MATH 314 Assignment #8

due on Wednesday, November 23, 2016

1. For each of the following functions, determine the interval(s) where the function is increasing or decreasing, and find all maxima and minima.

   (a) $f(x) := 4x - x^4$, $x \in \mathbb{R}$.

   (b) $g(x) := \frac{x^2}{1+x^2}$, $x \in \mathbb{R}$.

   (c) $u(x) := \sqrt{x} - x/2$, $x \geq 0$.

   (d) $v(x) := \frac{x}{1+|x|}$, $x \in \mathbb{R}$.

2. Establish the following inequalities.

   (a) For $0 < t < 1$, prove that $x^t \leq tx + (1-t)$ for all $x > 0$.

   (b) Prove that $a^t b^{1-t} \leq ta + (1-t)b$ for $a \geq 0$, $b \geq 0$, and $0 < t < 1$.

3. Let g be the function given by $g(x) := \ln[(1+x)/(1-x)]$ for $-1 < x < 1$.

   (a) Find the Taylor series of g about 0.

   (b) Find the interval of convergence of the power series in (a).

   (c) Use the power series in (a) to evaluate $\ln 2 = g(1/3)$ accurate to four decimal places.

4. Let f be the function on $\mathbb{R}$ defined by

$$f(x) := \begin{cases} x^2 \sin \frac{1}{x} & \text{for } x \in \mathbb{R} \setminus \{0\}, \\ 0 & \text{for } x = 0. \end{cases}$$

   (a) Find $f'(x)$ for for $x \in \mathbb{R} \setminus \{0\}$.

   (b) Prove that f is differentiable at 0 and that $f'(0) = 0$.

   (c) Show that f' is not continuous at 0.

5. Let $u(x) := \arctan x$ and $v(x) := 1/(1+x^2)$ for $x \in (-\infty, \infty)$.

   (a) Find the Taylor series of v about 0 and its interval of convergence.

   (b) Find the Taylor series of u about 0 and its interval of convergence.

   (c) Compute $v^{(6)}(0)$ and $v^{(7)}(0)$.

   (d) Compute $u^{(6)}(0)$ and $u^{(7)}(0)$.