

# Identification of delay dominant recycle systems

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## Abstract

A new identification method for single-input-single-output delay dominant recycle systems is presented in this paper. Identification of recycle systems is similar to that of closed loop systems. However, identification of recycle systems poses certain challenges in that the input-equivalent signal in the closed loop identification is not available for recycle systems. Therefore, special identification routines are required to ensure consistency of recycle and forward models. It is shown that consistent estimates of delay dominant recycle systems can be obtained by treating a delayed output as one of the inputs. Asymptotic variance expressions for the estimates of forward and recycle models are provided. These are then used in designing an optimal excitation signal for recycle systems. The results are illustrated through an industrial example.

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**Keywords:** Recycle systems, system identification, asymptotic consistency, asymptotic variance.

## 1 Introduction

Chemical processes which operate with material and energy recycle have attracted a lot of attention recently because of their dynamics and the consequent challenges they pose from a control point-of-view (Morud and Skogestad 1996). Research on the empirical model identification (using system identification routines), dynamics and control of recycle processes is of great practical importance because of the wide use of these systems in the process industry. While most of the work published on recycle processes concentrates on studying their dynamics and control, there have been relatively few publications which address the challenges in identifying dynamic models for recycle systems (Kwok *et al.* 2001).

Recycle systems consist of a forward path model and a recycle model with either a negative or a positive feedback (Denn and Levi 1982, Morud and Skogestad 1996). Hence, recycle systems are similar to closed loop systems. The dynamic behavior of recycle systems can be totally different from that of systems with no recycle. Recycle streams generally change the poles and zeros of the forward path

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leading to different phenomena such as slow response, stair-case like step response or even instability. Luyben (1994) has shown that recycle streams can sometimes lead to a “snowball effect” especially for certain control configurations. The behavior of recycle systems has been well studied in the literature and it is known that due to their atypical behavior, special control algorithms are needed to achieve good closed loop performance.

If both forward and recycle models are available, it is possible to achieve good closed loop performance by designing appropriate controllers (Lakshminarayanan and Takada 2001, Samyudia *et al.* 2000). Given the models, it is possible to remove the recycle effect on the forward model and design an appropriate controller (Scali and Ferrari 1999). Chodavarapu and Zheng (2001) present a method for tuning PID controllers with very little knowledge of the recycle model characteristics. One of the methods suggested by them for identifying the forward model is to remove the recycle stream temporarily for identification. However, in a real industrial setting it is rarely practical to do so.

A wide variety of chemical processes exhibit large time delays in the recycle streams. Heat integration, for instance, is an important feature in most industrial processes as it increases the efficiency of energy utilization. In a number of chemical processes, such as crude pre-heat train in refineries, heat is exchanged between feed and product streams. However, this integration makes the process dynamics very complex especially because of a large delay between the feed and product streams. Many such examples can be found in the chemical industry. In this paper, a recycle process from the bleaching operation in a Bleached Chemi-Thermo-Mechanical Pulp process is used for illustrative purposes.

Hence, in this paper the focus is on time delay dominant recycle systems. When identifying models for recycle processes with dominant time delays it is important to take the complexity of the model structure into account. These systems have staircase-shaped step response. Hence attempts to use black-box identification techniques can lead to identification of inconsistent models which approximate the staircase responses using high-order step responses. Considering this behavior of typical delay dominant recycle systems, an approach which can provide consistent estimates of both forward and recycle models is presented in this paper.

This paper is divided into five sections. Section 1 gives an overview of the literature on recycle systems and provides certain motivational arguments for studying identification of recycle systems. In section 2 the problem of identification of recycle systems is formulated. A method for consistent estimation of forward and recycle models is presented in section 3. Expressions for asymptotic variance and optimal input are also provided in this section. An industrial example is provided in section 4 and some concluding remarks are given in section 5.

## 2 Problem Formulation

System identification is a well developed area of research and there are a number of books on the techniques involved in identification of consistent models from process data (Ljung 1999, Söderström and Stoica 1989). In the traditional system identification methods open loop process data is used for estimating unbiased models. On the other hand, there are also methods for identifying models from closed loop data (Forssell and Ljung 1999, Huang and Shah 1997). The problem in closed-loop identification is to estimate both the process model and the controller using the setpoint and process input/output data. This can be done in a number of ways *viz.*, two step closed loop identification method, joint input-output identification and projection methods (Forssell and Ljung 1999).

