Model Predictive Control: Developing nonlinear models from operating data

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Outline

1. Motivation
2. Nonlinear Model Predictive Control
3. Gain Scheduling
4. Developing nonlinear models from operating data
5. Illustrative example – Nonlinear reactor
6. Concluding Remarks

Introduction

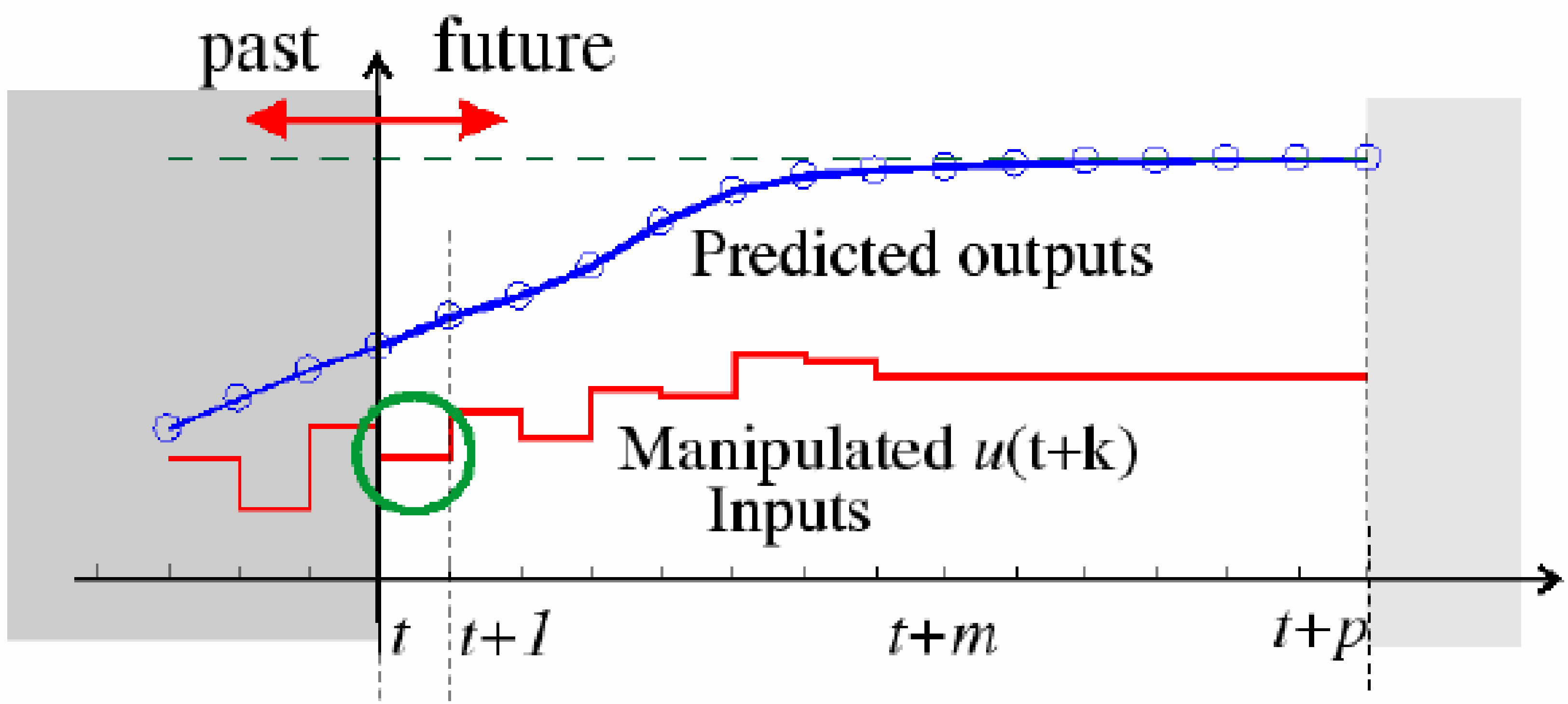
- Model Predictive Control (MPC)
  - Controls & optimizes multivariable processes with significant interactions.
  - Predicts future outputs using linear plant model.
  - Computes optimal manipulated variable movements, subject to constraints to achieve control objectives.
  - Widely used in chemical processes, e.g. petroleum refineries.
  - Also being applied to more nonlinear processes with considerable scope for performance improvement, e.g. polymer processes,...

MPC Technology

- Main steps in currently used MPC technology.
  - Model identification using open-loop step tests.
  - Controller configuration and initialization.
  - Performance verification using off-line tests.
  - Online deployment and tuning.
- MPC performance depends on model quality.
  - Model-Plant Mismatch (MPM) leads to performance degradation.
- MPM causes
  - Changes in operating conditions, feed from multiple sources, change in throughput, feed composition, seasonal variations etc.
  - Physical changes in the process, design changes, equipment degradation, etc.

Overview of MPC

- Model Predictive Control (MPC) strategy
  - Use linear process model to predict behavior of process variables (PVs) & choose future movements of manipulated variables (MVs) to achieve control objectives, subject to constraints.



Motivation

- Consider application of MPC to nonlinear reactor

$u_1 = D$

$u_2 = S_f$

$y_1 = X$

$y_2 = P$

$$\frac{dX}{dt} = -DX + \mu(P,S)X$$

$$\frac{dS}{dt} = D(S_f - S) - \frac{1}{Y_{x/s}}\mu(P,S)X$$

$$\frac{dP}{dt} = -DP + (\alpha\mu(P,S) + \beta)X$$

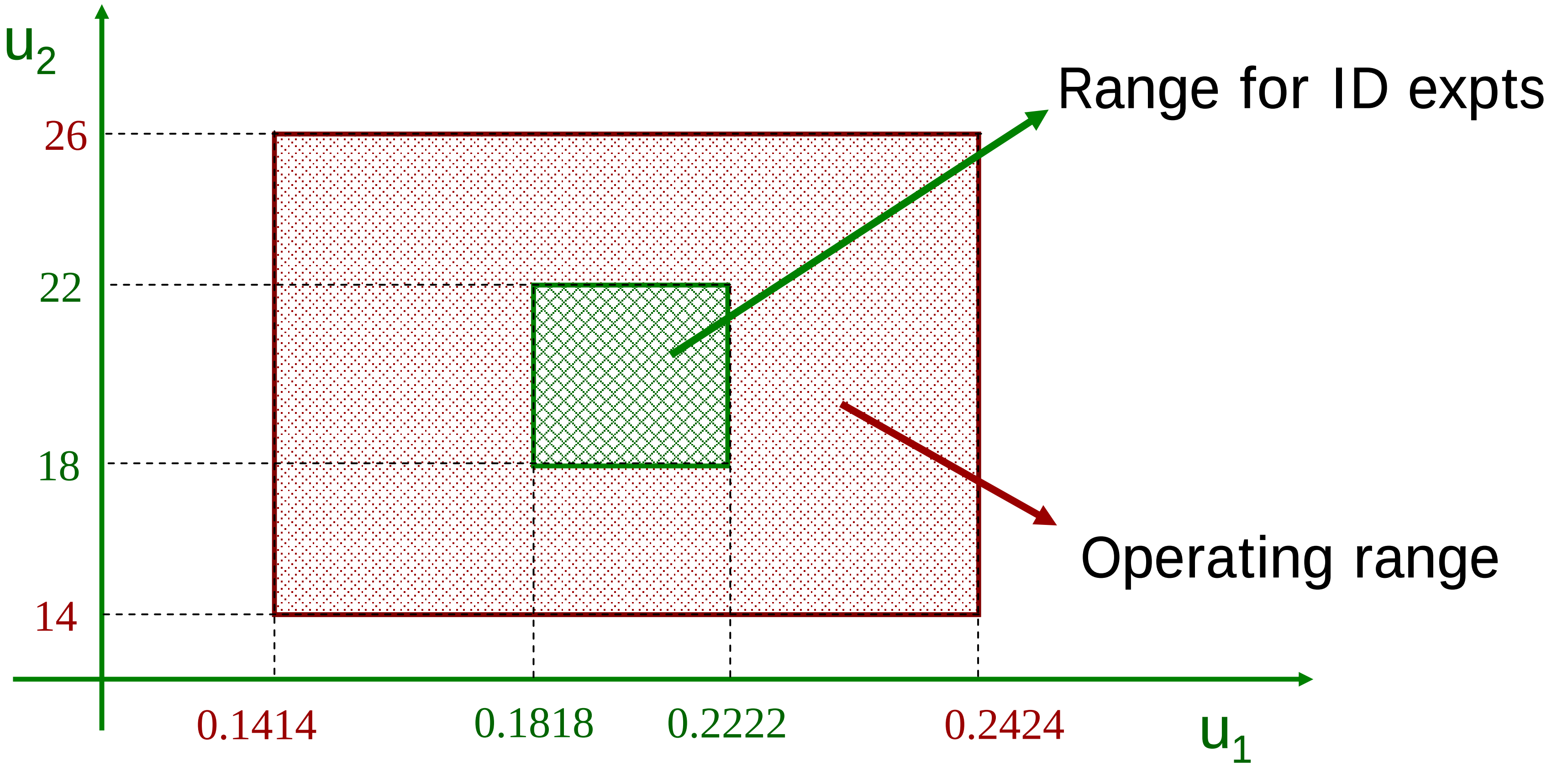
$$\mu(P,S) = \frac{\mu_m\left(1 - \frac{P}{P_m}\right)S}{K_m + S + \frac{S^2}{K_i}}$$

