MATH 314 Assignment #1

due on Wednesday, September 14, 2016

1. Let A, B, C, and X be sets. Prove the following statements:
   (a) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$.
   (b) $X \setminus (A \cap B) = (X \setminus A) \cup (X \setminus B)$.

2. Use the principle of mathematical induction to prove the following statements:
   (a) $1 + 3 + \cdots + (2n - 1) = n^2$ for all $n \in \mathbf{N}$.
   (b) $2^n > n^2$ for all $n \geq 5$.

3. Let A, B, C be sets, and let $f : A \to B$ and $g : B \to C$ be functions. Prove the following statements.
   (a) If f and g are injective, then $g \circ f$ is injective.
   (b) If f and g are surjective, then $g \circ f$ is surjective.
   (c) If f and g are bijective, then $g \circ f$ is bijective.
   (d) If f and g are bijective, then $(g \circ f)^{-1} = f^{-1} \circ g^{-1}$.

4. Let a and b be two elements of an ordered commutative ring. Prove the following statements.
   (a) $|a| - |b| \leq |a - b|$.
   (b) $\big||a| - |b|\big| \leq |a - b|$.
   (c) $2 \max\{a, b\} = (a + b) + |a - b|$.
   (d) $2 \min\{a, b\} = (a + b) - |a - b|$.

5. Let a, b, c, and d be elements of an ordered field. Prove the following statements.
   (a) If $bd > 0$, then $a/b < c/d \Leftrightarrow ad - bc < 0$.
   (b) If $bd > 0$ and $a/b < c/d$, then
   $$\frac{a}{b} < \frac{a + c}{b + d}.$$