Time-Frequency Representation of Microseismic Signals using the Synchrosqueezing Transform

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• Introduction to TFR.
• From the CWT to the Synchrosqueezing Transform.
• Two synthetic examples. (STFT vs SST)
• Two real cases in microseismicity. (STFT vs SST)
• One real case in seismic reflection data. (Spectral energy attribute: STFT vs SST)
• Conclusions.
New Time-Frequency Tool

• Research objective:
  – Introduce two novel high-resolution approach for time-frequency analysis.
    • Better Time & Frequency Representation.
    • Allow signal reconstruction from individual components.

• Possible applications:
  – Instantaneous frequency and modal reconstruction.
  – Multimodal signal analysis.
  – Nonstationary signal analysis.

• Main problem
  – All classical methods show some spectral smearing (STFT, CWT).
  – EMD allows for high res T-F analysis.
  – But Empirical means lack of Math background.

• Value proposition
  – Strong tool for spectral decomposition and denoising. One more thing ... It allows mode reconstruction.
Why are we going to the T-F domain?

• Study changes of frequency content of a signal with time.
  – Useful for:
    • attenuation measurement (Reine et al., 2009)
    • direct hydrocarbon detection (Castagna et al., 2003)
    • stratigraphic mapping (ex. detecting channel structures) (Partyka et al., 1998).
    • Microseismic events detection (Das and Zoback, 2011)

• Extract sub features in seismic signals
  – reconstruct band-limited seismic signals from an improved spectrum.
  – improve signal-to-noise ratio of the attributes (Steeghs and Drijkoningen, 2001).
The Heisenberg Box

Time Domain

- FT $\rightarrow$ Frequency Domain
- STFT $\rightarrow$ Spectrogram (Naive TFR)
- CWT $\rightarrow$ Scalogram

All of them share the same limitation:
The resolution is limited by the Heisenberg Uncertainty principle!

$$\sigma_t \sigma_f \geq \frac{1}{4\pi}$$

There is a trade-off between frequency and time resolutions

Gabor Uncertainty Principle


http://www.aip.org/history/heisenberg/p08.htm
The Heisenberg Box

- Localization: How well two spikes in time can be separated from each other in the transform domain. (Axial Resolution)

\[
\sigma_t \sigma_f \geq \frac{1}{4\pi}
\]

- Frequency resolution: How well two spectral components can be separated in the frequency domain.

\[
\sigma_t = \pm 5 \text{ ms} \\
\sigma_f = \frac{1}{(4\pi \times 5 \text{ ms})} \approx \pm 16 \text{ Hz}
\]

Heisenberg Uncertainty Principle

Choice of analysis window:
Narrow window $\rightarrow$ good time resolution
Wide window (narrow band) $\rightarrow$ good frequency resolution

Extreme Cases:
$\delta(t)$ $\rightarrow$ excellent time resolution, no frequency resolution

$\delta(f) = 1$ $\rightarrow$ excellent frequency resolution (FT), no time information

$$F[\delta(t)] = \int_{-\infty}^{\infty} \delta(t)e^{-j\omega t} dt = e^{-j\omega t} \bigg|_{t=0} = 1$$
Constant Q

\[ Q = \frac{f_c}{B} \]
Time-frequency representations

- Non-parametric methods
  - From the time domain to the frequency domain
    - Short-Time Fourier Transform - STFT
    - S-Transform - ST
    - Continuous Wavelet Transform – CWT
    - Synchrosqueezing transform – SST

- Parametric methods
  - Time-series modeling (linear prediction filters)
    - Short-Time Autoregressive method - STAR
    - Time-varying Autoregressive method - KS (Kalman Smoother)
The Synchrosqueezing Transform (SST)

- CWT (Daubechies, 1992)

\[ W_s(a,b) = \frac{1}{\sqrt{a}} \int s(t) \psi^* \left( \frac{t-b}{a} \right) dt \]

Instantaneous frequency is the time derivative of the instantaneous phase (Taner et al., 1979)

\[ \omega(t) = \frac{d\theta(t)}{dt} \]

- SST (Daubechies, 2011)

the instantaneous frequency \( \omega_s(a, b) \) can be computed as the derivative of the wavelet transform at any point \((a, b)\).

\[ \omega_s(a, b) = \frac{-j}{W_s(a, b)} \frac{\partial W_s(a, b)}{\partial b} \]

Last step: map the information from the time-scale plane to the time-frequency plane.