2 Measured Outputs y_1 & y_2

2 Manipulated variables u_1 & u_2

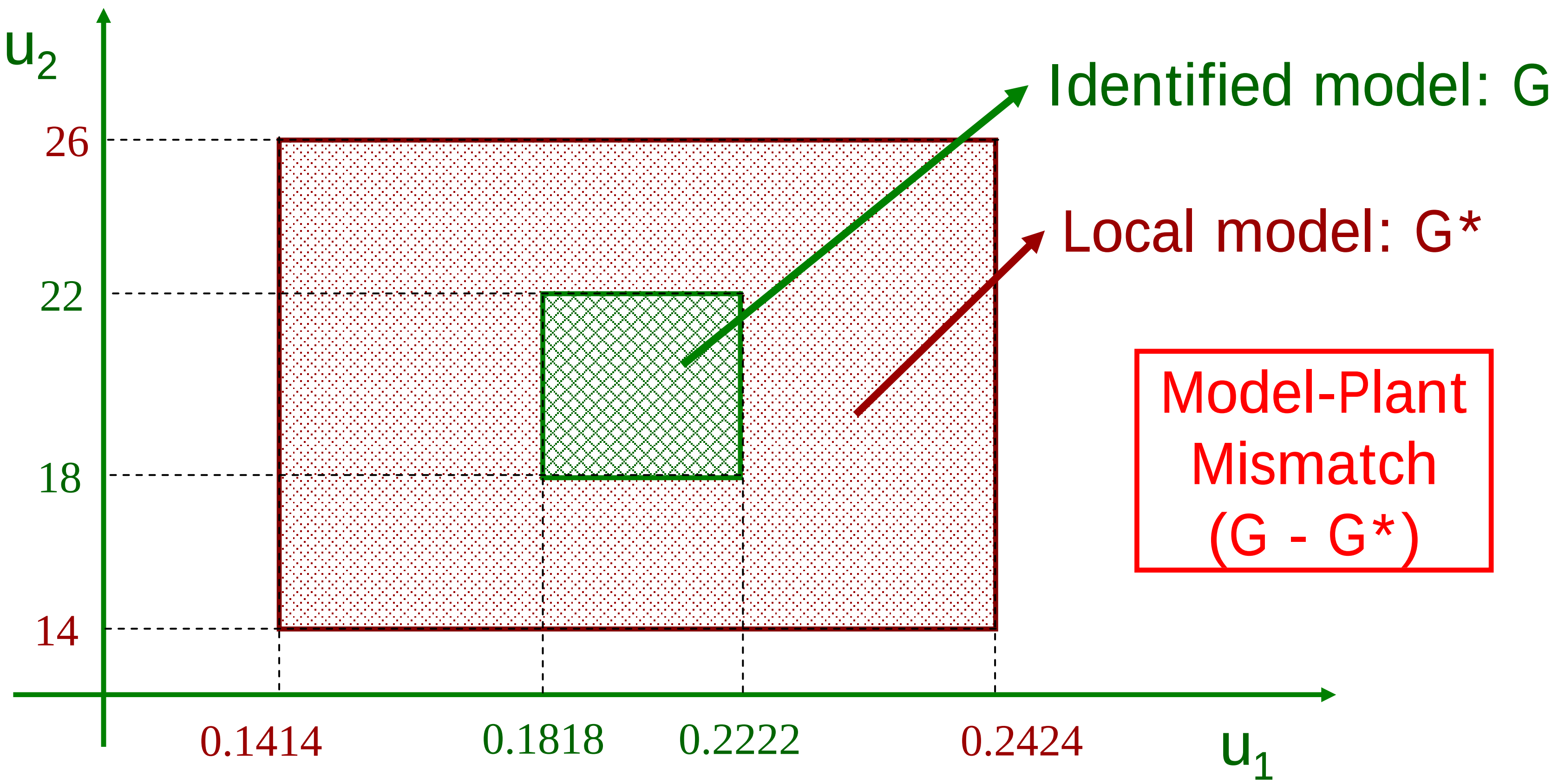
Motivation

- Typically MV range during model ID does not match complete operating range.



Motivation

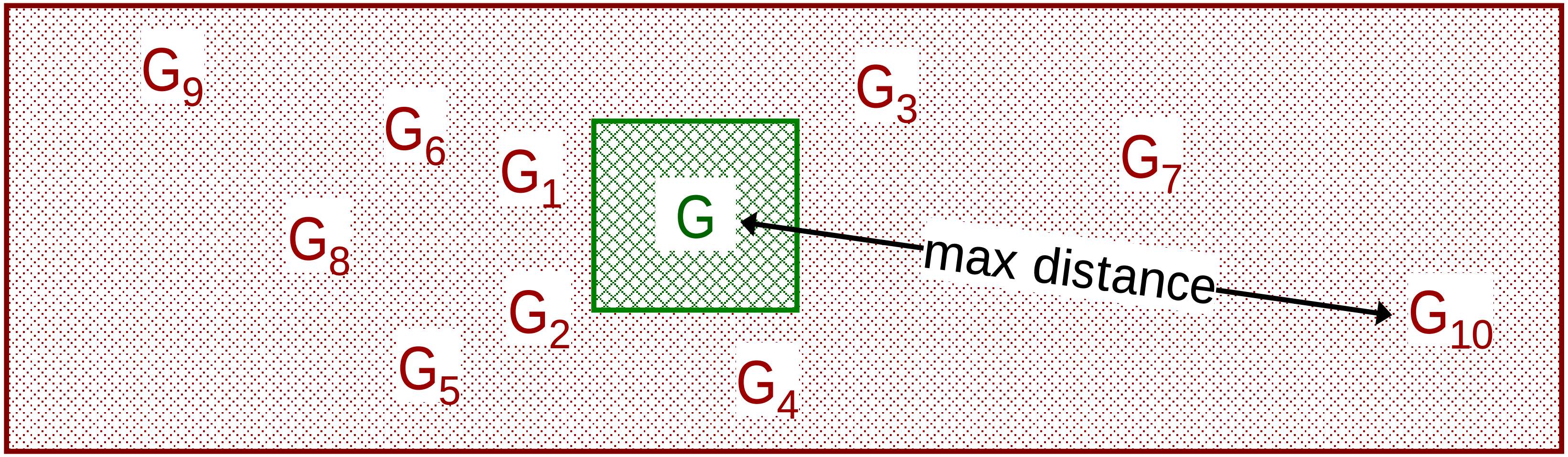
- Model identified over small range may not be best for all operating conditions.



MPM degrades MPC performance

- Model-Plant Mismatch (MPM)
  - There is a mismatch between:
    - G*: Best local linear model for current operating conditions
    - G: Linear model obtained during identification experiment, i.e., model used by linear MPC.
    - MPM = (G – G*)
- How does linear MPC handle MPM
  - Controller is tuned to suit *worst-case scenario*.
  - Controller should *work* when local plant conditions are farthest from MPC model in some sense.
  - Result: Detuned and extremely sluggish controller performance.

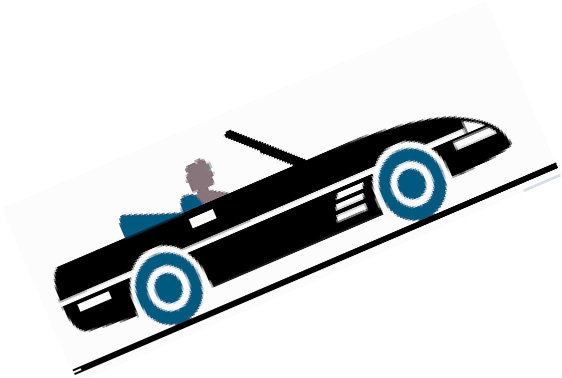
MPM degrades MPC performance



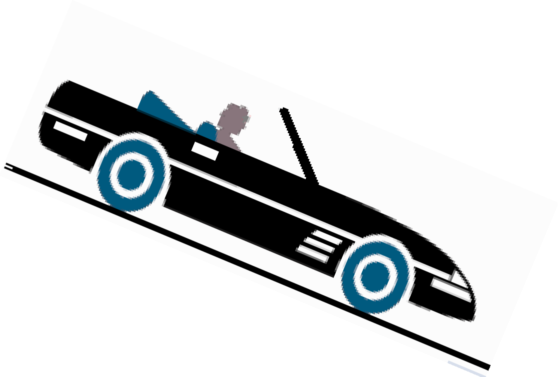
- Assume that G is farthest from G_{10} in terms of properties.
- Then controller is tuned so that system will work when G = G_1 and also G = G_{10}.
- One-size-fits-all policy results in poor performance

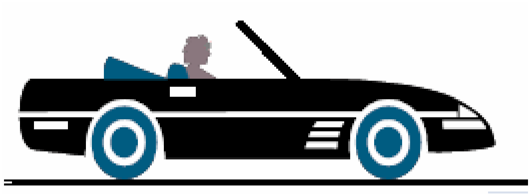
An example – driving a car...

