

DETECTION AND ISOLATION OF MODEL-PLANT MISMATCH FOR MULTIVARIATE DYNAMIC SYSTEMS

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Abstract: This paper proposes a new scheme to detect and isolate model-plant mismatch (MPM) for multivariate dynamic systems. The background of our study is the increasing demands on MPM detection and isolation for performance assessment of model predictive controllers (MPCs). In this paper, the MPM problem is formulated in terms of discrete time state space model which is widely used in MPCs. Three MPM detection indices (MDIs) are proposed to detect the MPM. Also a logic framework is proposed to isolate the system matrices that have MPM. A numerical example is presented to demonstrate the applicability of the proposed scheme.

Keywords: Fault detection, Fault isolation, Monitoring, Multivariate systems

1. INTRODUCTION

Model predictive controllers (MPCs) have been used widely for advanced process control in the process industries. Accordingly, assessment of MPC's performance has been drawn increasing research attention in recent years. Since the ground breaking work of Harris (1989), many techniques for controller performance assessment have been proposed (Huang and Shah, 1999). Poor performance of the MPC can be due to poor tuning, ill-posed constraints, model-plant mismatch (MPM), etc. Since MPM is a reality in most industrial processes, a practical controller performance assessment method should be able to detect and isolate the cause of poor performance as being due to degradation of model quality. The objective in this study is to focus on the detection and isolation of MPM. As will be shown later, the detection and diagnosis of MPM can be typically

formulated as a parametric fault problem in the process under consideration.

As agreed in the community of fault detection and isolation (FDI), faults can be classified as sensor faults, actuator faults, additive process faults and parametric faults (Gertler, 1988). While tremendous research and application effort has been made on FDI for sensor and actuator faults (Gertler, 1988; Li and Shah, 2002), relatively few methods on parametric FDI have been developed. Stoustrup and Niemann (1999) proposed a detection and isolation method for parametric faults in continuous time (CT) systems represented by state space models, assuming that only the system matrix \mathbf{A} can have parametric faults, while \mathbf{B} and \mathbf{C} matrices are fault-free. Li and Jiang (2004) also proposed a scheme for detection and isolation parametric faults in CT systems, assuming only one of the system matrices has parametric faults at each time.

This paper proposes a novel scheme for detection and isolation of MPM by extending the method

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of Li and Jiang (2004). The paper is organized as follows. In section 2, we formulate the MPM problem based on the discrete time state space model. Section 3 describes how to generate three different MPM detection indices (MDIs) and how to use them to isolate the faults. Section 4 presents a numerical example. The paper ends with concluding remarks in Section 5.

2. PROBLEM FORMULATION

Without loss of generality, we consider a multi-inputs-multi-outputs(MIMO) process, which is represented by the following discrete time state space model:

$$\begin{aligned} \mathbf{x}(k+1) &= \mathbf{A}\mathbf{x}(k) + \mathbf{B}\mathbf{u}(k) + \mathbf{p}(k) \\ \mathbf{y}(k) &= \mathbf{C}\mathbf{x}(k) + \mathbf{o}(k) \end{aligned} \quad (1)$$

In equation (1), $\mathbf{x}(k) \in R^n$ is the state variable vector; $\mathbf{u}(k) \in R^l$ is the input vector to the process; $\mathbf{y}(k) \in R^m$ is the measured output vector; $\mathbf{o}(k) \in R^m$ is the relevant measure noise; $\mathbf{p}(k) \in R^n$ is the process disturbance; and $\{\mathbf{A}, \mathbf{B}, \mathbf{C}\}$ are system matrices with appropriate dimensions. It is assumed that $\mathbf{u}(k) \in R^l$ is noise-free, and further more, $\mathbf{p}(k) \in R^n$ and $\mathbf{o}(k) \in R^m$ are two independent Gaussian distributed white noise vectors with respective covariance matrices \mathbf{R}_o and \mathbf{R}_p . Denote the nominal values of $\{\mathbf{A}, \mathbf{B}, \mathbf{C}\}$ as $\{\mathbf{A}_o, \mathbf{B}_o, \mathbf{C}_o\}$, which can be usually identified from training data using subspace identification method. If the system matrices deviates from their nominal value, one writes it as:

$$\mathbf{A} = \mathbf{A}_o + \Delta\mathbf{A}, \mathbf{B} = \mathbf{B}_o + \Delta\mathbf{B}, \mathbf{C} = \mathbf{C}_o + \Delta\mathbf{C}, \quad (2)$$

where $\{\Delta\mathbf{A}, \Delta\mathbf{B}, \Delta\mathbf{C}\}$ are the deviations of the system matrices, contain zero and non-zero elements. Using the above expression, one can rewrite the state space model as follows:

$$\begin{aligned} \mathbf{x}(k+1) &= \mathbf{A}_o\mathbf{x}(k) + \mathbf{B}_o\mathbf{u}(k) + \mathbf{p}(k) \\ &\quad + \Delta\mathbf{A}\mathbf{x}(k) + \Delta\mathbf{B}\mathbf{u}(k) \\ \mathbf{y}(k) &= \mathbf{C}_o\mathbf{x}(k) + \mathbf{o}(k) + \Delta\mathbf{C}\mathbf{x}(k) \end{aligned} \quad (3)$$

