

**PARAMETER AND DELAY ESTIMATION OF
CONTINUOUS-TIME MODELS FROM
IRREGULARLY SAMPLED OUTPUT**Salim Ahmed* , Biao Huang*¹ and Sirish L. Shah*** Department of Chemical and Materials Engineering
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Abstract: Linear filter approach might be the most commonly used method for continuous-time identification. Recently we have proposed a new linear filter method for simultaneous estimation of time delay and other parameters of continuous-time models in (Ahmed *et al.*, 2006). The proposed method involves choice of filter parameters and the filter structure is restricted to all real pole form. In this paper, the linear filter method is presented for a general structure of the filter. Also the filter parameters are updated iteratively. Next, an algorithm is prescribed to make this method applicable when the output is sampled irregularly. To demonstrate the performance of the proposed algorithm, results from a simulation example as well as an experimental example are presented. *Copyright ©2006 IFAC.*

Keywords: Linear filter, time delay, irregular data.

1. INTRODUCTION

Time delay estimation is an important part of system identification. In process control, it is even more important to consider time delay because of the common occurrence of the delay and its significant bearings on the achievable performance of control systems. However, both in continuous-time(CT) and discrete-time(DT) identification, the development of time delay estimation methods lags behind the advancement of the estimation techniques for other model parameters. For example, linear filter methods are commonly used for CT model parameter estimation and a significant development have taken place over the last few decades, see e.g. (Fairman, 1971; Garnier *et al.*, 2003; Saha and Rao, 1983; Wang and Gawthrop, 2001; Young, 2002). In linear filter approach, the most commonly used algorithm to estimate the time delay is based on a comprehensive search routine as used in (Rao and Sivaku-

mar, 1976; Saha and Rao, 1983; Young, 2002) where process parameters are estimated for a set of time delays within a certain range and a predefined cost function is calculated for every set of estimated parameters. Finally the delay that gives the optimum value of the cost function is chosen. This procedure is computationally expensive specially for rapidly sampled data. Other popular approaches are approximation of the delay by a rational transfer function such as the Padé approximation as in Agarwal and Canudas (1987) and the Laguerre expansion or by a polynomial approximation. Such approaches require estimation of more parameters and an unacceptable approximation error may occur for systems having large delay (Wang and Zhang, 2001). Most of the methods to directly estimate the delay along with other model parameters are based on step response data see e.g. (Wang and Zhang, 2001; Ingimundarson and Hagglund, 2001). It is well recognized that a step input may not provide a sufficient excitation.

An important issue related to many of the time

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delay estimation methods is that the delay is expressed in terms of number of sampling interval. The problem arises when the sampling interval is not constant. For such a case, the time delay becomes time varying and most of the methods fail to estimate such a parameter.

A more general problem that prevents many identification algorithms to be applicable in real industrial processes is the irregularity in data. By irregular data we refer to a data set that contains measurements of all variables at some time instants, but at others, measurements of only some variables are available. The unavailable elements are often termed as missing data. In chemical processes, data can be missing for two basic reasons: failures in the measurement devices and errors in data management. Sometimes data may be missing because of the strategy of sampling. For example, from a time and cost consideration, concentrations are less frequently measured than temperatures and pressures. For such multi-rate data the intermediate values of the slow sampled variables may be considered as missing. In some cases, where it requires time consuming lab analysis, frequency of measurements may be irregular (Imtiaz *et al.*, 2004).

In this paper, the linear filter method proposed in (Ahmed *et al.*, 2006) is presented for a more general filter structure. Also in this approach the filter parameters are updated iteratively within the iteration steps for time delay estimation. Thus it does not need any additional step in the algorithm and the user needs to specify only the initial values of the filter parameters. The final estimates of the process parameters are found to have little or no effect on the initial choice of filter parameters. For identification from irregular data we prescribe a procedure based on the idea of iterative prediction.