Worst-case scenario: Driving car on mountainous road



Here, we use lower gear

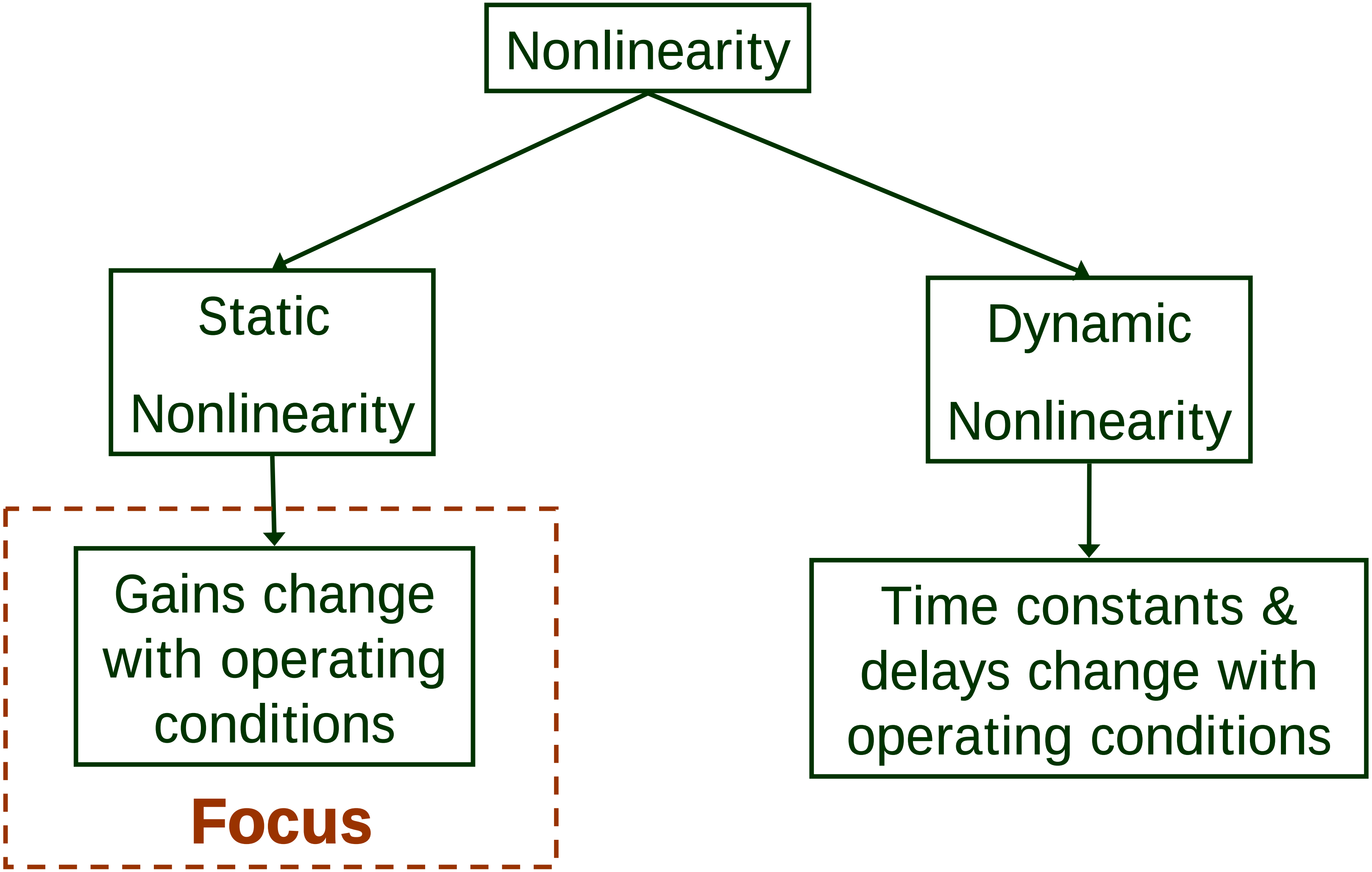




Here, we use higher gear

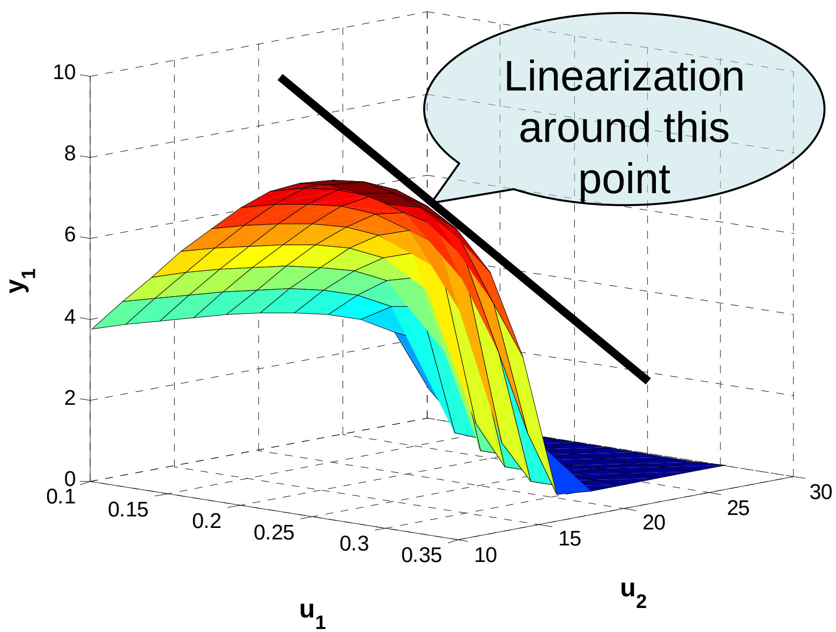
Imagine using lower gear for everyday driving!!!

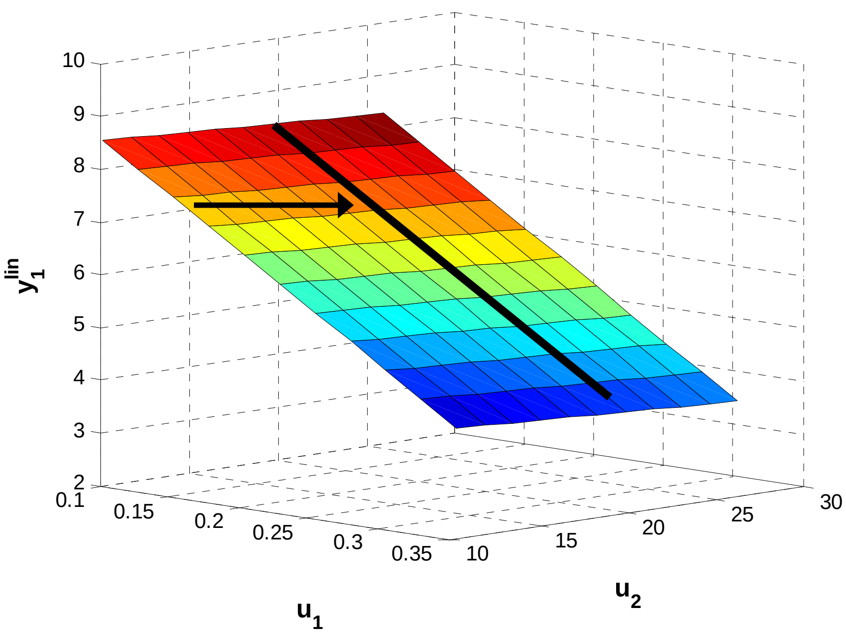
Types of nonlinearity



Static nonlinearity in reactor example

Steady-state behavior of y_1 is significantly nonlinear

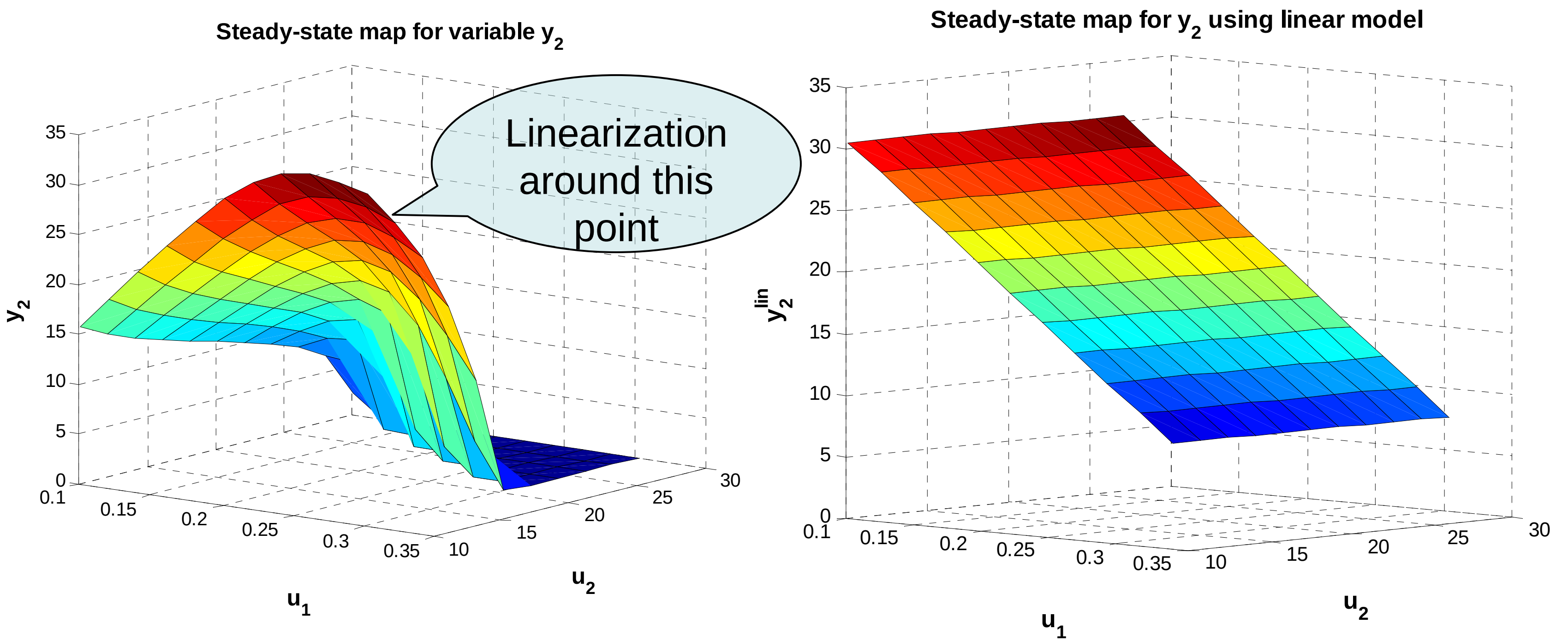




Using linear MPC could have significant effect on performance

Static nonlinearity in reactor example

Steady-state behavior of y_2 is significantly nonlinear



Using linear MPC could have significant effect on performance

Nonlinear Model Predictive Control

- One possibility: Use 1st principles model for predictions in MPC.