Equation (3) can be further manipulated into

$$\begin{aligned} \mathbf{x}(k+1) &= \mathbf{A}_o\mathbf{x}(k) + \mathbf{B}_o\mathbf{u}(k) + \mathbf{p}(k) + \\ &\quad \mathbf{a}(k) + \mathbf{b}(k) \\ \mathbf{y}(k) &= \mathbf{C}_o\mathbf{x}(k) + \mathbf{o}(k) + \mathbf{c}(k) \end{aligned} \quad (4)$$

with the new notations:

$$\mathbf{a}(k) = \Delta\mathbf{A}\mathbf{x}(k), \mathbf{b}(k) = \Delta\mathbf{B}\mathbf{u}(k), \mathbf{c}(k) = \Delta\mathbf{C}\mathbf{x}(k),$$

In equation (4), $\{\mathbf{a}(k), \mathbf{b}(k), \mathbf{c}(k)\}$ account for the effects of $\{\Delta\mathbf{A}, \Delta\mathbf{B}$ or $\Delta\mathbf{C}\}$ acting on the considered system.

3. MPM DETECTION

With equation (1)-(3), the MPM detection is to indicate if any elements in $\Delta\mathbf{A}$, $\Delta\mathbf{B}$ or $\Delta\mathbf{C}$ begin

to become non-zero, also to indicate the time instant at which MPM begin to occur. In equation (4), one can verify that $a(k) \neq 0$ indicates $\Delta\mathbf{A} \neq 0$, $b(k) \neq 0$ indicates $\Delta\mathbf{B} \neq 0$ and $c(k) \neq 0$ indicates $\Delta\mathbf{C} \neq 0$. Therefore MPM detection is equivalent to indicating if $a(k)$, $b(k)$ or $c(k)$ is non-zero.

3.1 Preliminary

Performing algebraic manipulation on equation (4), one can obtain the following stacked equation:

$$\begin{aligned} \mathbf{y}_s(k) &= \Gamma_s^o \mathbf{x}(k-s) + \mathbf{H}_s^o \mathbf{u}_s(k) + \mathbf{c}_s(k) + \mathbf{o}_s(k) \\ &+ \mathbf{G}_s^o \mathbf{p}_{s-1}(k-1) + \mathbf{G}_s^o [\mathbf{a}_{s-1}(k-1) + \mathbf{b}_{s-1}(k-1)] \end{aligned} \quad (5)$$

where $\Gamma_s^o = [\mathbf{C}_o^T \ (\mathbf{C}_o \mathbf{A}_o)^T \dots (\mathbf{C}_o \mathbf{A}_o^{s-1})^T]^T \in R^{m_s \times n}$ is the extended observability matrix with s being the order of the parity space;

$$\mathbf{H}_s^o = \begin{bmatrix} \mathbf{0} & \dots & \dots & \mathbf{0} & \mathbf{0} \\ \mathbf{C}_o \mathbf{B}_o & \mathbf{0} & \dots & \vdots & \vdots \\ \mathbf{C}_o \mathbf{A}_o \mathbf{B}_o & \mathbf{C}_o \mathbf{B}_o & \ddots & \mathbf{0} & \mathbf{0} \\ \vdots & \vdots & \ddots & \mathbf{0} & \mathbf{0} \\ \mathbf{C}_o \mathbf{A}_o^{s-1} \mathbf{B}_o & \mathbf{C}_o \mathbf{A}_o^{s-2} \mathbf{B}_o & \dots & \mathbf{C}_o \mathbf{B}_o & \mathbf{0} \end{bmatrix} \in R^{m_s \times l_s}$$

and

$$\mathbf{G}_s^o = \begin{bmatrix} \mathbf{0} & \dots & \dots & \mathbf{0} \\ \mathbf{C}_o & \mathbf{0} & \dots & \vdots \\ \mathbf{C}_o \mathbf{A}_o & \mathbf{C}_o & \ddots & \mathbf{0} \\ \vdots & \vdots & \ddots & \mathbf{0} \\ \mathbf{C}_o \mathbf{A}_o^{s-1} & \mathbf{C}_o \mathbf{A}_o^{s-2} & \dots & \mathbf{C}_o \end{bmatrix} \in R^{m_s \times ns},$$

are two lower triangular block Toeplitz matrices.

Also in equation (5), $\mathbf{y}_s(k) = [\mathbf{y}^T(k-s) \ \mathbf{y}^T(k-s+1) \ \dots \ \mathbf{y}^T(k)]^T \in R^{m_s}$ with $m_s = m(s+1)$, is the stacked output vector; $\mathbf{a}_{s-1}(k-1) \in R^{ns}$, $\mathbf{b}_{s-1}(k-1) \in R^{ns}$, $\mathbf{c}_s(k) \in R^{m_s}$, $\mathbf{o}_s(k) \in R^{m_s}$, $\mathbf{p}_{s-1}(k-1) \in R^{ns}$, $\mathbf{u}_s(k) \in R^{l_s}$ are also stacked vectors similarly defined as $\mathbf{y}_s(k)$. The choice of the order of parity space s is not exclusive and the selection of s can effect the performance of fault detection according to Ding *et al.* (1999). However, the optimal determination of s is out of the scope of this paper, and in this paper s is selected to be equal to n , according to Chow and Willsky (1984).