2. THE LINEAR FILTER METHOD

Let us consider a linear single input single output (SISO) system with time delay described by

$$\mathbf{a}_n \mathbf{y}^{(n)}(t) = \mathbf{b}_m \mathbf{u}^{(m)}(t - \delta) + e(t) \quad (1)$$

where,

$$\mathbf{a}_n = [a_n \ a_{n-1} \ \dots \ a_0] \in \mathbb{R}^{1 \times (n+1)}$$

$$\mathbf{b}_m = [b_m \ b_{m-1} \ \dots \ b_0] \in \mathbb{R}^{1 \times (m+1)}$$

$$\mathbf{y}^{(n)}(t) = [y^{(n)}(t) \ y^{(n-1)}(t) \ \dots \ y^{(0)}(t)]^T$$

$$\mathbf{u}^{(m)}(t - \delta) = [u^{(m)}(t - \delta) \ \dots \ u^{(0)}(t - \delta)]^T$$

$y^{(i)}$ and $u^{(i)}$ are i^{th} order time derivatives of y and u and $e(t)$ is the error term. Taking Laplace

transformation on both sides of eqn(1), considering that both input and output are initially at rest, we can write

$$\mathbf{a}_n \mathbf{s}^n Y(s) = \mathbf{b}_m \mathbf{s}^m U(s) e^{-\delta s} + E(s) \quad (2)$$

$Y(s)$, $U(s)$ and $E(s)$ are the Laplace transforms of $y(t)$, $u(t)$ and $e(t)$, respectively, and

$$\mathbf{s}^n = [s^n \ s^{n-1} \ \dots \ s^0]^T \in \mathbb{R}^{(n+1) \times 1} \quad (3)$$

Now, consider a causal filter described in Laplace domain as $F(s)$. If we apply the filtering operation on both sides of eqn(2) we end up with the formulation

$$\mathbf{a}_n \mathbf{s}^n F(s) Y(s) = \mathbf{b}_m \mathbf{s}^m F(s) U(s) e^{-\delta s} + F(s) E(s) \quad (4)$$

To estimate the time delay along with other parameters, in (Ahmed *et al.*, 2006) a filter of the form $F(s) = \frac{\beta^n}{s(s+\lambda)^n}$ is proposed where the parameters λ and β are to be specified by the user. Here, we propose a filter having a first order integral dynamics along with a lag dynamics which is the denominator of the process transfer function i.e.

$$F(s) = \frac{1}{sA(s)} \quad (5)$$

where, $A(s) = \mathbf{a}_n \mathbf{s}^n$ is the denominator of the process transfer function. The objective of using such a filter structure is to have the delay, δ , as an element of the parameter vector. The integrator generates an integration term of delayed input. This integrated delayed input signal, which represent a certain area under the input curve, can be expressed by subtracting two sub-areas from the integrated input signal. By doing so, δ becomes an explicit parameter in the estimation equation. To describe the necessary mathematical formulation let us define $\underline{Y}(s)$ as

$$\underline{Y}(s) = \frac{Y(s)}{A(s)} \quad (6)$$

By defining $\underline{U}(s)$ in the same way as $\underline{Y}(s)$ is defined in eqn(6), we can express eqn(4) as

$$\mathbf{a}_n \mathbf{s}_+^{n-1} \underline{Y}(s) = \mathbf{b}_m \mathbf{s}_+^{m-1} \underline{U}(s) e^{-\delta s} + \xi(s) \quad (7)$$

where, the subscript $(\bullet)_+$ means that the \mathbf{s}^{n-1} vector has been augmented by $\frac{1}{s}$, i.e.,

$$\mathbf{s}_+^{n-1} = \left[s^{n-1} \ s^{n-2} \ \dots \ s^0 \ \frac{1}{s} \right] \quad (8)$$

Now using partial fraction expansion the transfer function of the filter, $1/sA(s)$, can be expressed as

$$\frac{1}{sA(s)} = \frac{C(s)}{A(s)} + \frac{1}{s} \quad (9)$$

where, $C(s) = -(a_n s^{n-1} + a_{n-1} s^{n-2} + \dots + a_1)$. Eqn(9) can be used to represent the filtered input as

$$\begin{aligned} U_F(s) &= \frac{C(s)}{A(s)} U(s) + \frac{1}{s} U(s) \\ &= C(s) \underline{U}(s) + U_I(s) \end{aligned} \quad (10)$$

