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Optimal Predictive Control

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Abstract

Model Predictive Control denotes a class of optimal control techniques applied heavily in the petrochemical industries. The multitude of algorithms contained within this class possesses modifications in three main categories: the prediction model, the objective function and the determination of the control law. Depending on the control schemes chosen, varying degrees of differences and similarities will exist. An overview of the MPC structure is reviewed with illustrative examples concerning the popular Dynamic Matrix Control and Generalized Predictive Control algorithms with intent of demonstrating the use of this type of optimal control technique as related to the chemical process control and system identification fields of research.
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INTRODUCTION

The constantly changing landscape of engineering industrial systems necessitates the need for more complex control techniques in order to adequately synergize operations with both corporate and economical goals. As a consequence of global competition and more stringent environmental and safety regulations, process control has become increasingly important in the process industry. The advent of modern technology has allowed increasing computer processing speeds at decreasing costs. Consequently, advanced control techniques have developed over time to take advantage of these modern technologies.

The three-mode controller with proportional, integral and derivative (PID) feedback control became available in the 1930s for use in electronic equipment [13]. PID control is still widely used for basic control purposes; however, it is very limited for more difficult multivariable problems. First-generation model predictive control (MPC) concepts were pioneered by two industrial research groups in response to more complex and highly integrated processes: Dynamic Matrix Control (DMC) by Shell Oil [7] and a related approach by Adersa [12].

The term MPC designates a wide range of optimal control algorithms explicitly using a process model to solve a minimization problem for determining a control input [8]. A 2003 survey by Qin and Badgwell [10] revealed various early predictive control methods including, IDentification and COMmand (IDCOM), Internal Model Control (IMC), Extended Horizon Adaptive Control (EHAC), Model Algorithm Control (MAC), etc. Many of these algorithms are commercially available [2] including those well known in industry, for example, Honeywell’s RMPCT (Robust Multivariable Predictive Control Technology) and Shell’s SMOC (Shell Multivariable Optimizing Control). All these methodologies have certain elements in common, but the approach to certain elements differentiates them.

The purpose of this paper is to explore the optimal control techniques used in the process engineering industry today. Focus will be on a comparison of the DMC and Generalized Predictive Control (GPC) [5, 6] algorithms through their framework, advantages, disadvantages and use in industry; a general comparison of algorithms is also included. Given the context, a general introduction to MPC will be discussed to familiarize the reader with basic concepts and the aforementioned control algorithms will be compared as they are introduced.
1 Model Predictive Control

Model predictive control denotes a class of optimal control strategies explicitly using a process model for two main tasks: the prediction of future process behaviour and the calculation of the control action required to drive the predicted output as close as possible to the reference signal (set point). An optimization problem is solved in order to calculate the optimal control input at each time instant. According to [10] and [13], the main overall objectives of MPC include the prevention of input and output constraint violations, avoidance of excessive input magnitudes, control of specific output variables to optimal set points without disturbing other variables and controlling as many processes as possible under sensor/actuator restrictions. The application of MPC is widely demonstrated across many industries, with success ranging from the cement industry to robots [4] to common chemical processes such as crude oil distillation columns [11].

Model predictive control can be understood from an intuitive perspective. The foundations of MPC, namely prediction, planning and action are utilized in commonplace occurrences. For instance, academic performance relies on these principles. It is often the case allocations of marks are readily available in the class syllabus. A student can predict short term achievements in anticipation of reaching a final desired grade. Short term goals can be represented by weekly assignment marks. Comparison of the predicted performance with the short term target will subject the student to plan ahead. If an assignment mark is lower than targeted, extra effort into future assignments will be expected subject to constraints – assignments must still be completed by the due date and performance in other classes must be unperturbed regardless of the amount of effort required. Provided the decided new work schedule will result favourably towards the ultimate objective, corrective action will be taken. However, only the immediate action can be realized regardless of how far ahead planning was made. Following results of the next assignment, the same procedure of prediction and planning is repeated in order to determine a new work schedule. The procedure continues until the final desired grade is achieved. Other common analogies include playing chess or driving [2].

From a theoretical perspective, all controllers belonging to the MPC family can thus be characterized by the following strategy [8]. The future outputs, \( \hat{y} \) for the prediction horizon \( N \) are predicted at each discrete time instant, \( t \). These outputs rely on the \( a \ priori \) inputs and
calculated future control signals sent to the process. In order to keep the process as close as possible to the reference signal future inputs, $u_t$ are calculated by optimizing a specified criterion. The immediate control action is taken and the future actions are discarded. The strategy is repeated using updated information.