Define $\mathbf{z}_s(k) = [\mathbf{y}_s^T(k) \ \mathbf{u}_s^T(k)]^T \in R^{(m_s+l_s)}$ and introduce $\tilde{\mathbf{H}}_s = [\mathbf{I}_{m_s} | -\mathbf{H}_s^o] \in R^{m_s \times (m_s+l_s)}$, with \mathbf{I}_{m_s} being the m_s by m_s identity matrix. Also denote $\tilde{\mathbf{G}}_s^o = [\mathbf{G}_s^o | \mathbf{I}_{m_s}] \in R^{m_s \times (ns+m_s)}$ as the augmented fault gain matrix for $\{\mathbf{a}_{s-1}(k-1)$,

$\mathbf{b}_{s-1}(k-1), \mathbf{c}_s(k)$ }. Then equation (5) can be rewritten as

$$\begin{aligned} \tilde{\mathbf{H}}_s \mathbf{z}_s(k) &= \Gamma_s^o \mathbf{x}(k-s) + \mathbf{G}_s^o \mathbf{p}_{s-1}(k-1) \\ &+ \mathbf{o}_s(k) + \tilde{\mathbf{G}}_s^o \begin{bmatrix} \mathbf{a}_{s-1}(k-1) + \mathbf{b}_{s-1}(k-1) \\ \mathbf{c}_s(k) \end{bmatrix} \end{aligned} \quad (6)$$

3.2 Detection of mismatch in $\{\mathbf{A}, \mathbf{B}, \mathbf{C}\}$

In this subsection, we propose a MPM detection index η_{ABC} , which can be used to detect any mismatch in matrices \mathbf{A} , or/and \mathbf{B} , or/and \mathbf{C} .

We select a matrix $\mathbf{W}_o \in R^{(m_s-n) \times m_s}$ located in the null space of $\Gamma_s^o \in R^{(m_s \times n)}$, i.e. $\mathbf{W}_o \Gamma_s^o = 0$, and multiplying the stacked equation (6) by \mathbf{W}_o , one can obtain:

$$\begin{aligned} \mathbf{W}_o \tilde{\mathbf{H}}_s \mathbf{z}_s(k) &= \mathbf{W}_o [\mathbf{G}_s^o \mathbf{p}_{s-1}(k-1) + \mathbf{o}_s(k)] \\ &+ \mathbf{W}_o \tilde{\mathbf{G}}_s^o \begin{bmatrix} \mathbf{a}_{s-1}(k-1) + \mathbf{b}_{s-1}(k-1) \\ \mathbf{c}_s(k) \end{bmatrix} \end{aligned} \quad (7)$$

The optimal design of \mathbf{W}_o can be referred to Li and Shah (2002) for details.

Define

$$\begin{aligned} \mathbf{e}_s(k) &\equiv \mathbf{W}_o \tilde{\mathbf{H}}_s \mathbf{z}_s(k) = \mathbf{W}_o [\mathbf{G}_s^o \mathbf{p}_{s-1}(k-1) + \mathbf{o}_s(k)] \\ &+ \mathbf{W}_o \tilde{\mathbf{G}}_s^o \begin{bmatrix} \mathbf{a}_{s-1}(k-1) + \mathbf{b}_{s-1}(k-1) \\ \mathbf{c}_s(k) \end{bmatrix} \end{aligned} \quad (8)$$

as the primary residual vector (PRV) for detecting mismatch in matrices \mathbf{A} , or/and \mathbf{B} , or/and \mathbf{C} , and $\mathbf{W}_o \tilde{\mathbf{H}}_s \mathbf{z}_s(k)$ is the computational form. The following facts can be established for the PRV.

Remark 1. In the mismatch-free case, $\mathbf{a}_{s-1}(k-1) = 0, \mathbf{b}_{s-1}(k-1) = 0$ and $\mathbf{c}_s(k) = 0$, the PRV is reduced to $\mathbf{e}_s^*(k) = \mathbf{W}_o [\mathbf{G}_s^o \mathbf{p}_{s-1}(k-1) + \mathbf{o}_s(k)]$, which is a moving average (MA) process of the noise vector: $\mathbf{p}_{s-1}(k-1)$ and $\mathbf{o}_s(k)$. Also, $\mathbf{e}_s^*(k)$ can be proved to be a zero mean Gaussian distributed random noise vector from the distributions of $\mathbf{o}(k)$ and $\mathbf{p}(k)$ with a covariance matrix $\mathbf{R}_{s,e}$ (Johnson and Wichern, 1998). The internal form of $\mathbf{R}_{s,e}$ is:

$$\mathbf{R}_{s,e} = \mathbf{W}_o (\mathbf{G}_s^o \mathbf{R}_{s,p} \mathbf{G}_s^{oT} + \mathbf{R}_{s,o}) \mathbf{W}_o^T \quad (9)$$

where $\mathbf{R}_{s,p} = \mathbf{I}_s \otimes \mathbf{R}_p \in R^{ns \times ns}$, $\mathbf{R}_{s,o} = \mathbf{I}_{s+1} \otimes \mathbf{R}_o \in R^{m_s \times m_s}$, and \otimes stands for the kronecker tension product.