$$\frac{dX}{dt} = -DX + \mu(P,S)X$$

$$\frac{dS}{dt} = D(S_f - S) - \frac{1}{Y_{x/s}}\mu(P,S)X$$

$$\frac{dP}{dt} = -DP + (\alpha\mu(P,S) + \beta)X$$

$$\mu(P,S) = \frac{\mu_m\left(1 - \frac{P}{P_m}\right)S}{K_m + S + \frac{S^2}{K_i}}$$

Difficulties

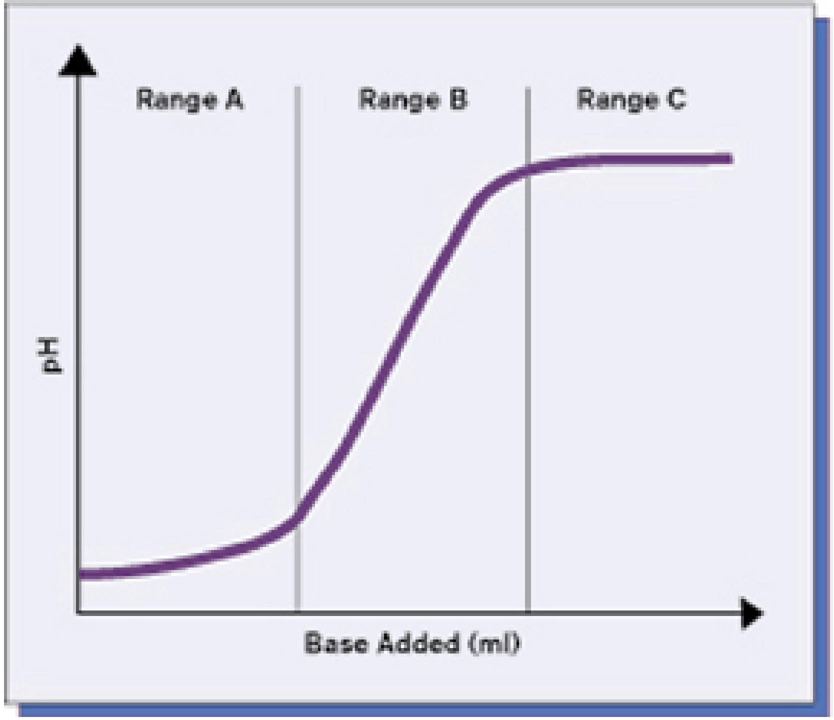
- 1st principles model not known for many processes.
- Parameter estimation not easy.
- Computationally intensive.

Gain Scheduling

- Most feedback control techniques, rely on principle of linearity
  - X% change in MV $\Rightarrow$ Y% change in PV.
  - Ratio or gain between X and Y is fixed.
- Unfortunately, not all processes are linear.
  - However, even non-linear processes can be approximated as linear if X & Y are small.
- Gain Scheduling:
  - Use different controllers in different operating regimes.
  - Determine current operating regime using a *scheduling* variable.
- Controllers which use nonlinear transformations are also doing gain scheduling in some sense.

Gain Scheduling: pH control example

- Consider chemical process where a base is added to a solution to increase its pH.
- Controller with fixed-gain model performs poorly.

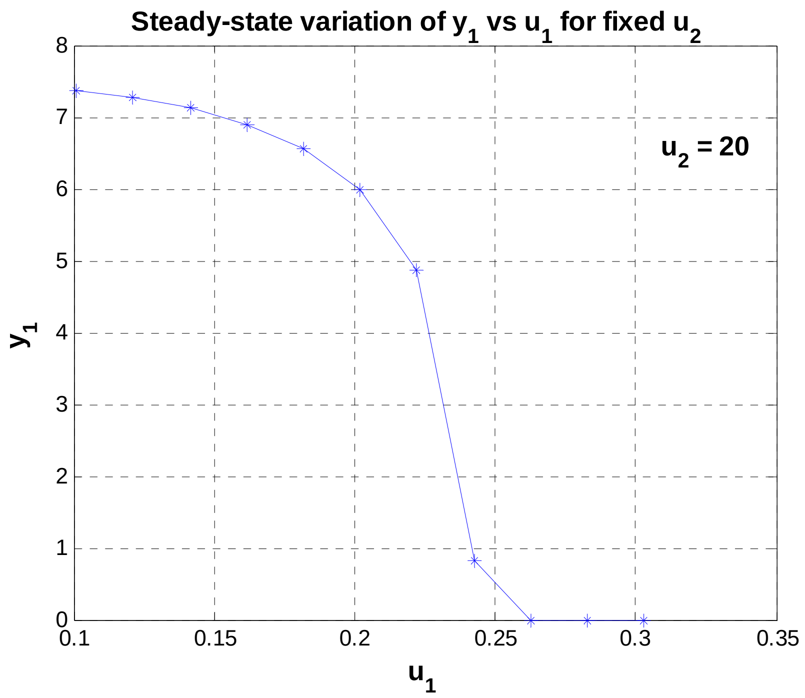


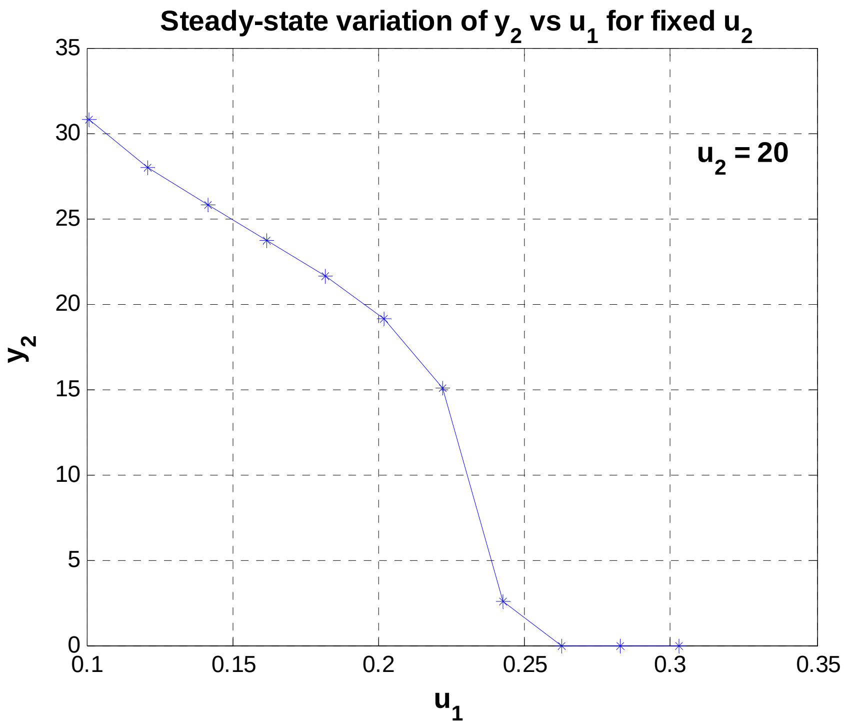
Alternative

- Use base addition as *scheduling* variable.
- Range A or C, gain is low $\Rightarrow$ controller can be aggressive.
- Range B, gain is high $\Rightarrow$ controller should be conservative.

Returning to our nonlinear reactor...