Even though the problem of identification of recycle systems reduces to that of closed loop systems, recycle systems pose certain challenges in identification. Traditional closed-loop identification methods

utilize three signals - setpoint, process input and process output. On the other hand for recycle systems, a signal equivalent to the process input is generally not available. In general, it is not possible to obtain consistent estimates of the forward and recycle models.

A typical recycle system is shown in fig.1 where  $G_f(q)$  is the true forward process model and  $G_r(q)$  is the true recycle model and  $H(q)$  is the noise model. The process input is represented by  $u(t)$ , process output by  $y(t)$  and the noise, represented by  $e(t)$ , is assumed to be normally distributed and white. For the sake of simplicity the process is assumed to be SISO.  $q$  represents the forward shift operator. The estimated model consists of an estimated process model represented by  $\hat{G}_f(q, \theta)$ , an estimated recycle model represented by  $\hat{G}_r(q, \theta)$  and an estimated noise model,  $\hat{H}(q, \theta)$  where  $\theta$  is the parameter vector. It is assumed that  $\hat{H}$  and its inverse are both stable and that all the inputs and the outputs are quasi-stationary signals. The estimated models are obtained from a data of length  $N$ .

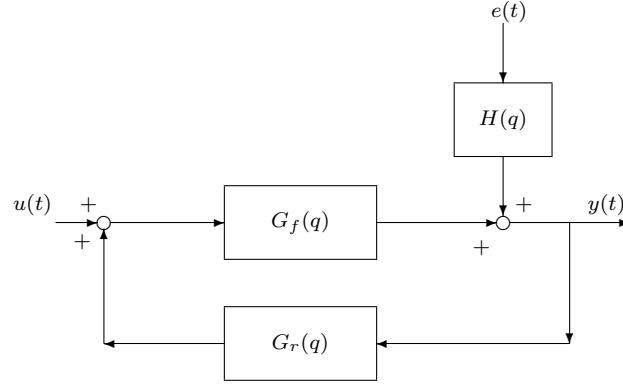


Figure 1: Block diagram of a typical recycle system

Let the true model be (see fig.1)

$$\begin{aligned} y(t) &= G_f u(t) + G_f G_r y(t) + H e(t) \\ &:= P u(t) + Q y(t-d) + H e(t) \\ &:= \Gamma U(t) + H e(t) \end{aligned} \quad (1)$$

$P$  and  $Q$  are defined in an obvious way.  $d = d_f + d_r$  is the sum of the forward path delay,  $d_f$  and the recycle path delay,  $d_r$  and  $U(t) = [u(t) \quad y(t-d)]^T$ .  $e(t)$  is white noise with variance  $\sigma_e$ . Also, assume that  $u(t)$  and  $e(t)$  are independent. The following additional notation is introduced for convenience.

$$\tilde{\Gamma}(q) := \Gamma - \hat{\Gamma} \quad \text{and} \quad \tilde{H}(q) := H - \hat{H}. \quad (2)$$

$\tilde{P}$  and  $\tilde{Q}$  are similarly defined. The data collected from identification experiments on the real process are denoted by

$$Z^N = \{u(1), y(1), \dots, u(N), y(N)\}. \quad (3)$$

**Example 2.1** A typical recycle system and its dynamic behavior is illustrated through this example. Consider the following recycle process

$$G_f = \frac{0.2}{1 - 0.1q^{-1}} q^{-4} \quad G_r = \frac{0.9}{1 - 0.5q^{-1}} q^{-26}. \quad (4)$$

Both forward and recycle models have relatively fast dynamics and the total delay in the recycle loop is much larger than the settling time of either model. The step response of this process is shown in

fig.2. The stair-case like step response is due to large delay in the recycle stream and comparatively fast dynamics in the forward and recycle streams. ■