Remark 2. In the presence of any MPM, i.e., either $\mathbf{a}(k) \neq 0, \mathbf{b}(k) \neq 0$ or $\mathbf{c}(k) \neq 0$,

$$\mathbf{e}_s(k) \equiv \mathbf{e}_s^*(k) + \mathbf{e}_s^f(k) \quad (10)$$

where $\mathbf{e}_s^f(k) = \mathbf{W}_o \tilde{\mathbf{G}}_s^o \begin{bmatrix} \mathbf{a}_{s-1}(k-1) + \mathbf{b}_{s-1}(k-1) \\ \mathbf{c}_s(k) \end{bmatrix}$

is the mismatch-contributed part in the PRV. In this case, the mean of $\mathbf{e}_s(k)$ is no longer zero (but the covariance is the same as in the mismatch-free case). Instead, the mean of $\mathbf{e}_s(k)$ is $\mathbf{e}_s^f(k)$.

Therefore, mismatch detection can be done by checking if the mean of the PRV is zero. Clearly, $\mathbf{e}_s(k)$ follows a multivariate Gaussian distribution with mean $\mathbf{e}_s^f(k)$ and covariance $\mathbf{R}_{s,e}$, i.e. $\mathbf{e}_s(k) \sim \mathcal{N}(\mathbf{e}_s^f(k), \mathbf{R}_{s,e})$.

Instead of using the PRV, one use the square weighted residual (SWR) as the MPM detection index. The SWR is defined as:

$$\eta_{ABC}(k) = (e_s(k))^T \mathbf{R}_{s,e} e_s(k) \quad (11)$$

η_{ABC} follows a chi-square distribution with $(m_s - n)$ degrees of freedom (Johnson and Wichern, 1998). Then given a confidence limit $\chi_{m_s-n}^2(\alpha)$, e.g. $\alpha = 1\%$, $\eta_{ABC}(k) \geq \chi_{m_s-n}^2(\alpha)$ indicates that matrix \mathbf{A} , or/and matrix \mathbf{B} , or/and matrix \mathbf{C} have deviated from the nominal value, while $\eta_{ABC}(k) \leq \chi_{m_s-n}^2(\alpha)$ implies there is no mismatch in matrix \mathbf{A} , matrix \mathbf{B} and matrix \mathbf{C} . To reduce the effect of transient and noise in the measured data, an exponentially weighted moving average (EWMA) filter can be applied to the PRV $e_s(k)$ first. Then the filtered PRV can be used to calculate the MPM detection index η_{ABC} .

3.3 Detection of mismatch in $\{\mathbf{A}, \mathbf{C}\}$

In this subsection, we propose a MPM detection index η_{AC} , which can be used to detect any mismatch in matrices \mathbf{A} , or/and \mathbf{C} . Rewrite (4) as:

$$\begin{aligned} \mathbf{x}(k+1) &= \mathbf{A}_o \mathbf{x}(k) + \mathbf{B} \mathbf{u}(k) + \mathbf{p}(k) + \mathbf{a}(k) \\ \mathbf{y}(k) &= \mathbf{C}_o \mathbf{x}(k) + \mathbf{o}(k) + \mathbf{c}(k) \end{aligned} \quad (12)$$

where $\mathbf{B} = \mathbf{B}_o + \Delta \mathbf{B}$. Doing the same derivation as for (5), one can obtain:

$$\begin{aligned} \mathbf{y}_s(k) &= \Gamma_s^o \mathbf{x}(k-s) + \mathbf{H}_s \mathbf{u}_s(k) + \mathbf{c}_s(k) + \mathbf{o}_s(k) \\ &+ \mathbf{G}_s^o \mathbf{p}_{s-1}(k-1) + \mathbf{G}_s^o \mathbf{a}_{s-1}(k-1) \end{aligned} \quad (13)$$

where $\mathbf{H}_s = \mathbf{H}_s^o|_{\mathbf{B}_o=\mathbf{B}}$ is unknown since \mathbf{B} is unknown matrix. Pre-multiplying \mathbf{W}_o on the both side of equation (13), where $\mathbf{W}_o \Gamma_s^o = 0$ is defined before, one can obtain:

$$\begin{aligned} \mathbf{W}_o \mathbf{y}_s(k) &= \mathbf{W}_o \mathbf{H}_s \mathbf{u}_s(k) + \mathbf{W}_o [\mathbf{G}_s^o \mathbf{p}_{s-1}(k-1) \\ &+ \mathbf{o}_s(k)] + \mathbf{W}_o [\mathbf{G}_s^o \mathbf{a}_{s-1}(k-1) + \mathbf{c}_s(k)] \end{aligned} \quad (14)$$