Applying eqn(9) and (10), we can restructure eqn(7) to give a standard least-square form

$$\begin{aligned} \underline{Y}_I(s) = & -\bar{\mathbf{a}}_n \mathbf{s}^{n-1} \underline{Y}(s) + \bar{\mathbf{b}}_m \mathbf{s}^{m-1} \underline{U}(s) e^{-\delta s} \\ & + b_0 [C(s) \underline{U}(s) + U_I(s)] e^{-\delta s} + \zeta(s) \end{aligned} \quad (11)$$

where,

$\bar{\mathbf{a}}_n$: \mathbf{a}_n with its last column removed, $\bar{\mathbf{a}}_n \in \mathbb{R}^{1 \times n}$
 $\bar{\mathbf{b}}_m$: \mathbf{b}_m with its last column removed, $\bar{\mathbf{b}}_m \in \mathbb{R}^{1 \times m}$

Taking inverse Laplace Transform, we get the time domain expression for any time $t = t_k$

$$\begin{aligned} \underline{y}_I(t_k) = & -\bar{\mathbf{a}}_n \underline{\mathbf{y}}^{n-1}(t_k) + \bar{\mathbf{b}}_m \underline{\mathbf{u}}^{m-1}(t_k - \delta) \\ & + b_0 [\underline{u}_c(t_k - \delta) + u_I(t_k - \delta)] + \zeta(t_k) \end{aligned} \quad (12)$$

with

$$\begin{aligned} \underline{y}_I(t_k) &= L^{-1} \left[\frac{\underline{Y}(s)}{s} \right] \\ \underline{u}_c(t_k - \delta) &= L^{-1} [C(s) \underline{U}(s) e^{-\delta s}] \\ u_I(t_k - \delta) &= L^{-1} \left[\frac{1}{s} U(s) e^{-\delta s} \right] \end{aligned}$$

At any time $t = t_k$, the integrated input and the integrated delayed input can be expressed as

$$\begin{aligned} u_I(t_k) &= \int_0^{t_k} u(t) dt \quad (13) \\ u_I(t_k - \delta) &= \int_0^{t_k} u(t) dt - \int_{t_k - \delta}^{t_k} [u(t) - u(t_k)] dt \\ &\quad - u(t_k) \delta \end{aligned} \quad (14)$$

Applying eqn(14) in eqn(12) and rearranging it to give a standard least square form we get

$$\begin{aligned} \underline{y}_I(t_k) = & -\bar{\mathbf{a}}_n \underline{\mathbf{y}}^{n-1}(t_k) + \mathbf{b}_m \underline{\mathbf{u}}_+^{m-1}(t_k - \delta) \\ & + b_0 u(t_k) \delta + \zeta(t_k) \end{aligned} \quad (15)$$

where,

$$\begin{aligned} \underline{\mathbf{u}}_+^{m-1}(t_k - \delta) &= \begin{bmatrix} \underline{u}^{m-1}(t_k - \delta) \\ \vdots \\ \underline{u}(t_k - \delta) \\ u_+(t_k - \delta) \end{bmatrix} \\ u_+(t_k - \delta) &= \underline{u}_c(t_k - \delta) + u_I(t_k) \\ &\quad - \int_{t_k - \delta}^{t_k} [u(t) - u(t_k)] dt \end{aligned}$$

Or equivalently

$$\gamma(t_k) = \phi^T(t_k) \theta + \zeta(t_k) \quad (16)$$

where,

$$\begin{aligned} \gamma(t_k) &= \underline{y}_I(t_k) \\ \phi(t_k) &= \begin{bmatrix} -\underline{\mathbf{y}}^{n-1}(t_k) \\ \underline{\mathbf{u}}_+^{m-1}(t_k - \delta) \\ u(t_k) \end{bmatrix} \\ \theta &= [\bar{\mathbf{a}}_n \quad \mathbf{b}^m \quad b_0 \delta]^T \end{aligned}$$