The MPC algorithms hold many differences and similarities depending on the configurations of three main aspects: the prediction model, the objective function and obtaining the control law. A comparison of MPC algorithms based mainly on these three elements and brief inclusion of industrial applications will be discussed with the commonly used dynamic matrix control and generalized predictive control algorithms used as specific illustrations. The intended nature of this paper is not just to collect and present various available information, but to interpret the results in a unified framework. Consequently, this necessitated slight alteration or reinterpretation in order to maintain context and coherence in notation.

### 1.1 Prediction Model

The process model is the foundation of virtually all control schemes. Without an appropriate model, controller performance can be significantly compromised as predictions are strictly dependent on an accurate identification. In general, the process and disturbance models make up the overall model. The process model describes the input-output relationship and the disturbance model approximates extraneous effects such as nonmeasurable inputs and inherent noise. Since the process output follows the trajectory of the setpoint as opposed to one setpoint at a specific instant, the prediction model must be fairly accurate multiple steps ahead. There are several model structures utilized in the variants of the MPC methodology and the advantages and disadvantages of each will be discussed.

The finite impulse response (FIR) model utilizes the output of the process in reaction to a unit impulse response. The form states the output at a given time instant can be expressed by a linear combination of past input values. The model for a single-input-single-output (SISO) case can be written as [2]

$$y_t = \sum_{i=1}^{\infty} h_i u_{t-i}$$

(1)
Where $h_i$ denotes the sampled output at time instant, $i$. The sum is often truncated to consider points inclusive to the settling time, $N_s$. The backshift operator, denoted $z^{-1}$ is commonly used to represent past time instants and the prediction using the FIR model can be represented as

$$
\hat{y}_{t+k|t} = \sum_{i=1}^{N_s} h_i u_{t+k-i|t}
$$

(2)

However, settling time is process dependent and can vary widely in magnitude. The number of parameters required may cause an issue both of storage and over-parameterization. Additionally, it is apparent the model can only be successfully applied to stable processes. The main advantage of this structure is the clear representation between the input variables and the process outputs. Because of the black-box structure, no previous information about the process is necessary as it is only concerned with the obtainable impulse response. Complex dynamics, particularly non-minimum phase problems, can be described using discrete time instants. A similar model, the step response model is used in the DMC algorithm and is detailed in section 2.

MPC algorithms using this form include MAC and Extended Prediction Self Adaptive Control (EPSAC).

The transfer function model is used to represent the relationship between input and output for a linear-time-invariant process. The model takes the form [13]

$$
y_t = \frac{B(z^{-1})}{A(z^{-1})} u_t = \frac{b_1z^{-1} + b_2z^{-2} + b_3z^{-3} \ldots + b_nbz^{-nb}}{1 + a_1z^{-1} + a_2z^{-2} + a_3z^{-3} \ldots + a_{na}z^{-na}} u_t
$$

(3)

The predictor is represented by

$$
\hat{y}_{t+k|t} = \frac{B(z^{-1})}{A(z^{-1})} u_{t+k|t}
$$

(4)

Unlike the FIR, both stable and unstable processes can be represented by a transfer function with the added advantage of requiring only a few parameters. Limitations in these parameters however, may restrict the sufficiency of modelling some systems. Two other main disadvantages of this type of prediction model exist. First, a priori information about the structure of the process is necessary, particularly the orders of A and B. Second, the transfer function is limited to linear differential equations because the mathematical application (Laplace transformation) required can only be applied to linear equations. This effectively means time-varying processes are unidentifiable with this structure and nonlinear systems may be insufficiently represented or overly complicated to represent. A variety of MPC control schemes
incorporate the transfer function model including special cases of GPC, EPSAC, Multistep Multivariable Adaptive Control (MUSMAR) and Unified Predictive Control (UPC).

Dynamic models derived using the physical properties of systems typically consists of several ordinary differential equations (ODE). This class of ODE model structures is known as the state space model and can be summarized as [3]

\[
x_{t+1} = Ax_t + Bu_t \\
y_t = Cx_t
\]

Where A, B, C represent the system, input and output matrices, respectively. The prediction model is stated as

\[
\hat{y}_{t+k|t} = C\hat{x}_{t+k|t}
\]

The main advantage of this structure is the applicability of many available modern control theories and techniques. Depending on the control methodology applied, the physical meaning of the state basis may be lost when translating from a different state space model. In other words, certain control designs require translating the initial state space model to an arbitrary model possessing specific properties for determining stability or controllability, for instance. The state space model also lends itself to many classes of systems including linear, nonlinear, lumped parameter, discrete and continuous processes. Another advantage is the ease of extension to multivariable systems by denoting the input and output as vectors and introducing the corresponding state matrices. The disadvantage of this structure concerns the availability of state information. Calculations may be complicated if states are not accessible and the design of an observer is required. The Predictive Functional Control (PFC) utilizes this structure.