With N samples of data, we suppose faults occurs at n_f th sample which actually can be detected by η_{ABC} . Then one can select an integer k_f from the range $[(n_f + s + 1) \ N]$ and construct the Hankel data matrices

$$\begin{aligned} \mathbf{Y}_{s,k_f,N} &= [y_s(k_f) \ y_s(k_f+1) \ \cdots \ y_s(N)] \\ \mathbf{U}_{s,k_f,N} &= [u_s(k_f) \ u_s(k_f+1) \ \cdots \ u_s(N)] \end{aligned} \quad (15)$$

based on $N - k_f + 1$ samples collected after the occurrence of the faults. In the same way, one can obtain:

$$\begin{aligned}
\mathbf{P}_{s-1,k_f-1,N-1} &= [p_{s-1}(k_f-1) \quad p_{s-1}(k_f) \quad \cdots \\
&\quad \quad \quad p_{s-1}(N-1)] \\
\mathbf{A}_{s-1,k_f-1,N-1} &= [a_{s-1}(k_f-1) \quad a_{s-1}(k_f) \quad \cdots \\
&\quad \quad \quad a_{s-1}(N-1)] \\
\mathbf{C}_{s,k_f,N} &= [c_s(k_f) \quad c_s(k_f+1) \quad \cdots \quad c_s(N)] \\
\mathbf{O}_{s,k_f,N} &= [o_s(k_f) \quad o_s(k_f+1) \quad \cdots \quad o_s(N)]
\end{aligned}$$

Then equation (14) can be transformed into:

$$\begin{aligned}
\mathbf{W}_o \mathbf{Y}_{s,k_f,N} &= \mathbf{W}_o \mathbf{H}_s \mathbf{U}_{s,k_f,N} + \mathbf{W}_o \mathbf{C}_{s,k_f,N} \\
&\quad + \mathbf{W}_o \mathbf{G}_s^o \mathbf{A}_{s-1,k_f-1,N-1} + \mathbf{W}_o \mathbf{O}_{s,k_f,N} \\
&\quad \quad \quad + \mathbf{W}_o \mathbf{G}_s^o \mathbf{P}_{s-1,k_f-1,N-1}
\end{aligned} \tag{16}$$

Define $\mathbf{U}_{s,k_f,N}^\perp = \mathbf{I}_{N-k_f+1} - \mathbf{U}_{s,k_f,N}^T (\mathbf{U}_{s,k_f,N} \times \mathbf{U}_{s,k_f,N}^T)^{-1} \mathbf{U}_{s,k_f,N}$, where \mathbf{I}_{N-k_f+1} is an $(N - k_f + 1) \times (N - k_f + 1)$ identity matrix. Note that $\mathbf{U}_{s,k_f,N}^\perp$ is located in the right null space of $\mathbf{U}_{s,k_f,N}$, i.e. $\mathbf{U}_{s,k_f,N} \times \mathbf{U}_{s,k_f,N}^\perp = \mathbf{0}$. It is assumed that $\mathbf{U}_{s,k_f,N} \mathbf{U}_{s,k_f,N}^T$ is non-singular. This assumption is true when the input signals are persistent excitation. Post-multiplying $\mathbf{U}_{s,k_f,N}^\perp$ on both sides of equation (16) gives:

$$\begin{aligned}
\mathbf{W}_o \mathbf{Y}_{s,k_f,N} \mathbf{U}_{s,k_f,N}^\perp &= \mathbf{W}_o \mathbf{C}_{s,k_f,N} \mathbf{U}_{s,k_f,N}^\perp \\
&\quad + \mathbf{W}_o \mathbf{G}_s^o \mathbf{A}_{s-1,k_f-1,N-1} \mathbf{U}_{s,k_f,N}^\perp \\
&\quad + \mathbf{W}_o \mathbf{G}_s^o \mathbf{P}_{s-1,k_f-1,N-1} \mathbf{U}_{s,k_f,N}^\perp \\
&\quad \quad \quad + \mathbf{W}_o \mathbf{O}_{s,k_f,N} \mathbf{U}_{s,k_f,N}^\perp
\end{aligned} \tag{17}$$

where the faults introduced by matrix \mathbf{B} and \mathbf{D} have been entirely removed. As $(N - k_f - 1) \rightarrow \infty$, asymptotically,

$$\begin{aligned}
\mathbf{P}_{s-1,k_f-1,N-1} \mathbf{U}_{s,k_f,N}^\perp &= \mathbf{P}_{s-1,k_f-1,N-1} \\
-\mathbf{P}_{s-1,k_f-1,N-1} \mathbf{U}_{s,k_f,N}^T &\times (\mathbf{U}_{s,k_f,N} \times \mathbf{U}_{s,k_f,N}^T)^{-1} \\
\times \mathbf{U}_{s,k_f,N} &\rightarrow \mathbf{P}_{s-1,k_f-1,N-1}
\end{aligned}$$

$$\begin{aligned}
\mathbf{O}_{s,k_f,N} \mathbf{U}_{s,k_f,N}^\perp &= \mathbf{O}_{s,k_f,N} \\
-\mathbf{O}_{s,k_f,N} \mathbf{U}_{s,k_f,N}^T &\times (\mathbf{U}_{s,k_f,N} \times \mathbf{U}_{s,k_f,N}^T)^{-1} \mathbf{U}_{s,k_f,N} \\
\rightarrow \mathbf{O}_{s,k_f,N} &
\end{aligned}$$