Eqn(16) can be written for $t_k = t_{d+1}, t_{d+2} \dots t_N$ where $t_d \leq \delta < t_d + 1$ and be combined to give

$$\mathbf{\Gamma} = \mathbf{\Phi} \theta + \zeta \quad (17)$$

From θ we can directly get $\bar{\mathbf{a}}_n$ and \mathbf{b}_m . δ is obtained as $\delta = \theta(n + m + 2) / \theta(n + m + 1)$. But

to estimate θ solving eqn(17), we need to know $A(s)$ and δ , which are of course unknowns. This problem can be solved by applying an iterative procedure that adaptively adjust an initial choice of $A(s)$ and δ until they converge. The least-square estimate of θ that minimizes the sum of the squared errors is given by

$$\hat{\theta}^{LS} = [\mathbf{\Phi}^T \mathbf{\Phi}]^{-1} \mathbf{\Phi}^T \mathbf{\Gamma} \quad (18)$$

However, the least-square solution does not give unbiased estimate in the presence of general forms of measurement noise such as colored noise. A popular bias elimination procedure is to use the instrumental variable (IV) method. A bootstrap estimation of IV type where the instrumental variable is built from an auxiliary model (Young, 1970) is considered here. The instrumental variable is defined as

$$\psi(t_k) = \begin{bmatrix} -\hat{\mathbf{x}}^{n-1}(t_k) \\ \underline{\mathbf{u}}_+^{m-1}(t_k - \delta) \\ u(t_k) \end{bmatrix} \quad (19)$$

where $\hat{x}(t) = \hat{G}(p, \hat{\theta}^{LS}) u(t - \delta)$ and $\hat{G}(p, \hat{\theta}^{LS})$ is the process model estimated from least-square solution. The IV-based bias-eliminated parameters are given by

$$\hat{\theta}^{IV} = [\mathbf{\Psi}^T \mathbf{\Phi}]^{-1} \mathbf{\Psi}^T \mathbf{\Gamma} \quad (20)$$

The IV estimate can also be calculated in a recursive or recursive/iterative manner. The iterative iteration procedure is summarized in *Algorithm 1*.

Algorithm 1: Iterative procedure for parameter and delay estimation.

Step 1: Initialization Choose an initial estimate $A_0(s)$ and δ_0 .

Step 2: LS step $i = 1$ Construct $\mathbf{\Gamma}$ and $\mathbf{\Phi}$ by replacing $A(s)$ and δ by $A_0(s)$ and δ_0 and get the LS solution of θ as

$$\hat{\theta}^{LS} = (\mathbf{\Phi}^T \mathbf{\Phi})^{-1} \mathbf{\Phi}^T \mathbf{\Gamma} \quad (21)$$

$\hat{\theta}^1 = \hat{\theta}^{LS}$. Get $\hat{A}_1(s)$, the process numerator $\hat{B}_1(s)$ and $\hat{\delta}_1$ from $\hat{\theta}^1$.

Step 3: IV step $i = i + 1$. Construct $\mathbf{\Gamma}$, $\mathbf{\Phi}$ and $\mathbf{\Psi}$ by replacing $A(s)$, $B(s)$ and δ by $\hat{A}_{i-1}(s)$, $\hat{B}_{i-1}(s)$ and $\hat{\delta}_{i-1}$ and get the IV solution of θ as

$$\hat{\theta}^i = (\mathbf{\Psi}^T \mathbf{\Phi})^{-1} \mathbf{\Psi}^T \mathbf{\Gamma} \quad (22)$$

Obtain $\hat{A}_i(s)$, $\hat{B}_i(s)$ and $\hat{\delta}_i$ from $\hat{\theta}^i$ and repeat **step 3** until \hat{A}_i and $\hat{\delta}_i$ converge.

Step 4: Termination When \hat{A}_i and $\hat{\delta}_i$ converge, the corresponding $\hat{\theta}^i$ is the final estimate of parameters.