Various other model structures exist for system identification, for example the ARIMA model introduced in detail in section 2 is commonly used in the GPC algorithm. First principles models will be the most accurate method of identification/prediction as they are based on the intrinsic properties of a system. However, these models are extremely time-consuming to construct and often many parameters are unavailable or infeasible to measure. Lastly, the application of neural networks and fuzzy logic has also been applied to chemical industries, particularly for modelling nonlinear systems such as continuous stirred tank reactors. Nonlinear models are more complicated to identify and require solving a more complicated optimization problem.
Depending on the choice of the prediction or identification model, several advantages and disadvantages will inherently manifest. A lot of the advantages stem from the simplicity of the models and their extension to represent different classes of processes. On the other hand, many of the disadvantages are due to the processes themselves, especially in regards to the physical properties. The different types of MPC algorithms will seek to consider these aspects and presumably claim benefits to their intended or preferred applications.

1.2 Objective Functions

The objective function can be expressed in general as [2, 9]

\[
J = \sum_{j=N_t}^{N_2} [r_{t+j} - \hat{y}(t + j|t)]^T Q_j [r_{t+j} - \hat{y}(t + j|t)] + \sum_{j=1}^{N_u} [\Delta u_{t+j-1}]^T R_j [\Delta u_{t+j-1}] \tag{7}
\]

The main purpose of this quadratic form is to determine the best control input allowing the future output to follow the set point. In order to determine the optimal effort needed to attain the desired trajectory, the control input is penalized using a weighting matrix, \( R_j \). \( N_1 \) and \( N_2 \) are the minimum and maximum prediction horizons, respectively and \( N_u \) is the control horizon. A large minimum prediction horizon will indicate initial errors up to \( N_1 \) are unimportant; a large value of \( N_2 - N_1 + 1 \) suggests errors are significant over a larger time span. Many of the formulated quadratic problems are solved using conventional methods including solution to the least-squares problem or utilization of the Levenberg-Marquardt and Quasi-Newton algorithms for quadratic functions.

Constraints are inherent in all processes. In many control systems, inputs cannot be arbitrarily large and may have limitations both in magnitude and response rate. Valve stiction, for example, may limit the opening position and rate of change. It may also be damaging to equipment if changes in magnitude occur too quickly. Similarly, output states are almost always bounded as well. For instance, pressure and temperature must be within a certain range because of safety concerns. Economical, environmental or safety limitations, as well as equipment restrictions may introduce constraints on the process variables. Consequently, the optimization problem necessitates inclusion of these constraints. Equation (8) shows the constraints on the input, output and bounds of both the magnitude and rate of the control signal [2, 9].
Due to the unpredictability of the actual output, constraints can only be applied to the predicted output, emphasizing the necessity for a good prediction model. The forms of the DMC and GPC objective functions and their method of accounting for constraints are introduced later.

1.3 Control Law

The control actions are determined by minimizing the objective function, $J$. The typical least squares problem can be solved by taking the partial derivatives of the objective function with respect to $\Delta u_t$ and solving the resulting equations with respect to zero. In the case of constraints, iterative methods may be necessary and the solution may not be trivial due to the high degree of freedom. A common simplification is to assume the control input becomes constant after a designated time, $N_u$.

To elaborate on the differences and similarities between the MPC algorithms, the design procedures for the DMC and GPC algorithms will be outlined.

2 A Comparison

2.1 Dynamic Matrix Control

Dynamic matrix control was developed in the early 1970s by Cutler and Ramaker of Shell Oil with an initial application in 1973 [10]. DMC is largely used in engineering industries, particularly in the petrochemical sector.

The DMC algorithm uses a prediction model analogous to the FIR model. Changes in a process output are related to past input changes via weighted sum. The truncated step response model for stable systems is given by Camacho [2]

$$y_t = \sum_{i=1}^{N_s} g_i \Delta u_{t-i} + \sum_{i=N_s+1}^{\infty} g_i \Delta u_{t-i} = \sum_{i=1}^{N_s} g_i \Delta u_{t-i} + z_t$$

(9)

Where $g_i$ denotes the sampled output and $\Delta u_t = u_t - u_{t-1}$. The predictor is given as
\[ \hat{y}_{t+k|t} = \sum_{i=1}^{N_s} g_i \Delta u_{t+k-i|t} + Z_{t+p} \] (10)