Significant portion of steady state behavior map can be fit using simple polynomial models





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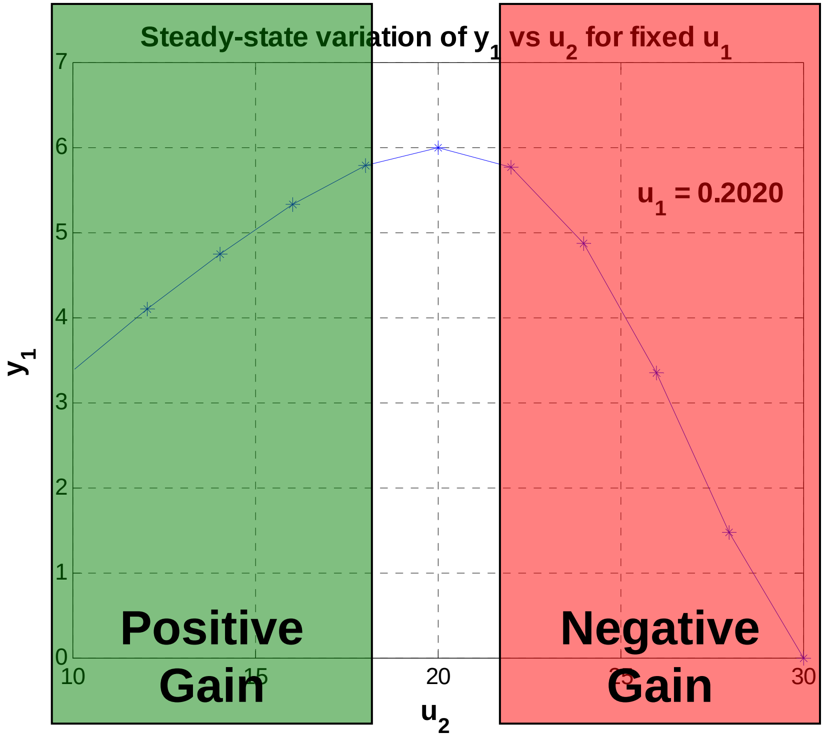
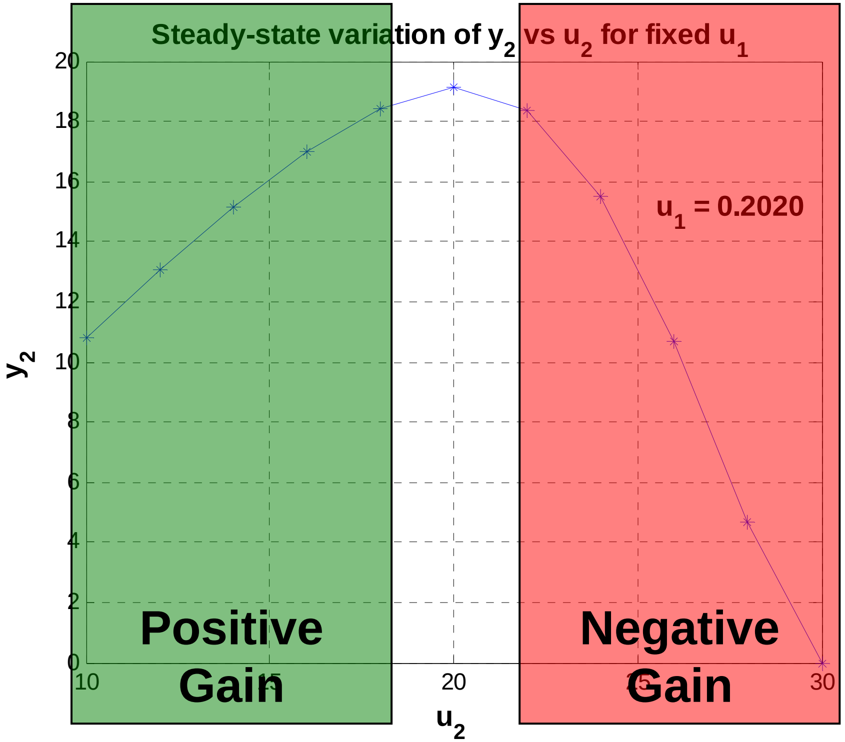




Obtaining steady-state maps of processes

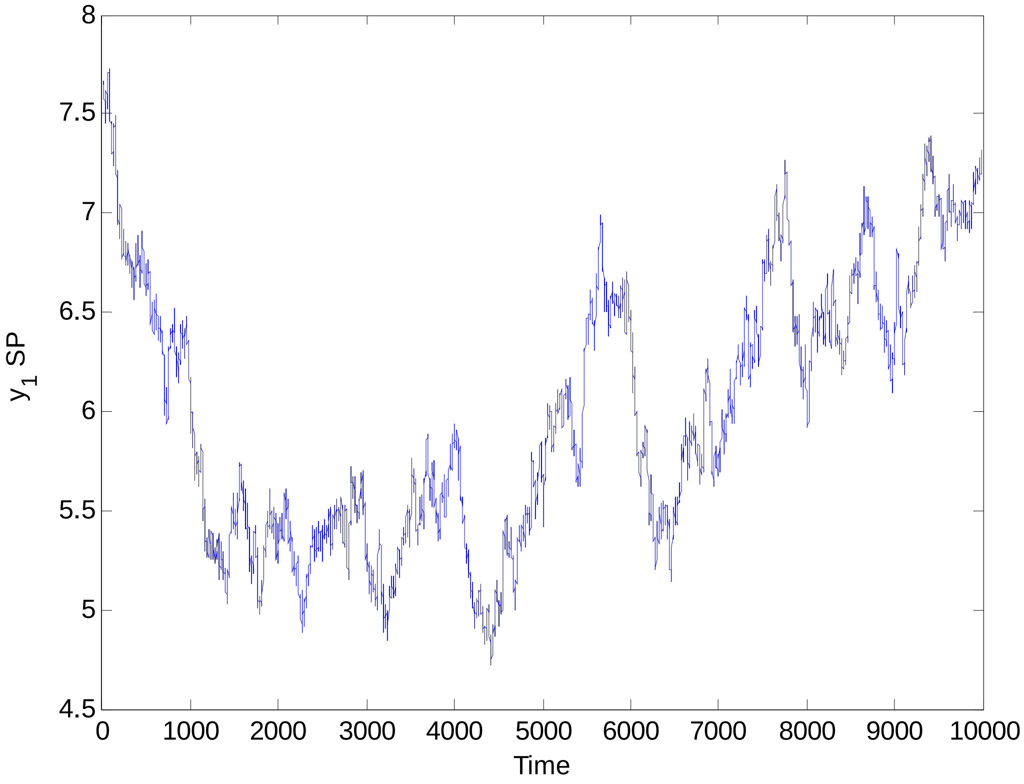
- Problem: Steady-state values of outputs (PVs) for different combinations of input (MV) values are not known beforehand.
- For obtaining the steady-state maps in earlier slides, we have used a 1st principles model.
- However, we can obtain these steady-state maps using historical operating data.
- Proposed Solution: Data-driven approach for obtaining steady-state maps using historical operating data.

Data-driven approach for static nonlinearity

- Step 1: Collect historical data covering various operating conditions for plant.
- Step 2: Perform low-pass filtering of data such that the dynamics of the process are filtered out, by choosing filter settling time greater than process settling times.
- Step 3: Fit simple polynomial models to low-pass filtered data.
- Step 4: Using the coefficients to assess strength of nonlinearity.

Nonlinear reactor simulation

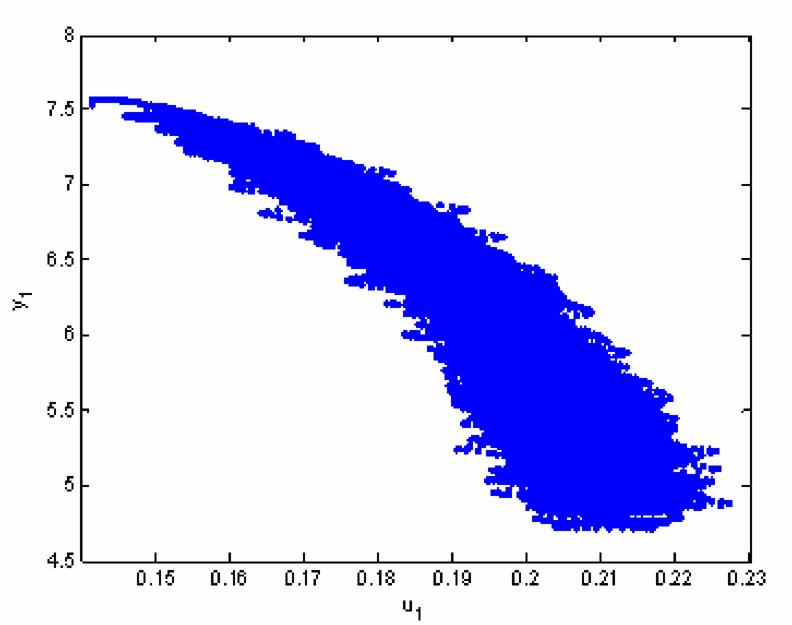
- Step 1: Collecting historical data covering various operating conditions for plant.

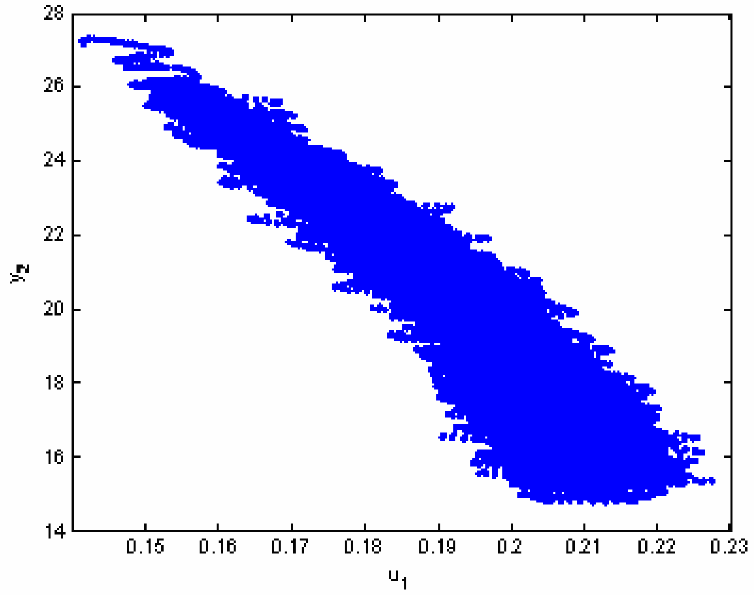


Nonlinear reactor is operated under linear MPC with a reference signal shown, covering plant operating points over long time period.