\((b, a) \rightarrow (b, \omega_s(a, b))\), this operation is called “synchrosqueezing”
Synchrosqueezing depends on the continuous wavelet transform and reassignment.

Seismic signal $s(t)$
Mother wavelet $\psi(t) \rightarrow f, \Delta f$

Continuous Wavelet Transform (CWT) $W_s(a, b)$

Instantaneous Frequency (IF) $w_s(a, b)$

Reassignment step:
Compute Synchrosqueezed function $T_s(f, b)$

Extract dominant curves from $T_s(f, b)$

Time-Frequency Representation

Reconstruct signal as a sum of modes

Reassignment procedure:
Placing the original wavelet coefficient $W_s(a, b)$ to the new location $W_s(w_s(a, b), b) \rightarrow T_s(f, b)$

Extracting curves
Noiseless synthetic signal $s(t)$ as the sum of the following components:

$$s_1(t) = 0.5 \cos(10\pi t), \quad t = 0: 6 \text{ s}$$
$$s_2(t) = 0.8 \cos(30\pi t), \quad t = 0: 6 \text{ s}$$
$$s_3(t) = 0.7 \cos(20\pi t + \sin(\pi t)), t = 6: 10.2 \text{ s}$$
$$s_4(t) = 0.4 \cos(66\pi t + \sin(4\pi t)), t = 4: 7.8 \text{ s}$$

$$f(t) = \frac{d\theta(t)}{2\pi \, dt}$$

$$f_{s1}(t) = 5$$
$$f_{s2}(t) = 15$$
$$f_{s3}(t) = 10 + \cos(\pi t)/2$$
$$f_{s4}(t) = 33 + 2 \times \cos(4\pi t)$$
Synthetic Example 1: STFT vs SST

Noiseless synthetic signal $s(t)$ as the sum of the following components:

\[
\begin{align*}
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The challenging synthetic signal:
- 20 Hz cosine wave, superposed 100 Hz Morlet atom at 0.3 s
- two 30 Hz zero phase Ricker wavelets at 1.07 s and 1.1 s,
- three different frequency components between 1.3 s and 1.7 s of respectively 7, 30 and 40 Hz.
The challenging synthetic signal:
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1 - Difference between the original signal and the sum of the modes

2- Mean Square Error (MSE)

\[ MSE = \frac{1}{N} \sum_{n=0}^{N-1} |s(t) - \hat{s}(t)|^2 \]
The challenging synthetic signal:
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- three different frequency components between $1.3$ s and $1.7$ s of respectively $7$, $30$ and $40$ Hz.

MSE = $0.0021$
Real Example: TFR. 5 minutes of data

![Graph showing TFR and synchrosqueezing transform (SST).]
Real Example. Rolla Exp. Stage A2

- **S-wave**
- **P-wave**
- **Noise**
Real Example. Rolla Exp. Stage A2

Waves freq content hardly discernible

Separation of P and S modes in time-frequency
Real Example. Rolla Exp. Stage A2

SST is one tool for:
- TFR
- Modal reconstruction
- Denoising
More applications of SST: seismic reflection data

Seismic dataset from a sedimentary basin in Canada

Original data. Inline = 110

- Erosional surface
- Channels
- Strong reflector
More applications of SST: seismic reflection data

CMP 81- Located at the first channel

![Graph of seismic reflection data](image-url)
More applications of SST: seismic reflection data

CMP 81 - Located at the first channel

Synchrosqueezing transform

![Graph showing seismic reflection data with time and frequency axes]
More applications of SST: seismic reflection data

Time slice at 420 ms

Frequency decomposition

SST - 20 Hz
SST - 40 Hz
SST - 60 Hz
More applications of SST: seismic reflection data

C80 Spectral Energy
Van der Baan, Fomel & Perz. TLE, 2010
Conclusions

• SST provides good TFR.
  – Recommended for post processing and high precision evaluations.
  – Attractive for high-resolution time-frequency analysis of microseismic signals.
• SST allows signal reconstruction
  – SST can extract individual components.

• Applications of Signal Analysis and Reconstruction
  – Instrument Noise Reduction,
  – Signal Enhancement
  – Pattern Recognition
Acknowledgment

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