: 27), who writes that Aristotle’s “way of handling validity is to take the valid moods of the first figure as basic and to establish the validity of moods in the remaining figures by reducing them to moods of the first figure.”

8 I do not defend this claim here. I discuss whether narrow syllogisms are inferences or theorems, whether the syllogistic is a logic or a theory, and whether the syllogistic is well represented as a Fitch-style natural deduction system in my [author 1].

9 The other methods used to establish the validity of second and third figure syllogisms are indirect proof and exposition. For discussion of the latter, see Smith (1982).

10 Reading huparchei with the manuscripts, as opposed to Alexander’s reported akolouthei, adopted in the OCT.

11 Ross (1949: 302) claims that Aristotle’s method of contrasted instances merely cites empirical facts to show that no conclusion results of necessity from a particular combination of premisses, but “gives no reason for this, e.g. by pointing out that an undistributed middle or an illicit process is involved.” Ross’s assumption, that empirical evidence cannot establish invalidity, but that rather a formal explanation ought to be given, rests on the Formal conception of the topic-neutrality of logic.

12 The General conception of topic-neutrality is arguably consistent with Aristotle’s own use of the term ‘logical’ (logikôs and its cognates). Aristotle’s meaning of such terminology is controversial. Ross [note on 1029b13], for example, holds that ‘logical’ “probably always refers to linguistic inquires or considerations.” Simplicius (in Phys. 440.19-441.2), on the other hand, argues that Aristotle’s intention in calling a puzzle ‘logical’ at Phys. 3.3 (202a21-22) is that the puzzle proceeds from generalities rather than from principles peculiar and appropriate to the subject. Burnyeat (2001: 19-23) endorses and defends Simplicius’ view of Aristotle’s use of this terminology.

13 I’ve translated the to X locutions as references to the expressions, a common use of this locution. The claim that the two expressions are the same (tauton) is the claim that the
two are equivalent in the sense of having the same conditions under which each expresses a true thought. The mereological interpretation of the categorical propositions is not uncommon throughout the *Prior Analytics*. Other uses of the A *en holo einai* B locution include: 25b33, 30a2, and 53a21. Use of the corresponding *hos meros* locution include: 42a10, 42a16, 49b37, 64a17, 64b12, and 69a14.

Aristotle mentions a third quantity, indeterminate propositions, but these are not obviously a class of propositions distinct from universal and particular propositions. Rather, Aristotle is pointing out that some object language sentences are ambiguous with respect to their quantity and need to be disambiguated as either a particular or a universal proposition.

See, for example, Smiley (1973) and Corcoran (1974).

The transitivity of the part relation was questioned by Rescher (1955). Simons (1987) characterizes any genuine mereology as antisymmetric: if one thing is a part of another, and that other a part of the first, then the one and the other are identical. However, this characterization is now controversial. Cotnoir (forthcoming), for example, argues that extensionality entails antisymmetry, so a non-extensional mereology is not antisymmetric. The argument of the paper is neutral on the question whether the quantitative part relation is extensional or non-extensional. So I will not discuss whether the quantitative part relation is antisymmetric.

There are passages where Aristotle allows for there to be unique members of a kind. For example, at 1023b29-32, Aristotle includes a singular god among the species of the genus *living thing*. This is not to say that there is a universal term which has as its referent a mereological sum composed of just one individual, since it is not obvious that Aristotle identifies universals and species. Nonetheless, the ‘by its nature’ proviso at 17a40 may allow for there to be exceptional cases of universals with just one instance: if so, then a weakly supplementary preorder models the norm.

This suggestion is not new. Logicians after Aristotle drew upon Aristotle’s account of a quantitative whole for a topical maxim permitting certain inferences. Peter of Spain (1988: 234), for example, writes: “A quantitative whole is a universal taken universally, as, for example, ‘every man’, ‘no man’. The Topic from a quantitative whole is a relationship of a quantitative whole to its part; and it is both constructive and destructive. For example, ‘Every man is running; therefore, Socrates is running’. Where does the Topic come from? From a quantitative whole. The maxim: Whatever is predicated of a quantitative whole is also predicated of any of its parts.” Aristotle himself would not express the notion of logical consequence as resting on a topical maxim, since topical maxims are guidelines for argument which admit of exceptions.

On Euler diagrams as a pedagogical tool for teaching the syllogistic, see for example, Armstrong and Howe (1990) and Bennett and Nolt (1994). Savio (1998) develops a complete decision procedure for the syllogistic based on Euler diagrams. These authors do not ascribe such purposes to Aristotle. Striker (1998) hints at a reading of 25b32-26a2 as a heuristic devise. However, the discussion in such ancient commentators as Ammonius suggests the diagrammatic representation of a syllogism as a linear array of terms connected by links representing the predicative ties holding between the terms in
the premises and conclusion. Kneale and Kneale (1962, 71-2) and Rose (1968) discuss this suggestion. Of course, the spatial relations in an Euler diagram do not merely represent containment relations; they are themselves containment relations. Euler diagrams represent by being not analogies but metonyms.

21 This is not to say that the perfection or evident validity of Barbara and Celerent consists solely in their correlation to mereological theorems. Darii and Ferio are perfect despite not being correlated to mereological theorems and despite being superfluous in the reduction of narrow syllogisms. The perfection of the first figure narrow syllogisms consists in possession of a middle term which is the predicate of one premise and the subject of the other.

22 See Lear (1979) and Scanlan (1983).

23 There are disanalogies between mathematical logic and the syllogistic. For example, Aristotle lacks a sharp distinction between syntax and semantics. For discussion, see Corcoran (1972: 214) Lear (1980: 11) and Smiley (1994: 28).

24 If Aristotle holds that a conjunction or a conditional does not express a single proposition, he may hold that conjunction introduction and *modus ponens* are not broad syllogisms. For discussion of the hypothetical syllogism, see Lear (1980).

25 Etchemendy occasionally gives the impression that the worry is that the alleged analysans depends on a contingency—namely, the size of the universe. However, the issue is not that a purported analysis of logical consequence relies on a contingent fact. As McGee (1992) notes, we may include sets into our ontology, to ensure the infinitude of the universe. Indeed, although we may disagree about whether our ontology ought to include sets, surely it is not a contingent matter: if there are sets, there are necessarily sets. The issue is that an alleged analysis of logical consequence relies on an extralogical fact. The claim that the size of the universe is an extralogical fact depends tacitly on an assumption about logicality. I have argued in this paper that the topic-neutrality of logic need not entail that logical truths obtain independently of any worldly content whatsoever. However, I do not claim that set theoretic theorems are true independently of the particular identities of things. And so I do not claim that the model theoretic definitions provide an account of logical consequence in terms of a General conception of topic-neutrality.