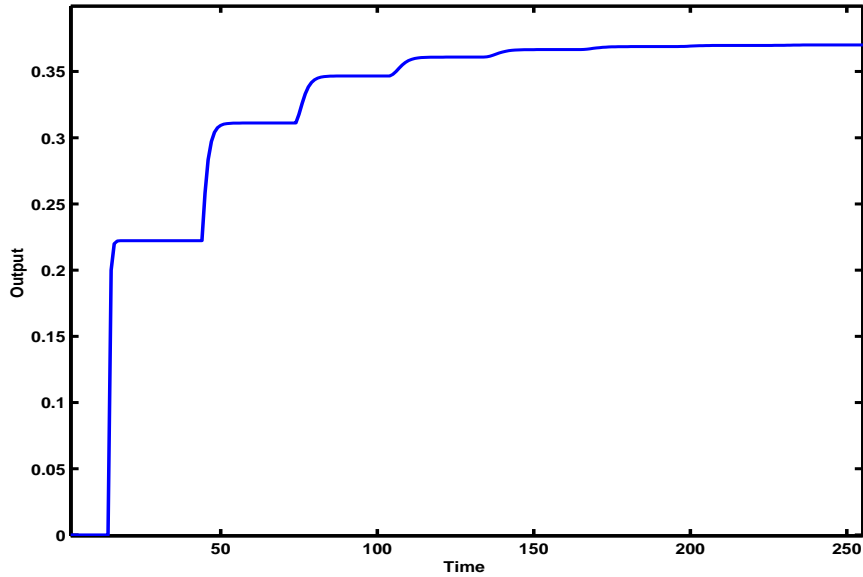


Figure 2: A typical stair-case like step response of a recycle system

### 3 Estimation of forward and recycle models

The identification exercise is divided into two parts - one of identifying the forward and recycle model delays and one of identifying the process dynamics. It is possible to pose the identification problem as an optimization problem over the space of delays and the model parameters of the dynamic portion. However, it will be a complicated nonlinear optimization problem and it is difficult to ensure consistency of the identified model. In this section, conditions under which asymptotic consistency of the identified model can be guaranteed are derived.

#### 3.1 Estimation of delay

A simple spectral method for estimating the forward and recycle delays is presented in this subsection. The estimated delays are then used in obtaining consistent estimates of the forward and recycle models. Write (1) as

$$y(t) = \frac{G_f}{1 - G_f G_r} u(t) + \frac{H}{1 - G_f G_r} e(t). \quad (5)$$

Let  $F(q)$  be a whitening filter of  $u(t)$  i.e.,

$$F(q)u(t) = w(t)$$

where  $w(t)$  is a white noise sequence independent of  $e(t)$ . Filtering both the output and the input with this filter, one can obtain

$$y_f = \frac{G_f}{1 - G_f G_r} w(t) + \frac{FH}{1 - G_f G_r} e(t)$$

A result which is useful in estimating the forward and recycle delays when the recycle delay is large, is now presented.

**Proposition 3.1** Assume that the impulse response coefficients of  $G_f$  and  $\frac{1}{1-G_f G_r}$  are monotonically decreasing. Also, assume that  $d$  is “large enough” compared to the time constant of the forward dynamic model. Then the cross-correlation between the filtered output and the input, written as  $R_{y_f w}(\tau)$  at lag  $\tau$ , attains local maxima at  $d_f$  and  $d_f + d$ .

**Proof:** The proof follows from the fact that  $w(t)$  is white and that the impulse response coefficients of  $G_f$  and  $\frac{1}{1-G_f G_r}$  have local maxima at  $d_f$  and at  $d_f + d$ .

□

**Remark 3.1** At first sight, the above result appears very restrictive. However, the assumption on monotonically decreasing impulse response coefficients is satisfied by many recycle processes in the chemical industry. This is a simple but very useful method in estimating consistent values of the delays in the forward and recycle streams.

**Example 3.1** Consider the recycle process in (4). Using the estimation method explained above, the cross-correlation between the filtered output and input is shown in fig. 3. A whitening filter,  $F(q)$ , for the input sequence is obtained by fitting an ARMA model (for instance using the function *arma* in Matlab). It is important to point out that this filter is not unique. Depending on the order of the filter used, one may get different realizations of noise,  $w(t)$ . However, the correlation plot shown is independent of the choice of  $F(q)$ . In this example a 10th order ARMA model is used for  $F(q)$ . ■

