MATH 314   Assignment #2

due on Wednesday, September 21, 2016

1. (a) Prove that there is no rational number r such that $r^2 = 3$.

   (b) Prove that $a + b\sqrt{2}$ is an irrational number for all rational numbers a and b with $b \neq 0$.

2. Let x and y be real numbers. Prove the following statements.

   (a) If $x > 0$, then there exists a unique natural number n such that $n - 1 < x \leq n$.

   (b) If $0 < y < 1$, then there exists a unique integer $n \geq 2$ such that

   $$\frac{1}{n} \leq y < \frac{1}{n-1}.$$

3. Write the following sets in interval notation:

   (a) $\{x \in \mathbb{R} : |x - 2| < 3\}$ (b) $\{x \in \mathbb{R} : |2x + 1| \geq 5\}$

   (c) $\{x \in \mathbb{R} : x^2 < 8\}$ (d) $\{x \in \mathbb{R} : x^3 \leq 8\}$

4. For each set below, find its maximum, supremum, minimum, and infimum if they exist.

   (a) $(0, 3]$ (b) $\{1 - 1/n : n \in \mathbb{N}\}$

   (c) $\mathbb{R} \setminus [1, \infty)$ (d) $\{n - (-1)^n : n \in \mathbb{N}\}$

5. Let A be a nonempty bounded subset of $\mathbb{R}$, and let $s := \sup A$.

   (a) Show that $s \in A$ if and only if $s = \max A$.

   (b) Let $-A := \{-x : x \in A\}$. Prove that $\inf(-A) = -s$.