Though there is no theoretical proof available, extensive simulation studies show that the iterative procedure converges monotonically except for non-minimum phase (NMP) processes. However, for NMP processes it exhibits monotonic divergence. Based on this, for such processes, we suggest an *ad hoc* procedure that defines $\Delta\delta = \delta_{i-1} - \delta_i$ and in the $(i + 1)$ -th stage of iteration the guessed value is taken as $\delta_i + \Delta\delta$. The iteration steps otherwise remain the same.

3. IDENTIFICATION FROM IRREGULAR DATA

3.1 The Algorithm

For identification with irregular data, we propose an algorithm based on iterative prediction. However, it is not possible to develop a single algorithm that can deal with every types of data irregularity. We will consider here a specific type of irregular data where the input is available at all time instants but not necessarily in regular interval while the output is available at some time instants and missing at others. As the initialization of the iterative procedure, a so called input only method is applied. A distinguished feature of these methods is that the current output is expressed in terms of only current and previous input. So the parameter estimation equation can be formulated only at the time instants when output is available. The estimated model is then used to predict the missing values to get a complete data set. Next, this complete data set is used to estimate the parameters of the continuous time model using the procedure described in section 2. This model is then used to predict the missing outputs. This procedure is carried on iteratively by replacing the prediction from previous model by that from the current one until some convergence criteria are met. The iteration algorithm is presented in *Algorithm 2* and the different steps of the iteration procedure are detailed below.

3.2 Input only modeling

A number of input only approaches, both in discrete-time and continuous-time, are available in literature. In this work, we adopt one of the orthogonal basis function approach, the Laguerre polynomial model in continuous time for the initial prediction. The use of Laguerre functions in identification goes back to Wiener (1956). In the transform domain, the Laguerre functions are given as (Lee, 1932)

$$L_j(s) = \sqrt{2p} \frac{(s-p)^j}{(s+p)^{j+1}} \quad (23)$$

Let $z_j(t)$ be the output of the j -th Laguerre function, with $u(t)$ as input, i.e.

$$Z_j(s) = L_j(s)U(s) \quad (24)$$

where, $Z_j(s)$ and $U(s)$ represent the Laplace transform of $z_j(t)$ and $u(t)$, respectively. The output of a stable plant with input $u(t)$ can be approximated by a truncated l -th order Laguerre polynomial model as

$$y(t) = \sum_{j=0}^l \alpha_j z_j(t) \quad (25)$$

where, $\alpha = [\alpha_0, \alpha_1 \dots \alpha_l]^T$, is the parameter vector for the Laguerre model. Theories and proofs

of the convergence of the Laguerre model can be found in (Makila, 1990; Parington, 1991; Wang and Cluett, 1995). In the initial prediction stage using Laguerre polynomial model, the estimation equation (eqn(25)) is formulated only at the time instants when the output is available i.e.,

$$y(t_i^{obs}) = \sum_{j=0}^l \alpha_j z_j(t_i^{obs}) \quad (26)$$

Next, eqn(26) can be formulated for t_i^{obs} with $i = 1, 2 \dots M$, where, M is the number of available output, to give an equation in least square form as

$$\mathbf{Y}_{obs} = \mathbf{Z}_{obs} \alpha \quad (27)$$

where,

$$\mathbf{Y}_{obs} = [y(t_1^{obs}) \ y(t_2^{obs}) \ \dots \ y(t_M^{obs})]^T \quad (28)$$

$$\mathbf{Z}_{obs} = \begin{bmatrix} z_0(t_1^{obs}) & z_1(t_1^{obs}) & \dots & z_l(t_1^{obs}) \\ z_0(t_2^{obs}) & z_1(t_2^{obs}) & \dots & z_l(t_2^{obs}) \\ \dots & \dots & \dots & \dots \\ z_0(t_M^{obs}) & z_1(t_M^{obs}) & \dots & z_l(t_M^{obs}) \end{bmatrix} \quad (29)$$

Finally the parameter vector can be estimated as

$$\hat{\alpha} = (\mathbf{Z}_{obs}^T \mathbf{Z}_{obs})^{-1} \mathbf{Z}_{obs}^T \mathbf{Y}_{obs} \quad (30)$$

Now, the missing elements of the output can be obtained as

$$\hat{y}_i^{mis} = \sum_{j=0}^l \hat{\alpha}_j z_j(t_i^{mis}) \quad (31)$$

The estimated value of the missing elements can then be inserted into the output vector to get a complete data set.