because $\mathbf{P}_{s-1,k_f-1,N-1} \mathbf{U}_{s,k_f,N}^T \rightarrow \mathbf{0}$ and $\mathbf{O}_{s,k_f,N} \mathbf{U}_{s,k_f,N}^T \rightarrow \mathbf{0}$ under open loop condition. This is because the input $\mathbf{u}(k)$ is independent of the white noises $\mathbf{p}(k)$ and $\mathbf{o}(k)$. Therefore, equation (17) can be written as:

$$\begin{aligned}
\mathbf{W}_o \mathbf{Y}_{s,k_f,N} \mathbf{U}_{s,k_f,N}^\perp &= \mathbf{W}_o \mathbf{C}_{s,k_f,N} \mathbf{U}_{s,k_f,N}^\perp \\
&\quad + \mathbf{W}_o \mathbf{G}_s^o \mathbf{A}_{s-1,k_f-1,N-1} \mathbf{U}_{s,k_f,N}^\perp + \mathbf{W}_o \mathbf{O}_{s,k_f,N} \\
&\quad \quad \quad + \mathbf{W}_o \mathbf{G}_s^o \mathbf{P}_{s-1,k_f-1,N-1}
\end{aligned} \tag{18}$$

Define $\mathbf{E}_{\mathbf{A},\mathbf{C}} \equiv \mathbf{W}_o \mathbf{Y}_{s,k_f,N} \mathbf{U}_{s,k_f,N}^\perp$ as the primary residual matrix (PRM) for detecting mismatch in matrices \mathbf{A} or/and \mathbf{C} . We denote $e_{s,ac}(k)$, $k \in [k_f, N]$, as the PRV at the k th sample instant for detecting mismatch in matrices \mathbf{A} or/and \mathbf{C} , where $e_{s,ac}(k)$ is the $(k - k_f + 1)$ th column of $\mathbf{E}_{\mathbf{A},\mathbf{C}}$.

Remark 3. If there is no mismatch in matrix \mathbf{A} and matrix \mathbf{C} , i.e. $\mathbf{W}_o \mathbf{C}_{s,k_f,N} \mathbf{U}_{s,k_f,N}^\perp = \mathbf{0}$ and

$\mathbf{W}_o \mathbf{G}_s^o \mathbf{A}_{s-1,k_f-1,N-1} \mathbf{U}_{s,k_f,N}^\perp = \mathbf{0}$, then $\mathbf{E}_{\mathbf{A},\mathbf{C}} = \mathbf{W}_o \mathbf{G}_s^o \mathbf{P}_{s-1,k_f-1,N-1} + \mathbf{W}_o \mathbf{O}_{s,k_f,N}$. In this case, it can be easily verified that $e_{s,ac}(k)$ is a zero mean Gaussian distributed random vector with covariance $\mathbf{R}_{s,e,ac}$, i.e. $e_{s,ac}(k) \sim \mathfrak{N}(\mathbf{0}, \mathbf{R}_{s,e,ac})$, where $\mathbf{R}_{s,e,ac} = \mathbf{R}_{s,e}$ which is derived in equation (9).

Remark 4. If there are mismatch in matrix \mathbf{A} or/and matrix \mathbf{C} , the mean of $e_{s,ac}(k)$ will be non-zero, while the covariance remains the same. Therefore, mismatch detection can be done by checking if the mean of the PRV is zero.

Again, PRV ($e_{s,ac}(k)$) is transformed into SWR ($\eta_{AC}(k)$) as the MPM detection index, which is calculated as:

$$\eta_{AC}(k) = (e_{s,ac}(k))^T \mathbf{R}_{s,e,ac}^{-1} e_{s,ac}(k)$$

$\eta_{AC}(k)$ follows a chi-square distribution with $(m_s - n)$ degrees of freedom. Then given a confidence limit $\chi_{m_s-n}^2(\alpha)$, $\eta_{AC}(k) \geq \chi_{m_s-n}^2(\alpha)$ indicates that matrices \mathbf{A} or/and \mathbf{C} have faults, while $\eta_{AC}(k) \leq \chi_{m_s-n}^2(\alpha)$ implies there is no fault in matrices \mathbf{A} and \mathbf{C} .

3.4 Detection of mismatch in $\{\mathbf{C}\}$

In this subsection, we propose a MPM detection index η_C , which can be used to detect any mismatch in matrix \mathbf{C} .

Select a transformation matrix $\mathbf{W}^o \in R^{(m_s - n_s) \times m_s}$ from the null space of $\Psi_s^o = [\Gamma_s^o | \mathbf{G}_s^o]$, and pre-multiply it on the both sides of equation (6), one can obtain:

$$\mathbf{W}^o \tilde{\mathbf{H}}_s \mathbf{z}_s(k) = \mathbf{W}^o \mathbf{c}_s(k) + \mathbf{W}^o \mathbf{o}_s(k) \tag{19}$$

Define

$$\mathbf{e}_{s,c}(k) \equiv \mathbf{W}^o \tilde{\mathbf{H}}_s \mathbf{z}_s(k) = \mathbf{W}^o \mathbf{c}_s(k) + \mathbf{W}^o \mathbf{o}_s(k) \tag{20}$$

as the PRV for detecting mismatch in matrix \mathbf{C} , where the right hand side (RHS) of the equation is the computational form, while the left hand side (LHS) is the internal form.