It is common to separate the predicted response into two parts to represent the free response where no control action is taken and the forced response or controlled response. Using vector notation, the predicted response can be represented as [9]

\[ \hat{y} = y^* + G\Delta u \] (11)

Where the free and forced response are given by Equations (12), respectively

\[ \hat{y}_{t+k|t} = \sum_{i=1}^{k} g_i \Delta u_{t+k-i} + y^*_{t+k} \] (12)

\[ y^*_{t+k} = \sum_{i=k+1}^{N_s} g_i \Delta u_{t+k-i} + Z_{t+p} \]

The matrix \( G \) is known as the Dynamic Matrix. For a derivation of the predicted response and form of the Dynamic Matrix, refer to Huang and Kadali (2008). Advantages and disadvantages of this prediction model are similar to the FIR case which includes its simplicity and intuitiveness; the need for a large number of parameters and the inability to model unstable systems. Both structures are classified as non-parametric models requiring little \textit{a priori} information about the process order or underlying structure. Over-parameterization stems from this fact. For example, a first-order process can be represented by three parameters (gain, time constant, dead–time); a typical FIR or step response structure can consist upwards of 30 coefficients to describe the same dynamics. However, the use of this representation allows the control law to be calculated analytically.

The objective function of the DMC algorithm (and MPC in general) is intended to drive the output as close as possible to the setpoint trajectory while penalizing control moves. This setup allows a less aggressive response with smaller input actions. As well, penalties on input moves provide a “numerical benefit in that they can be used to directly improve the conditioning of the numerical solution [10].” The technique also allows some degree of robustness to modelling errors. The original formulation of the DMC objective function was solved as a least-squares problem and early applications utilized Linear Programming techniques to solve for optimal operating points. However, dynamic disturbances had the potential to drive the input actions away from their optimal targets in order to keep outputs at optimal conditions.
Modifications to the algorithm accounted for the trade-off of moving one input off target versus another to reach optimal conditions. Prett and Gillette [10] also introduced time varying constraints to prevent violations of absolute input restrictions. However, it was not until the introduction of DMC using Quadratic Programming (QDMC) that the general issue of constraint handling was addressed. Reformulation of the problem allowed explicit representation of both input and output constraints. The Hessian matrix of the quadratic objective function is almost always positive definite for many control problems resulting in a convex problem that is easily dealt with numerically using readily available optimization algorithms. Numerical optimization occurs at each control interval. The form of the constrained DMC objective function for the multivariable case is summarized as [9]

\[
\min_{\Delta u} J(\Delta u) = (r - \hat{y})^T Q (r - \hat{y}) + \Delta u^T + R \Delta u
\]

subject to

\[
\hat{y} = y^* + Gu
\]

\[
\begin{align*}
    u_{\text{min}} & \leq u \leq u_{\text{max}} \\
    y_{\text{min}} & \leq \hat{y} \leq y_{\text{max}}
\end{align*}
\]

For the unconstrained case, the solution can be found analytically by using the principles of the first order necessary condition and solving for the desired control action.

\[
\Delta u = (G^T Q^T G + R)^{-1} G^T Q^T (r - y^*)
\]

Calculation of the inverse matrix is required at each sampling interval. This has the potential to be demanding both in storage and computing capacity. In practice for the unconstrained case, this is computed off-line but since the first control move is implemented, only the first row needs to be stored. The constrained case is slightly more complicated requiring online calculation of the gain matrix for active constraints.

A main advantage of the DMC algorithm is the ease of extension to multivariable problems as seen in Equation (13). Common applications of the DMC algorithm include control of fluid catalytic cracking reactors and temperature control of steam through fuel gas pressure in a pyrolysis furnace. The latter case demonstrates the use DMC in a multivariable environment where temperature is controlled at multiple locations in the furnace.

In summary, the main disadvantages of the DMC algorithm are found in the complexity of the prediction model and the inability of the model structure to identify unstable processes.
On the other hand, the intuitiveness of the structure setup makes the influence of each input apparent and no prior information about the system’s physical properties are necessary. Computationally, both the linear and quadratic problems are readily solved using available packages. Input and output constraints are dealt with explicitly through the objective function. However, the constrained quadratic programming problem can be computationally demanding, requiring numerical methods; and space consuming, requiring storage of the gain matrices for active constraints.