3.3 Criterion for convergence

The iterative procedure described here is based on the idea of iterative prediction. Consequently, a natural option for criterion of convergence is the prediction error. As the output has missing elements, we can define the mean squared error at i -th stage of iteration as

$$MSE_i^{obs} = \frac{1}{M} \sum_{k=1}^M [y(t_k^{obs}) - \hat{y}_i(t_k^{obs})]^2 \quad (32)$$

where, \hat{y}_i is the prediction of the model obtained in the i -th stage of iteration. Convergence of this MSE criterion is equivalent to the convergence of the model prediction and the model parameters.

4. SIMULATION RESULTS

To demonstrate the applicability of the proposed methods, we consider here a second order process having the following transfer function

$$G(s) = \frac{-4s + 1}{9s^2 + 2.4s + 1} e^{-0.615s} \quad (33)$$

A complete data set of 2000 samples is generated using a random binary signal (RBS) as input with

Algorithm 2: Algorithm for parameter estimation from irregular data.

Step 1: Initial Prediction Using only the observed output, estimate the parameters of the input only model using eqn(30). Predict the missing element of the output using eqn(31). Use these predicted values, \hat{y}_{mis}^0 to replace y_{mis} . $i = 0$.

Step 2: Iterative Prediction $i = i + 1$. Using *Algorithm1* Estimate the continuous time model parameters with the complete data set $y = [y_{obs} \ \hat{y}_{mis}^{i-1}]$. Use the estimated model, θ^i , to get the i -th prediction of the missing values, \hat{y}_{mis}^i . Replace \hat{y}_{mis}^{i-1} by \hat{y}_{mis}^i

Step 3: Comparison Compare MSE_{obs}^i with MSE_{obs}^{i-1} . If there is significant improvement, go back to step 2 and repeat the iteration.

Step 4: Termination When MSE_{obs}^i converges, the corresponding θ^i is the final estimate of parameters.

a uniform sampling interval of 30 milliseconds (ms). Discrete time white noise is added to get the noisy output signal. The signal to noise ratio (NSR) is 10%. Table 1 summarizes the parameter estimation results for 100 Monte Carlo simulations (MCS) when all the data are available.

Table 1. Estimation results when all data are available

Parameter	True value	Estimated parameters	
		Mean	Variance
a_2	9.00	9.0068	0.0387
a_1	2.40	2.4309	0.0465
b_1	-4.00	-4.0201	0.0570
b_0	1.00	1.0109	0.0068
δ	0.615	0.6302	0.0253

Next, to test the performance of the algorithm proposed for irregular data, we generate three sets of irregular data that differ in terms of their amount of data missing. Every 3rd samples are taken out to generate a data set for 33% missing data, every 2nd for 50% missing and every 2nd and 3rd for 67%. The model estimated using the iterative algorithm is compared with the model estimated using only the available data i.e. data at the time instants when both input and output are available. To compare different models with a single index we define a total error criterion that is measure of bias and variance together. We denote it by E_{total} where

$$E_{total} = \frac{1}{N_{\theta}} \sum_{i=1}^{N_{\theta}} \frac{(\hat{\theta}_i - \theta_i)^2 + \text{var}(\hat{\theta}_i)}{\theta_i^2} \quad (34)$$

θ_i is the true values of the i^{th} parameter and $\hat{\theta}_i$ is its estimated value. N_{θ} is the number of parameters. Figure 1 shows the total error for 100 MCS study. The estimated value is the mean of 100 estimates. The error corresponding to 0% missing data refers to the model estimated using the entire data set and can be taken as a benchmark. When 33% of the data are missing, the model estimated using only the available data has error comparable with the benchmark value and the iterative algorithm has little room to improve.