Remark 5. If there is no mismatch in matrix \mathbf{C} , i.e. $\mathbf{c}_s(k) = \mathbf{0}$, the PRV is reduced to $\mathbf{e}_{s,c}^*(k) = \mathbf{W}^o \mathbf{o}_s(k)$, which is a moving average (MA) process of the noise vector: $\mathbf{o}_s(k)$. Also, $\mathbf{e}_{s,c}^*(k)$ can be proved to be a zero mean Gaussian distributed random noise vector from the distributions of $\mathbf{o}(k)$ with a covariance matrix $\mathbf{R}_{s,e,c}$. The internal form of $\mathbf{R}_{s,e,c}$ is:

$$\mathbf{R}_{s,e,c} = \mathbf{W}^o \mathbf{R}_{s,o} \mathbf{W}^{oT} \tag{21}$$

Remark 6. In the presence of any mismatch in matrix \mathbf{C} , the mean of $\mathbf{e}_{s,c}(k)$ is no longer zero,

but the covariance remains the same. Therefore, mismatch detection can be done by checking if the mean of the PRV is zero.

Again, PRV ($e_{s,c}(k)$) is transformed into SWR ($\eta_C(k)$) as the MPM detection index, which is calculated as:

$$\eta_C(k) = (e_{s,c}(k))^T \mathbf{R}_{s,e,c} e_{s,c}(k)$$

$\eta_C(k)$ follows a chi-square distribution with $(m_s - n_s)$ degrees of freedom. Then given a confidence limit $\chi_{m_s - n_s}^2(\alpha)$, $\eta_C(k) \geq \chi_{m_s - n_s}^2(\alpha)$ indicates that matrix \mathbf{C} has faults, while $\eta_C(k) \leq \chi_{m_s - n_s}^2(\alpha)$ implies there is no fault in matrix \mathbf{C} .

3.5 Mismatch isolation logic

In last three subsections, three MPM detection indices (MDI) - η_{ABC} , η_{AC} and η_C are proposed which can be used to detect the mismatch in the system matrices. Beside simply detecting the mismatch in the system matrices, it is desirable to tell which matrices have faults. In this subsection, we would like to propose a logic framework to isolate faults using the three MDIs. First, denote $\eta_{ABC}(k) = 1$ if there is mismatch in matrices \mathbf{A} , or/and \mathbf{B} , or/and \mathbf{C} , and $\eta_{ABC}(k) = 0$ if there is no mismatch in these three matrices. Same, denote $\eta_{AC}(k) = 1$ if there is mismatch in matrices \mathbf{A} , or/and \mathbf{C} , and $\eta_{AC}(k) = 0$ if there is no mismatch; denote $\eta_C(k) = 1$ if there is mismatch in matrix \mathbf{C} , and $\eta_C(k) = 0$ if there is no mismatch. Table 1 contains the fault isolation logic framework.

Table 1. Fault isolation logic using $\eta_{ABC}(k)$, $\eta_{AC}(k)$, and $\eta_C(k)$

$\eta_{ABC}(k)$	$\eta_{AC}(k)$	$\eta_C(k)$	Fault Matrix
1	0	0	\mathbf{B}
1	1	0	\mathbf{A} or \mathbf{AB}
1	1	1	\mathbf{C} or \mathbf{AC} or \mathbf{BC} or \mathbf{ABC}

4. NUMERICAL EXAMPLE

In this section, a numerical example is provided to demonstrate and verify the correctness of the proposed scheme. The simulated process is a second order dynamic system. The system matrices in the discrete time domain are:

$$\mathbf{A}_0 = \begin{bmatrix} 0.6082 & -0.0100 \\ 2.2668 & 0.0364 \end{bmatrix}$$

$$\mathbf{B}_0 = \begin{bmatrix} 0.5602 & 0.0008 \\ 1.3760 & 0.3026 \end{bmatrix}$$

$$\mathbf{C}_0 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

For this process, \mathbf{C}_0 is only the sensor gain matrix which typically is unlikely to deviate. So in this example we assume there is no mismatch in \mathbf{C}_0 and we only consider three different mismatch cases: (1) mismatch in \mathbf{A} ; (2) mismatch in \mathbf{B} and (3) mismatch in both \mathbf{A} and \mathbf{B} .

For each case, a set of 3000 samples training data of $\{\mathbf{u}(k), \mathbf{y}(k)\}$ are collected and the fault is introduced at 500th sample. The inputs $\mathbf{u}(k)$ are simulated by pseudo random binary signals with small magnitude, and are assumed to be noise-free. However, the outputs $\mathbf{y}(k)$ for each case are corrupted by Gaussian distributed white noise $\mathbf{o}(k)$. If we denote the noise-free part of the outputs as $\mathbf{y}_0(k)$, then the measured outputs are expressed as $\mathbf{y}(k) = \mathbf{y}_0(k) + \mathbf{o}(k)$. Therefore, for each case, the noise-to-signal ratio (NSR) for each output variable can be expressed as:

$$NSR_i = [STD \text{ of } \mathbf{o}_i(k)] / [STD \text{ of } \mathbf{y}_{0i}(k)],$$

for $\forall i = 1, 2,$

where STD refers to standard deviation; $\mathbf{y}_{0i}(k)$ is the i th noise-free output sequence and $\mathbf{o}_i(k)$ is the corresponding white noise sequence.