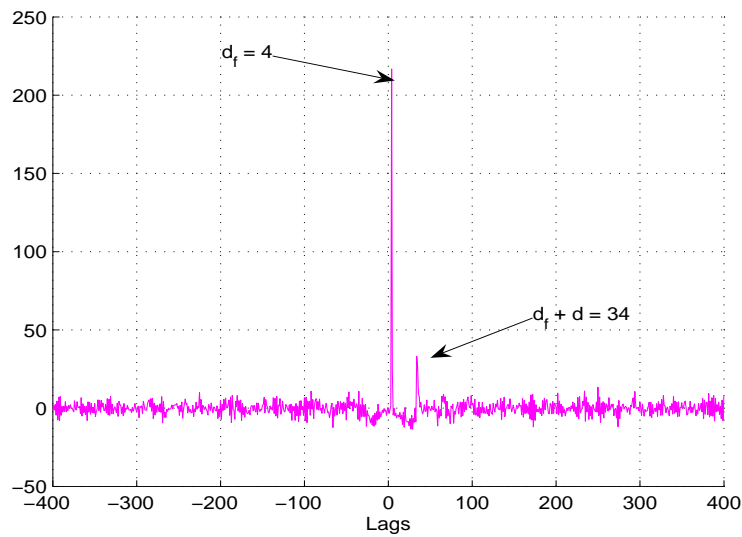


Figure 3: Cross-Correlation between filtered output and input

### 3.2 Asymptotic consistency

In this section it is shown that by treating lagged output as one of the inputs, it is possible to obtain consistent estimates of the forward and recycle models for delay dominant processes. Consider the

model equation in (1). Then the one-step ahead prediction error objective function for identification can be written as

$$\begin{aligned} V_N(\theta, Z^N) &= \frac{1}{N} \sum_{t=1}^N (y(t+1) - \hat{y}(t+1))^2 \\ &:= \frac{1}{N} \sum_{t=1}^N \varepsilon(t, \theta)^2 \end{aligned} \quad (6)$$

where  $\hat{y}(t+1)$  is the optimal one-step ahead predictor given by

$$\hat{y}(t+1) = \hat{H}^{-1} \hat{\Gamma} U(t+1) + (1 - \hat{H}^{-1}) y(t+1). \quad (7)$$

Then the prediction error estimate is given by

$$\hat{\theta}_N = \arg \min_{\theta \in D_{\mathcal{M}}} V_N(\theta, Z^N) \quad (8)$$

where  $D_{\mathcal{M}} \subset \mathbb{R}^n$  is a set in which the parameter vector is allowed to vary.  $n$  is the length of the parameter vector. The main result of this paper is now presented in the form a theorem.

**Theorem 3.1** Consider the model in (1) and the estimation method in (8). Also assume that

$$\tilde{P} := q^{-d_f} \sum_{k=0}^n p_k q^{-k} \quad \tilde{H} := \sum_{k=0}^m h_k q^{-k}$$

where  $d \geq n+1 \geq 1$  and  $d \geq m \geq 1$ . Then under the assumptions presented in the previous section and assuming that  $\hat{H}(q)$  is independent of the parameter vector,

$$\hat{\theta}_N \rightarrow D_c = \arg \min_{\theta \in D_{\mathcal{M}}} \int_{-\pi}^{\pi} |\hat{H}|^{-2} |\tilde{P}|^2 \Phi_u + |\hat{H}|^{-2} |\tilde{Q}|^2 \Phi_{y_d} + |\hat{H}|^{-2} |\tilde{H}|^2 \Phi_e d\omega \quad (9)$$

with probability one. Hence, it is possible to obtain consistent estimates of  $\tilde{P}$  and  $\tilde{Q}$ .

**Proof:** As the data length,  $N$ , tends to infinity, the prediction error identification objective function in (6) converges almost surely to (Forssell and Ljung 1999)

$$\begin{aligned} \lim_{N \rightarrow \infty} V_N(\theta, Z^N) &\rightarrow \bar{V}(\theta) \\ &:= \frac{1}{2\pi} \int_{-\pi}^{\pi} \left( \begin{bmatrix} \tilde{\Gamma} & \tilde{H} \end{bmatrix} \begin{bmatrix} \Phi_U(\omega) & \Phi_{Ue}(\omega) \\ \Phi_{eU}(\omega) & \Phi_e(\omega) \end{bmatrix} \begin{bmatrix} \tilde{\Gamma} \\ \tilde{H} \end{bmatrix}^* (\hat{H} * \Phi_e * \hat{H}^*)^{-1} \right) d\omega \end{aligned}$$