Univariate scatter plots can be misleading

- Plotting each MV vs. each PV is useful only if all other MVs are held constant.

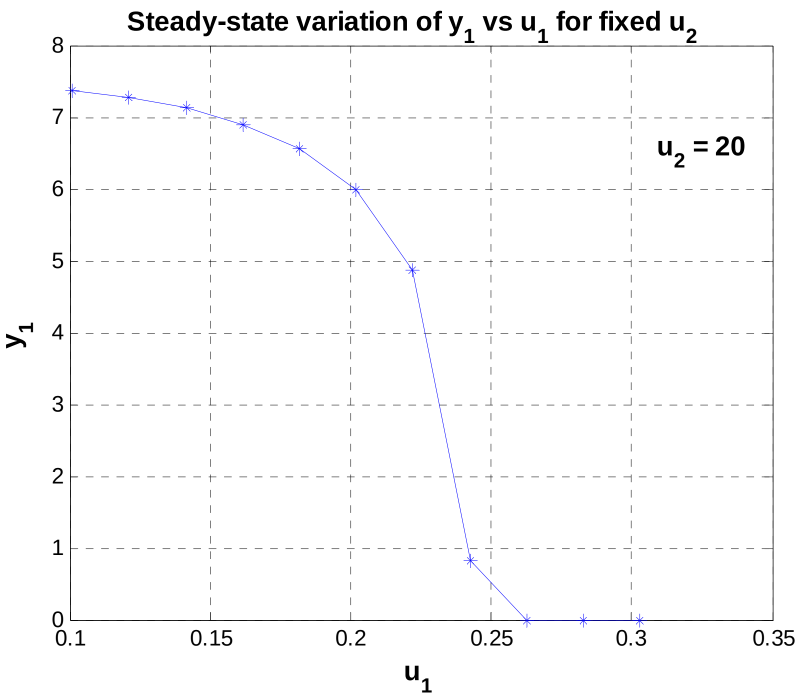


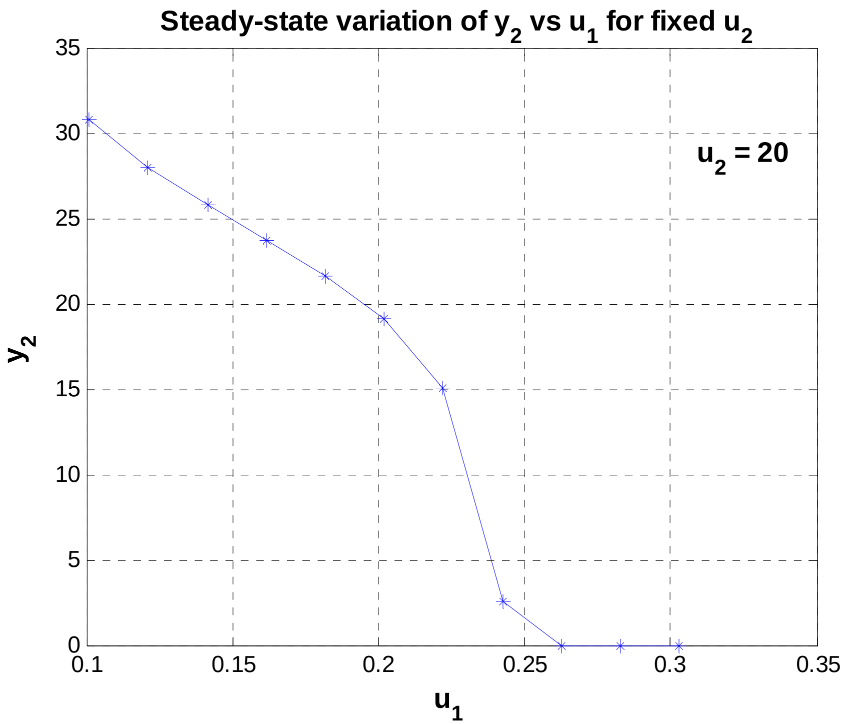


These scatter plots seem to reveal the nonlinearity between y_1 & u_1 and y_2 & u_1

Univariate Scatter Plots...

Scatter plots seem to match top portions of the steady state maps





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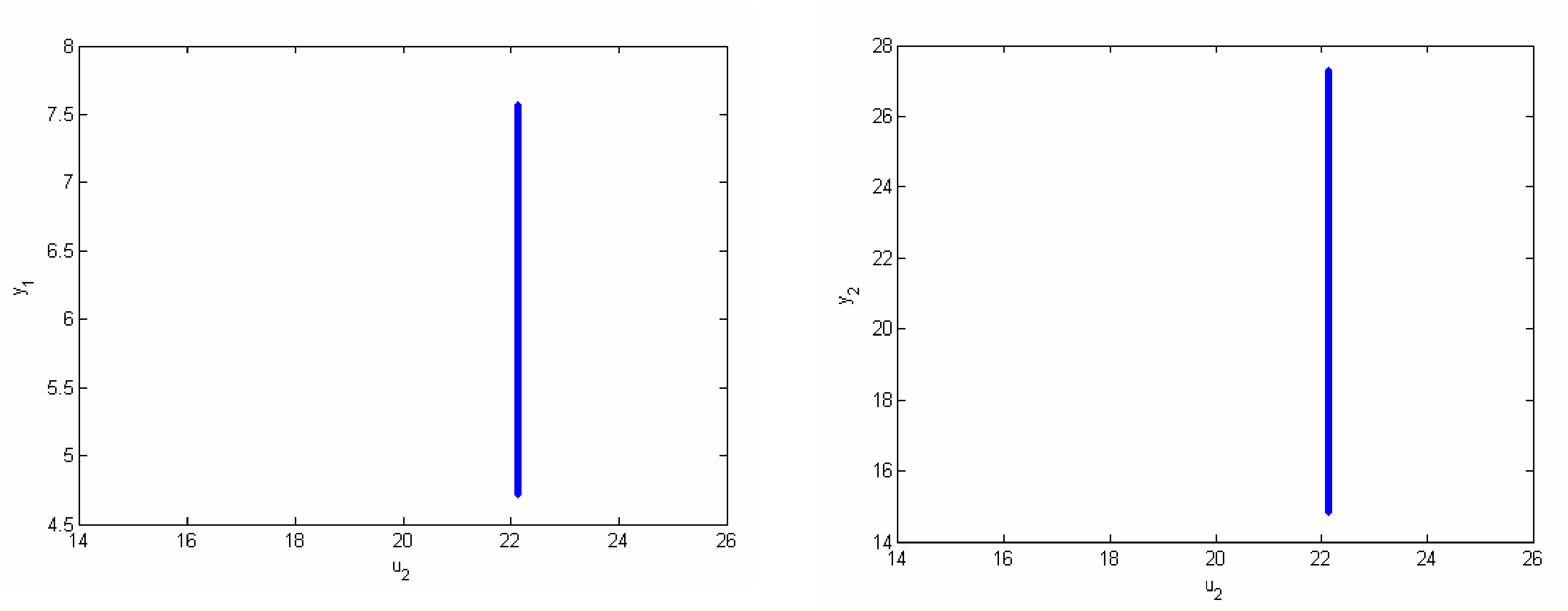
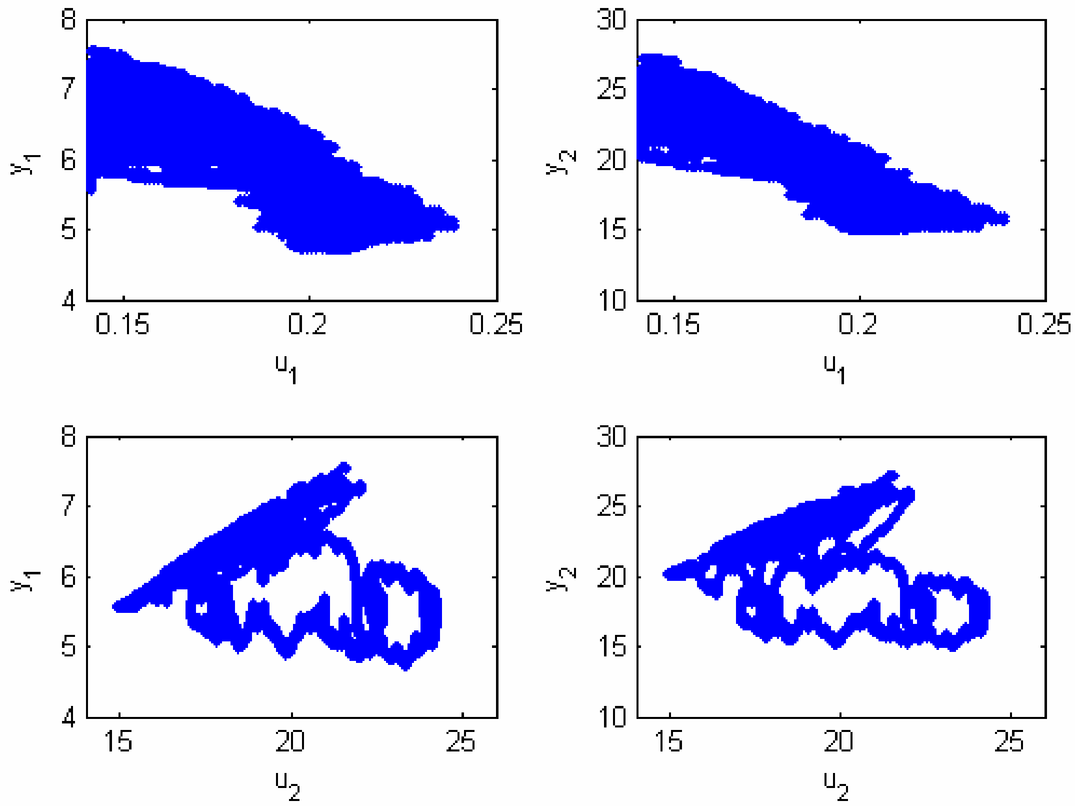


Scatter plots reveal the nonlinearity between y_1 & u_1 and y_2 & u_1 only because u_2 was held constant in this data set.