Case (1): We introduce 10% mismatch of $\mathbf{A}_0(k)$ to the process. The NSR for the two outputs are $NSR_1 = 11.19\%$ and $NSR_2 = 2.73\%$. In Figure 1, the mismatch detection indices $\eta_{ABC}(k)$ and $\eta_{AC}(k)$ have been scaled (or divided) by $\chi_{m_s - n_s}^2(\alpha)$, where $\alpha = 0.01$ and $\chi_6^2(0.01) = 16.812$ (same for the other two cases). The dash line in the plot is the scaled confidence limit whose value is 1. The $\eta_{ABC}(k)$ index clearly shows that after 500th sample, the mismatch occurs. Then starting from $kf=600$ th sample, we begin to plot $\eta_{AC}(k)$ and it clearly shows that there is mismatch in \mathbf{A} (since we have assumed no mismatch in \mathbf{C}). Thus we have confirmed that there is mismatch in \mathbf{A} .

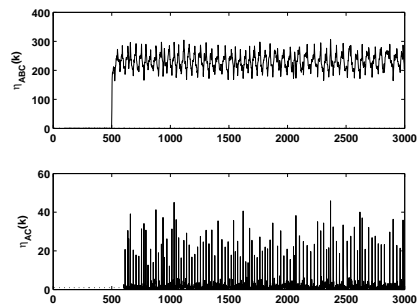


Fig. 1. Mismatch detection indices for case 1

Case (2): We introduce 10% mismatch of $\mathbf{B}_0(k)$ to the process. The NSR for the two outputs are $NSR_1 = 14.81\%$ and $NSR_2 = 4.12\%$. The scaled mismatch detection indices $\eta_{ABC}(k)$ and $\eta_{AC}(k)$ are plotted in Figure 2. The $\eta_{ABC}(k)$ index clearly shows that after 500th sample, the mismatch occurs. Then starting from $kf=600$ th sample, we

begin to plot $\eta_{AC}(k)$ and it clear shows that there is no mismatch in **A**. Thus the conclusion is that the mismatch happens only in **B**.

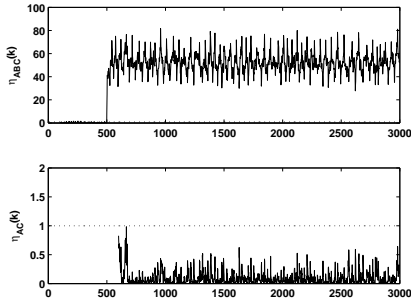


Fig. 2. Mismatch detection indices for case 2

Case (3): We introduce 10% mismatch in both $\mathbf{A}_0(k)$ and $\mathbf{B}_0(k)$. The NSR for the two outputs are $NSR_1 = 6.94\%$ and $NSR_2 = 1.74\%$. The scaled mismatch detection indices $\eta_{ABC}(k)$ and $\eta_{AC}(k)$ are plotted in Figure 3. The $\eta_{ABC}(k)$ index clear shows that after 500th sample, the mismatch occurs. Then starting from $kf=600$ th sample, we begin to plot $\eta_{AC}(k)$ and it clear shows that there is mismatch in **A** (since we have assumed no mismatch in **C**).

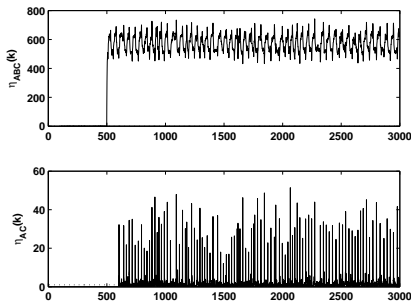


Fig. 3. Mismatch detection indices for case 3

Comparison of Case (1) and Case (3): In case (3), there is extra 10% mismatch in **B** compare to case (1). Thus the magnitude of $\eta_{ABC}(k)$ after 500th sample in Figure 3 is bigger than the one in Figure 1. This is because $\eta_{ABC}(k)$ measures mismatch both in **A** and **B**. However, the magnitudes of $\eta_{AC}(k)$ for both cases after 600th sample are quite close. This is because in both cases there is the same 10% mismatch of **A** and $\eta_{AC}(k)$ only considers the mismatch in **A**.

5. CONCLUSION

A new scheme for detection and isolation of model-plant-mismatch (MPM) in multivariate dynamic process has been developed. The MPM problem has been formulated in terms of the discrete time state space model. Three different mismatch detection indices (MDI) has been proposed

to detect the MPM. Also the MDIs can be used to isolate the system matrices that have mismatch. The whole approach is completely data driven.

A numerical example is presented to demonstrate the efficacy of the the new scheme. Three different mismatch faults, including (1) mismatch in system matrix **A**, (2) mismatch in system matrix **B** and (3) mismatch in both **A** and **B**, are simulated. The simulation results validates that the new scheme can effectively detect and isolate the mismatch in different situations.

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