This objective function can be expanded to

$$\bar{V}(\theta) := \frac{1}{2\pi} \int_{-\pi}^{\pi} \left( \begin{bmatrix} \tilde{P} & \tilde{Q} & \tilde{H} \end{bmatrix} \begin{bmatrix} \Phi_u(\omega) & \Phi_{uy_d}(\omega) & \Phi_{ue}(\omega) \\ \Phi_{y_d u}(\omega) & \Phi_{y_d}(\omega) & \Phi_{y_d e}(\omega) \\ \Phi_{eu}(\omega) & \Phi_{ey_d}(\omega) & \Phi_e(\omega) \end{bmatrix} \begin{bmatrix} \tilde{P} \\ \tilde{Q} \\ \tilde{H} \end{bmatrix}^* (\hat{H} * \Phi_e * \hat{H}^*)^{-1} \right) d\omega \quad (10)$$

where  $y_d$  is the delayed output,  $y(t-d)$ . Notice that the minimum of this objective function occurs at the true parameters of the true models *i.e.*, when  $\hat{P} = P$ ,  $\hat{Q} = Q$  and  $\hat{H} = H$ . If the spectral matrix at the center is non-diagonal, it is not possible to guarantee unbiased estimates of the process and noise models unless their correct structures are known. Typically, guessing the true structure of the noise

model is very difficult and as a result, often, minimization of the above objective function leads to biased estimates. However, for the problem under consideration, all the terms involving cross-spectra are identically zero. Thus allowing unbiased estimates. In order to prove this, expand the integral in (10) to,

$$\begin{aligned} \bar{V}(\theta) &:= \frac{1}{2\pi} \int_{-\pi}^{\pi} \left( |\tilde{P}|^2 \Phi_u + |\tilde{Q}|^2 \Phi_{y_d} + |\tilde{H}|^2 \Phi_e + \tilde{P} \Phi_{uy_d} \tilde{Q}^* + \tilde{P} \Phi_{ue} \tilde{H}^* + \tilde{Q} \Phi_{y_d u} \tilde{P}^* \right. \\ &\quad \left. + \tilde{Q} \Phi_{y_d e} \tilde{H}^* + \tilde{H} \Phi_{eu} \tilde{P}^* + \tilde{H} \Phi_{ey_d} \tilde{Q}^* \right) |\hat{H}|^{-2} \Phi_e d\omega. \end{aligned} \quad (11)$$

Now introduce the following notation,

$$\begin{aligned} T_1 &:= \frac{1}{2\pi} \int_{-\pi}^{\pi} \tilde{P} \Phi_{uy_d} \tilde{Q}^* |\hat{H}|^{-2} \Phi_e d\omega = \frac{1}{2\pi} \int_{-\pi}^{\pi} (\tilde{Q} \Phi_{y_d u} \tilde{P}^*)^* |\hat{H}|^{-2} \Phi_e d\omega \\ T_2 &:= \frac{1}{2\pi} \int_{-\pi}^{\pi} \tilde{Q} \Phi_{y_d e} \tilde{H}^* |\hat{H}|^{-2} \Phi_e d\omega = \frac{1}{2\pi} \int_{-\pi}^{\pi} (\tilde{H} \Phi_{ey_d} \tilde{Q}^*)^* |\hat{H}|^{-2} \Phi_e d\omega. \end{aligned} \quad (12)$$

**Term I:** Assume, without loss of generality, that  $\hat{H} = 1$ , *i.e.*, consider output-error structures. Also, without loss of generality assume that the input is white<sup>1</sup>. Then

$$\begin{aligned} T_1 &:= \frac{1}{2\pi} \int_{-\pi}^{\pi} \hat{H}^{-1} \tilde{P} \Phi_{uy_d} \tilde{Q}^* (\hat{H}^{-1})^* \Phi_e d\omega \\ &= \frac{1}{2\pi} \int_{-\pi}^{\pi} \tilde{P} \Phi_{uy_d} \tilde{Q}^* \Phi_e d\omega \end{aligned} \quad (13)$$

