

MATH 314 Assignment #5

due on Friday, October 14, 2016

1. Find the following limits.

(a) $\lim_{x \rightarrow 1} \frac{x^2 + 2x - 3}{x^2 - 1}$.

(b) $\lim_{x \rightarrow 4} \frac{\sqrt{x} - 2}{x - 4}$.

2. Find the following limits.

(a) $\lim_{x \rightarrow \infty} \frac{2 - 5x - 4x^2}{3x^2 + 1}$.

(b) $\lim_{x \rightarrow \infty} (\sqrt{x^2 + 2x} - x)$.

3. Let $f(x) = \sqrt{4 - x}$ for $x \leq 4$ and $g(x) = x^2$ for all $x \in \mathbb{R}$.

(a) Give the domains of the functions $f + g$, fg , $f \circ g$ and $g \circ f$.

(b) Find the values $f \circ g(0)$, $g \circ f(0)$, $f \circ g(1)$, $g \circ f(1)$, $f \circ g(2)$ and $g \circ f(2)$.

(c) Are the functions $f \circ g$ and $g \circ f$ equal?

(d) Are $f \circ g(3)$ and $g \circ f(3)$ meaningful?

4. Let f and g be two functions from \mathbb{R} to \mathbb{R} . Prove the following statements.

(a) If f is continuous, then the function $|f|$ is continuous.

(b) If f and g are continuous, then the function $\max\{f, g\}$ is continuous.

5. Let $f(x) := 1 + x^2$ and $g(x) := x(1 - x^2)$, $x \in \mathbb{R}$. Moreover, let h be the function defined by

$$h(x) := \begin{cases} 1 & \text{if } x \geq 0, \\ -1 & \text{if } x < 0. \end{cases}$$

(a) Prove that $f \circ h$ and $h \circ f$ are continuous functions from \mathbb{R} to \mathbb{R} .

(b) Find the set of discontinuity points of $g \circ h$ and $h \circ g$, respectively.