### 2.1 Generalized Predictive Control

GPC was developed by Clarke et al [5, 6] in 1987 and has garnered a quick reputation in both an industrial setting and academic environment. The GPC algorithm identifies a class of parametric model structures, the autoregressive and integrated moving average (ARIMAX) model denoted by [9], also known as the CARIMA model [2]

\[ A(z^{-1})y_t = B(z^{-1})u_{t-1} + \frac{C(z^{-1})}{1 - z^{-1}}e_t \]  

Typically, \( A \) and \( C \) are considered to be monic. The role of the differencing operator \((1 - z^{-1})\) is to include integral action of the controller by including an internal model of typical white noise disturbances. Thus, two immediate advantage of this model are apparent: the ability to represent random disturbances occurring at random instances (Brownian motion) and the offset-free steady-state control achieved via the integrator.

A control sequence is found to minimize the multistage GPC quadratic cost function of the form (univariate case) [9]

\[ J = \sum_{i=N_1}^{N_2} (r_{t+i} - \hat{y}(t+i|t))^2 + \sum_{i=N_1}^{N_u} \lambda(\Delta u_{t+i-1})^2 \]  

Where \( \lambda \) denotes the penalty on the control input. Refer to Huang and Kadali (2008) for selection of horizon parameters and derivation of solution.

Using this form of the quadratic function and model structure requires use of the Diophantine equation to obtain the output predictor. The following equation is analogous to Equation (12) of the DMC algorithm, denoting the free and forced response.

\[ \hat{y}_{t+k|t} = c_p \Delta u_{t+k-1} + y^*_{t+k} \]
Equation (11) also denotes the general predictor form of the GPC algorithm. But, the matrix $G$ is now derived from $\zeta_p$. The GPC control law for the unconstrained case is derived in the same manner as DMC and is found to be

$$\Delta u = (G^T G + \lambda I)^{-1} G^T (r - y^*)$$

Management of constraints is similar to that found with the DMC case. Where an $ad hoc$ approach was original taken, an alternative formulation to the original GPC method known as Constrained Receding Horizon Predictive Control (CRHPC) essentially computes a future control sequence so the predicted output is constrained by the exact reference value.

Similar advantages exist in GPC in its ease of extension to multivariable problems. Common application of the GPC scheme include composition control in an evaporator or effluent concentration and temperature control via feed and coolant flow rate in a jacketed stirred tank reactor.

The main advantage of the GPC control method is in the structure of the prediction model. The parametric ARIMAX model allows unstable processes to be identified and with less parameters. In addition, the form of the disturbance model compared to the step-type disturbance model of DMC allows for a broader representation of process perturbations. Calculation of the control law necessitates the use of the Diophantine equation and depending on the complexity of the problem, the computational load will vary. Obviously in the $n$–output, $m$–input multivariable case, at least $n$ recursive equations will need to be programmed and solved to determine the optimal control sequence.

In summary, the form of the MPC controller scheme used will largely depend on the intended use and type of system being studied. It also suffices to point out, the illustration of the two control schemes shows the MPC algorithms possess differences and similarities even in the slightest of manners.
Conclusion

Model Predictive Control denotes a type of advanced optimal control schemes used in industry today. The multitude of algorithms classified under this moniker possess slight differences and similarities in regards to three main aspects:

1. The prediction model
2. The objective function
3. The calculation of the optimal control sequence

To illustrate the modifications amongst the control schemes, the DMC and GPC algorithms were outlined. It was found the main differences regard the prediction model structure. A step response model is often over-parameterized leading to storage issues and application is limited to strictly stable processes. The strength of this approach requires minimal or no information about the process order or structure. The setup of the model also makes it intuitive to understand the corrective action taken. However, there are several industrial processes for which an approximate linear model provides an insufficient representation of the process dynamics. In these cases, the attractiveness of the step (or FIR) model is overshadowed by the naïve simplicity of the model structure. The ARIMAX structure used in GPC allows for a more general representation of processes but requires the use of the Diophantine equation to calculate a control law. Introduction of the integral action also allows offset-free steady-state control. The control law for the constrained case of both structures can be readily solved using Quadratic Programming methods. Computational and storage limitations depend on the implementation of the dynamic matrix, whether computed on-line or off-line. Both algorithms can be extended to handle input and output constraints for both univariate and multivariate problems. Applications of both algorithms are heavily concentrated in the oil and petrochemical industries due to its foundation in refining, but extension to other sectors including aerospace, food processing, metallurgy and pulp and paper exist.

Future work can consider nonlinear MPC to handle processes with frequent changes in operating conditions or switching dynamics. It is obvious system identification is the Achilles heel of model predictive control; though by no means is it strictly relegated to MPC. Regardless, a cumulative overview of the popular optimal control schemes applied in the petrochemical industries today was successfully demonstrated and discussed in this paper.
References