Now note that

$$\begin{aligned} y(t-d) &= \frac{P}{1-Qq^{-d}} q^{-d} u(t) + \frac{Hq^{-d}}{1-Qq^{-d}} e(t) \\ &:= q^{-d-d_f} \sum_{k=0}^{\infty} g_k q^{-k} u(t) + \frac{Hq^{-d}}{1-Qq^{-d}} e(t) \end{aligned} \quad (14)$$

for some impulse response coefficients,  $g_k$ . Therefore,

$$\begin{aligned} \Phi_{uy_d}(\omega) &= (\Phi_{y_d u}(\omega))^* \\ &= e^{i\omega(d+d_f)} \sum_{k=0}^{\infty} g_k e^{ik\omega} \Phi_u(\omega) \end{aligned} \quad (15)$$

Consider the case where  $\tilde{P} = q^{-d_f} \sum_{k=0}^n p_k q^{-k}$  and define  $\tilde{Q} = \sum_{k=0}^{\infty} q_k q^{-k}$  for some impulse response coefficients  $p_k$  and  $q_k$ . Then

$$\begin{aligned} T_1 &:= \frac{1}{2\pi} \int_{-\pi}^{\pi} \left( e^{-id_f \omega} \sum_{k=0}^n p_k e^{-ik\omega} \right) \left( e^{i(d+d_f)\omega} \sum_{k=0}^{\infty} g_k e^{ik\omega} \right) \left( \sum_{k=0}^{\infty} q_k e^{ik\omega} \right) \Phi_e \Phi_u d\omega \\ &= \frac{1}{2\pi} \int_{-\pi}^{\pi} \left( e^{id\omega} \sum_{k=0}^n p_k e^{-ik\omega} \right) \left( \sum_{k=0}^{\infty} g_k e^{ik\omega} \right) \left( \sum_{k=0}^{\infty} q_k e^{ik\omega} \right) \Phi_e \Phi_u d\omega \end{aligned} \quad (16)$$

Observe that if  $d \geq n+1$ , then the integrand in the above expression can be expressed as a summation of infinite exponentials with positive powers *i.e.*,

$$T_1 := \sum_{k=1}^{\infty} \frac{\Phi_e \Phi_u}{2\pi} \int_{-\pi}^{\pi} s_k e^{ik\omega} d\omega \quad (17)$$

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<sup>1</sup>Please note that if either the chosen noise model is not unity or if the input spectrum is not white, it is possible to filter the input and output with appropriate filters and achieve this condition.

for some constants  $s_k$ . The integral is zero for all positive  $k$ . Therefore,  $T_1$  is zero if  $d \geq (n + 1)$  and

$$\tilde{P} = q^{-d_f} \sum_{k=0}^n p_k q^{-k} \quad (18)$$

**Term II:** Now consider the term involving  $\Phi_{y_d e}$ . This term can be written as

$$\begin{aligned} T_2 &:= \frac{1}{2\pi} \int_{-\pi}^{\pi} \hat{H}^{-1} \tilde{Q} \Phi_{y_d e} \tilde{H}^* (\hat{H}^{-1})^* \Phi_e d\omega \\ &= \frac{1}{2\pi} \int_{-\pi}^{\pi} \tilde{Q} \Phi_{y_d e} \tilde{H}^* \Phi_e d\omega \end{aligned} \quad (19)$$

Now assume that

$$\tilde{H} := \sum_{k=0}^m h_k q^{-k}$$

i.e., the error in the impulse response coefficients of the noise model approaches zero after  $m$  impulse response coefficients. Then from (14),

$$\begin{aligned} \Phi_{y_d e} &= \frac{H(e^{i\omega}) e^{-id\omega}}{1 - Q(e^{i\omega}) q^{-d}} \Phi_e(\omega) \\ &:= e^{-id\omega} \sum_{k=0}^{\infty} r_k e^{-ik\omega} \Phi_e(\omega) \end{aligned}$$

Hence,

$$\begin{aligned} T_2 &= \frac{1}{2\pi} \int_{-\pi}^{\pi} \left( \sum_{k=0}^{\infty} q_k e^{-ik\omega} \right) \left( e^{-id\omega} \sum_{k=0}^{\infty} r_k e^{-ik\omega} \right) \left( \sum_{k=0}^m h_k e^{ik\omega} \right) \Phi_e^2(\omega) d\omega \\ &:= \sum_{k=1}^{\infty} \frac{\Phi_e \Phi_e}{2\pi} \int_{-\pi}^{\pi} \nu_k e^{-ik\omega} d\omega \end{aligned} \quad (20)$$