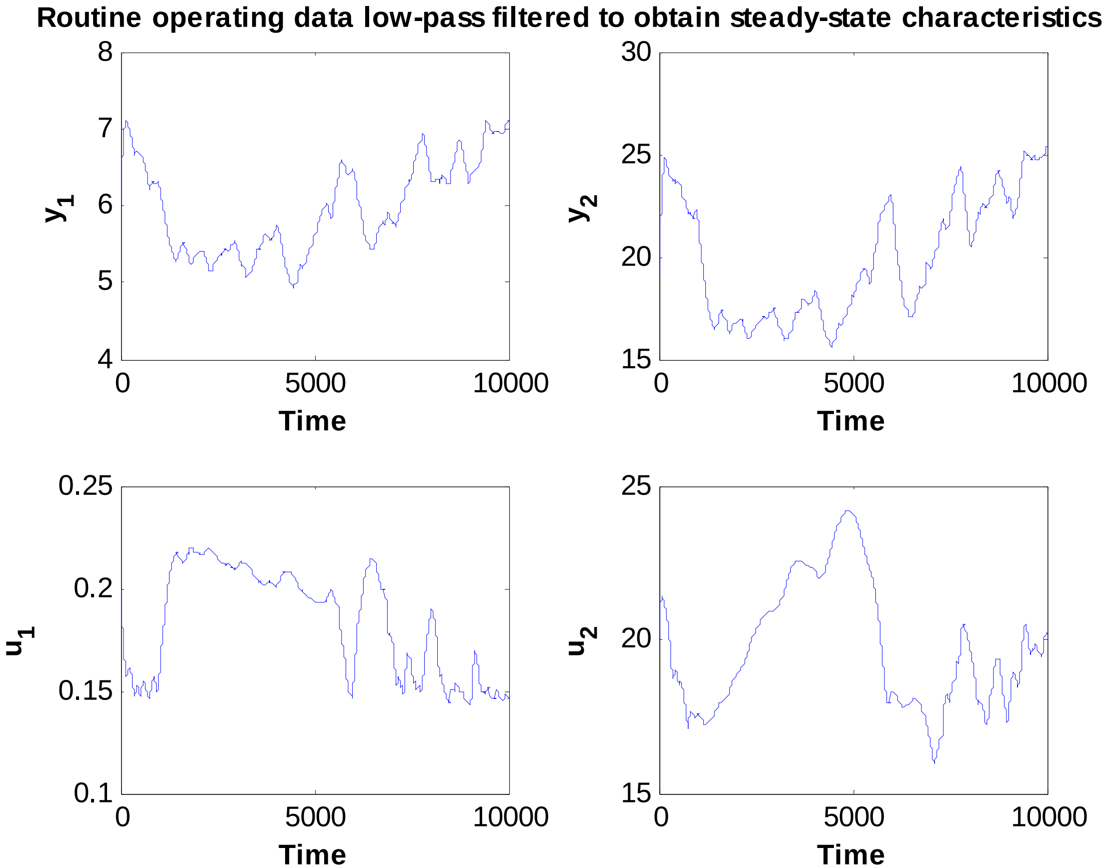
Univariate scatter plots can be misleading

- When all MVs move simultaneously, univariate scatter plots do not reveal nonlinearity properly.



Filtering out the dynamics

- Step2: Dynamics are filtered out using 1st order filters with settling times greater than process settling times.



Fitting simple polynomial models

$$y_1 = \alpha_0 + \alpha_{11}u_1 + \alpha_{21}u_1^2 + \alpha_{12}u_2 + \alpha_{22}u_2^2$$

$$y_2 = \beta_0 + \beta_{11}u_1 + \beta_{21}u_1^2 + \beta_{12}u_2 + \beta_{22}u_2^2$$

$$y_1 = -21.7868 + 87.3577\, u_1 - 296.1109\, u_1^2 + 2.1948\, u_2 - 0.0544\, u_2^2$$

$$y_2 = -58.8581 + 154.3242\, u_1 - 714.8317\, u_1^2 + 7.5110\, u_2 - 0.1860\, u_2^2$$

$$-21.9497 \le \alpha_0 \le -21.6240$$

$$86.3193 \le \alpha_{11} \le 88.3961 \qquad 2.1838 \le \alpha_{12} \le 2.2058$$

$$-298.9593 \le \alpha_{21} \le -293.2625 \qquad -0.0547 \le \alpha_{22} \le -0.0541$$

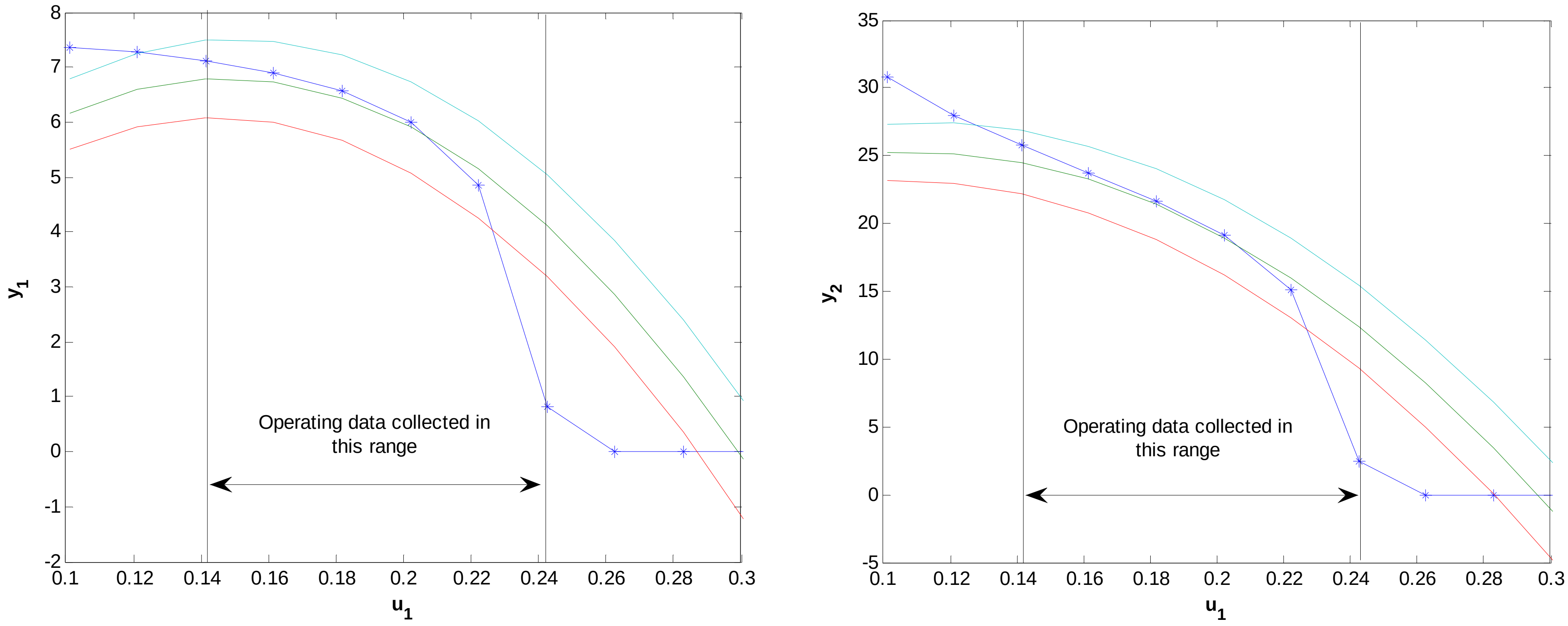
$$-59.4087 \le \beta_0 \le -58.3075$$

$$150.8135 \le \beta_{11} \le 157.8348 \qquad 7.4738 \le \beta_{12} \le 7.5482$$

$$-724.4615 \le \beta_{21} \le -705.2019 \qquad -0.1869 \le \beta_{22} \le -0.1851$$

Fitting simple polynomial models

- Comparison of steady-state behavior of approximate models with actual process shows good fit in operating range of process.