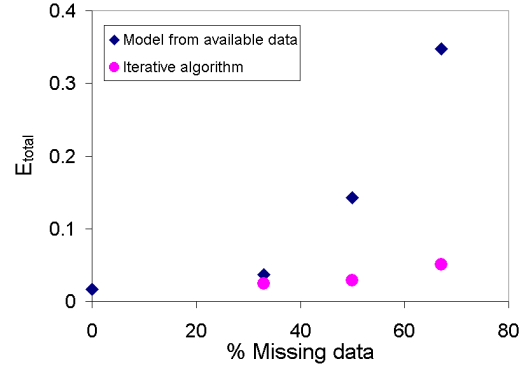


Fig. 1. Improvement of model quality using the iterative algorithm for the simulation example

This suggests that the available data are enough to give a good model. Consequently the error level of the model estimated using the iterative algorithm remains almost the same. However, when more data are missing the error corresponding to the model estimated using the available data is much higher than the benchmark value and the iterative algorithm reduces the error to a level comparable with the benchmark.

5. EXPERIMENTAL EVALUATION

The iterative prediction algorithm is evaluated using an experimental data set from a mixing process. The set-up consists of a continuous stirred tank used as a mixing chamber having two input streams fed from two tanks. A salt solution and pure water run from the feed tanks and mixed together in the mixing chamber. The output is the concentration of salt in the tank. The input is that in the feed. The electrical conductivity of the solutions are used as measure of their concentration. The volume of liquid in the stirred tank and its temperature are maintained at a constant level. The concentration of the feed is manipulated by adjusting the ratio of the flow rate of the feed salt solution and the flow rate of water. The total input flow i.e., the combined salt solution inlet and water inlet is kept constant which ensures a constant outlet flow. The input signal is a random binary signal. The sampling period is 20 seconds. A total of 955 data points are used for this study. To study the effect of % data missing and evaluate the performance of the iterative prediction algorithm, missing data were chosen on a random basis and the algorithm was applied. The study is carried out for 30%, 50% and 70% missing data. To generate a certain data set, say with 30% of its elements missing, 30% of the available output data are taken out on a random basis. The identification algorithm is then applied with the remaining 70% data points. The same procedure is applied 100 times with a different data set chosen each

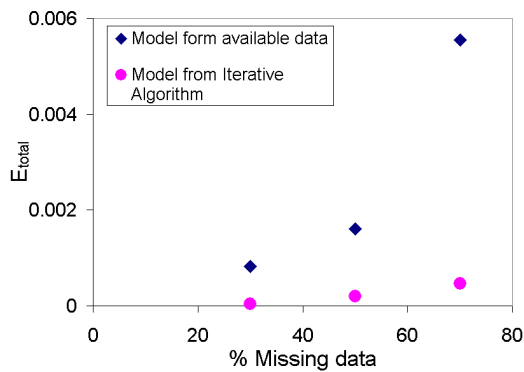


Fig. 2. Improvement of model quality using the iterative algorithm for the mixing process.

time containing 70% of the total data. Finally we get 100 estimates of the parameters. The total error is then calculated from the estimated mean and variance of the 100 estimates. To calculate the bias error, the model estimated using the entire data set is taken as the nominal or true value. Figure 2 shows the performance of the proposed iterative algorithm for the mixing process data. While the error levels for models estimated only the available data points are high, the iterative algorithms gives final estimates of the parameters with a much lower levels of error.

6. CONCLUSION

Identification from irregular data has been considered in discrete-time identification but mainly for multi-rate data. In continuous-time identification, it is assumed that the methods are capable of dealing with irregular data by nature. However, the inherent assumption of inter-sample behavior resulting in certain arbitrary interpolation introduces errors in the estimation of continuous-time parameters. In this paper, a simple algorithm is presented to deal with irregular output sampling. It has been demonstrated both using simulated and experimental data that the quality of the model estimated using the proposed model based prediction algorithm is much better than the quality of the model estimated using only the available output data. Also we present a recently proposed linear filter method for a more general filter structure.

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