The above integral is again zero for all positive  $k$ . Noticing that  $\Phi_{u_e}(\omega) = 0$  for all  $\omega$ , it is easy to see that (11) reduces to (9)

□

**Remark 3.2** The conditions on the forward model are clearly satisfied if it is a pure gain model with delay. They are approximately satisfied when the impulse response coefficients of  $\tilde{P}$  decay very fast compared to the length of the total delay,  $d$ . It is straightforward to show that other off-diagonal terms in the spectral matrix are also zero under these conditions. If all the off-diagonal terms are zero, then the identification objective function reduces to

$$\bar{V}(\theta) = \frac{1}{2\pi} \int_{-\pi}^{\pi} |\hat{H}|^{-2} |\tilde{P}|^2 \Phi_u + |\hat{H}|^{-2} |\tilde{Q}|^2 \Phi_{y_d} + |\hat{H}|^{-2} |\tilde{H}|^2 \Phi_e d\omega \quad (21)$$

From this expression, it is clear that even if the noise model is fixed to unity, the minimum occurs at  $\hat{P} = P$  and  $\hat{Q} = Q$  provided the model set includes the true model. Hence, the above identification algorithm results in unbiased estimates for delay dominant recycle processes. Also, note that  $\Phi_{y_d}(\omega)$  can be replaced by  $\Phi_y(\omega)$  in the above expression.

**Corollary 3.1** Assuming that the process output was zero for all times before a certain time in the past, as  $d \rightarrow \infty$ , (21) reduces to the open loop prediction error identification objective function i.e.,

$$\bar{V}(\theta) = \frac{1}{2\pi} \int_{-\pi}^{\pi} |\hat{H}|^{-2} |\tilde{P}|^2 \Phi_u + |\hat{H}|^{-2} |\tilde{H}|^2 \Phi_e d\omega \quad (22)$$

### 3.3 Asymptotic transfer function variance

Asymptotic transfer function variance estimates would provide insight into the spectral characteristics of the input signal that would lead to optimal estimates. Hence, in this section expressions for the variance of the transfer function estimate are provided. The identification method proposed in the previous section is similar to a closed loop identification exercise except for the use of lagged output as an input. The asymptotic variance results derived in Ljung (1999) can be used assuming that the true process is in the set of models considered for identification. Therefore,

$$\text{cov} \begin{pmatrix} \hat{P}(e^{i\omega}) \\ \hat{Q}(e^{i\omega}) \\ \hat{H}(e^{i\omega}) \end{pmatrix} \sim \frac{n}{N} \Phi_\nu(\omega) \begin{bmatrix} \Phi_u(\omega) & 0 & 0 \\ 0 & \Phi_{y_d}(\omega) & 0 \\ 0 & 0 & \Phi_e(\omega) \end{bmatrix}^{-1}. \quad (23)$$

where  $\nu(t) = He(t)$  and  $n$  is the model order. In other words,

$$\text{var}(\hat{P}(e^{i\omega})) = \text{var}(\hat{G}_f(e^{i\omega})) \sim \frac{n}{N} \frac{\Phi_\nu(\omega)}{\Phi_u(\omega)} \quad (24)$$

and

$$\text{var}(\hat{Q}(e^{i\omega})) = \text{var}(\hat{G}_f(e^{i\omega})\hat{G}_r(e^{i\omega})) \sim \frac{n}{N} \frac{\Phi_\nu(\omega)}{\Phi_{y_d}(\omega)} \quad (25)$$

The recycle model estimate is obtained by dividing  $\hat{Q}$  by  $\hat{P}$ . The variance of the recycle model can be obtained by linear approximation of this ratio.

$$\begin{aligned} \hat{G}_r &= \frac{\hat{Q}}{\hat{P}} \\ &= \frac{Q + \Delta Q}{P + \Delta P} \\ &\approx \frac{Q}{P} \left( 1 + \frac{\Delta Q}{Q} - \frac{\Delta P}{P} \right) \end{aligned} \quad (26)$$

where the approximation is obtained by ignoring second order terms. Again, ignoring the second order terms, the variance of recycle model can be written as

$$\text{var}(\hat{G}_r(e^{i\omega})) = \frac{1}{|G_f|^2} \text{var}(\hat{Q}) + \frac{|G_f G_r|^2}{|G_f|^4} \text{var}(\hat{G}_f) \quad (27)$$