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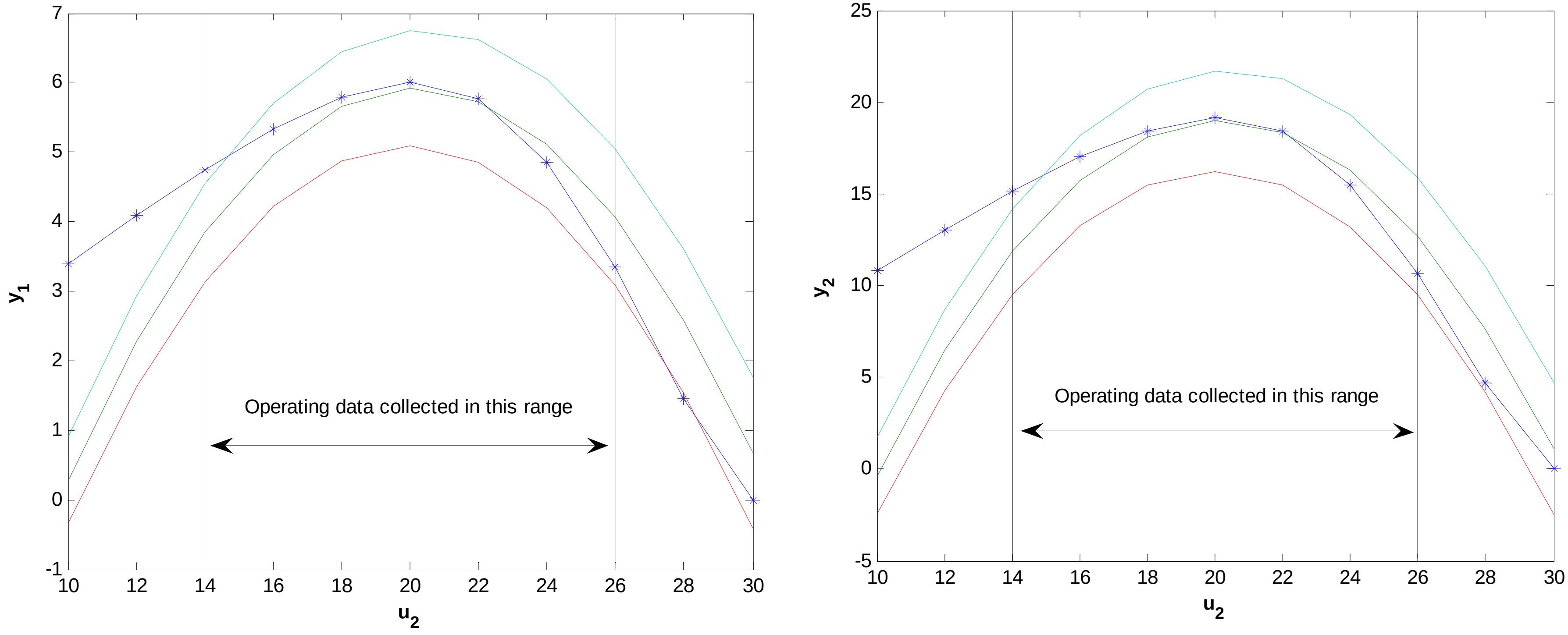
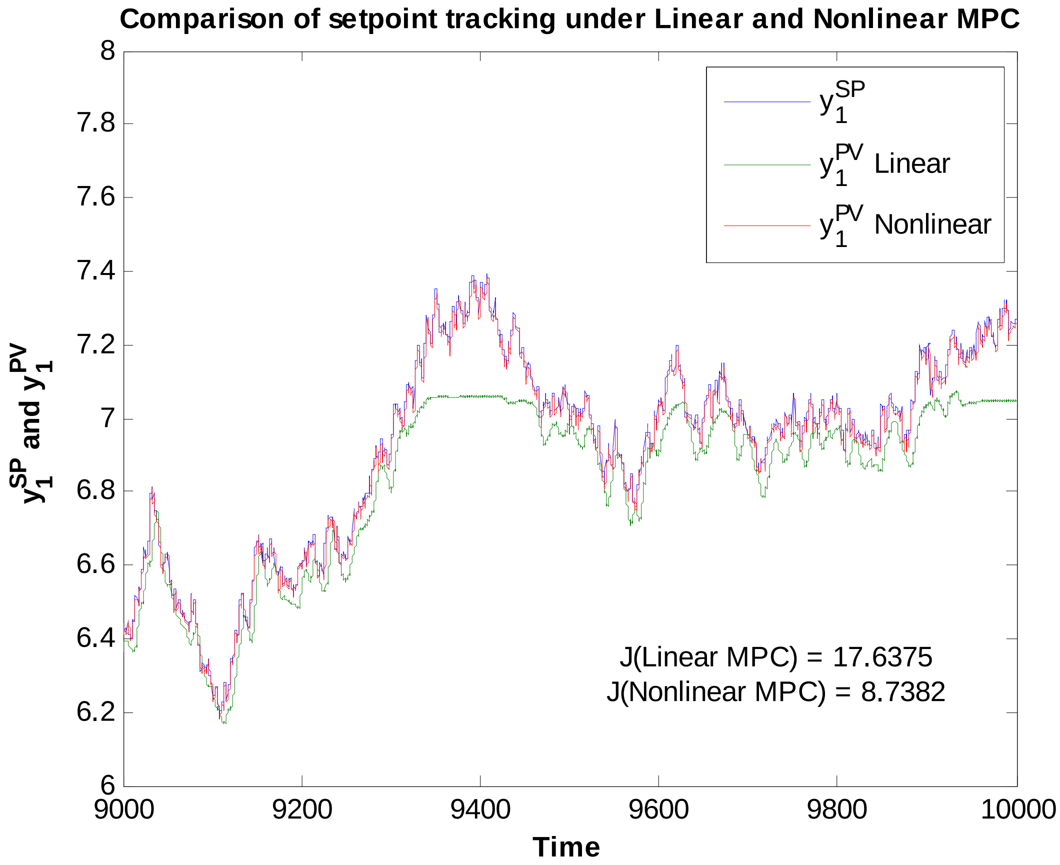


Improved MPC performance

- Nonlinear MPC outperforms Linear MPC in tracking output.



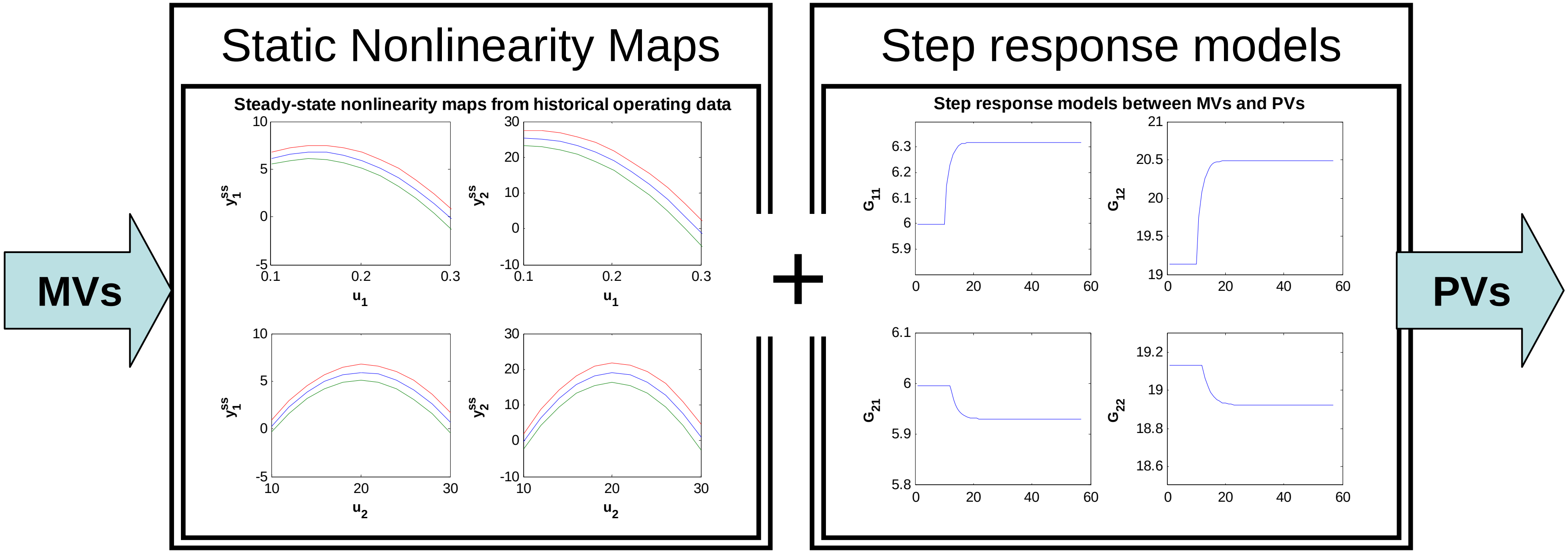
Objective fn values

Linear MPC ≈ 17.64

Nonlinear MPC ≈ 8.74

Summary

- Earlier: Step-response models identification experiments.
- Now: Also get Static Nonlinearity maps from routine operating data.



Concluding Remarks

- Proposed data-driven technique uses historical data to assess gain-nonlinearity in system.
- Technique fits polynomial models to low-pass filtered data and uses coefficients to determine strength of gain nonlinearity.
- Once the nonlinear models are available, they are used in controller for compensation in a gain-scheduling fashion.
- Technique has been demonstrated on nonlinear reactor.
- Future work will focus on approximating the gain surface using piece-wise linear functions.

Acknowledgements

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- CPC Group Members

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