The frequency arguments of the transfer functions in the above expression are ignored for brevity. Substituting for the variance of  $\hat{Q}$  in (27),

$$\text{var}(\hat{G}_r(e^{i\omega})) \sim \frac{n}{N} \frac{\Phi_\nu}{|G_f|^2} \left[ \frac{|1 - G_f G_r|^2}{|G_f|^2 \Phi_u + |H|^2 \Phi_e} + \frac{|G_r|^2}{\Phi_u} \right] \quad (28)$$

Assuming that the signal to noise ratio is large *i.e.*,

$$\frac{|H|^2 \Phi_e}{|G_f|^2 \Phi_u} \ll 1 \quad \forall \omega \quad (29)$$

and therefore,

$$\text{var}(\hat{G}_r(e^{i\omega})) \sim \frac{n}{N} \frac{\Phi_\nu}{|G_f|^2} \left[ \frac{|1 - G_f G_r|^2}{|G_f|^2 \Phi_u} + \frac{|G_r|^2}{\Phi_u} \right] \quad (30)$$

**Remark 3.3** The above expression for the variance of recycle model indicates that good excitation is needed in the frequency regions where  $|G_r|$  is large. Interestingly, in the regions where  $|G_f|$  is large, it helps in reducing the variance in  $\hat{G}_r$ . This can be explained as follows - a good excitation signal for the recycle model is one where  $y(t)$  has the necessary frequency contents in it for the identification of recycle model. Since  $y(t)$  is the input to the recycle model, the recycle model quality will be good in the frequencies that are *passed* by the forward model. On the other hand, in regions where  $|G_f|$  is small, the variance of  $\hat{G}_r$  is large. In other words, the variance of the recycle model is large outside the bandwidth of the forward model and this is a fundamental limitation on the quality of the estimated recycle model. Hence, a good identification input should have enough excitation in regions where  $|G_r|$  and  $|H|$  are expected to be large.

### 3.4 Optimal Input Design

The quality of an estimated model depends on the type of input used. Depending on the spectral characteristics of the input, different frequency components of the forward and recycle models are emphasized in the estimated models. In this section, expressions for optimal input, based on minimizing the mean-square error in the identified models, are derived.

In order to find an optimal input, one has to define an objective function with respect to which the input is optimal. The most natural objective function is the mean squared error in the outputs of the forward and recycle models. Let the output of the recycle model be denoted by  $y_r(t)$ . Then

$$y(t) = \frac{G_f}{1 - G_f G_r} u(t) + \frac{G_r H}{1 - G_f G_r} e(t) \quad (31)$$

$$y_r(t) = G_r y(t) \quad (32)$$

Henceforth, the frequency arguments are ignored whenever it is obvious, for simplicity in notation. A measure of model quality that depends on the data length can then be defined as

$$J^N = E[(y(t) - \hat{y}^N(t))^2] + E[(y_r(t) - \hat{y}_r^N(t))^2] \quad (33)$$

where  $\hat{y}^N(t)$  and  $\hat{y}_r^N(t)$  are the outputs of the forward and recycle models based on data length of  $N$ , obtained by replacing  $G_f, G_r, H$  in (31) and (32) by  $\hat{G}_f, \hat{G}_r, \hat{H}$  respectively. Using the notation,  $f(T) = \begin{bmatrix} \frac{G_f}{1 - G_f G_r} & \frac{G_r H}{1 - G_f G_r} \end{bmatrix}$ , where  $T = [G_f \ G_r \ H]$ ,

$$\begin{aligned} y(t) &= f(T)[u(t) \ e(t)]^T \\ \hat{y}^N(t) &= f(\hat{T})[u(t) \ e(t)]^T \end{aligned}$$

Now linearizing the expression for  $\hat{y}^N(t)$  around  $T$ ,

$$y(t) - \hat{y}^N(t) \approx \tilde{T} \frac{\partial f^T}{\partial T} [u(t) \ e(t)]^T.$$

where  $\tilde{T} = [\tilde{G}_f \ \tilde{G}_r \ \tilde{H}]$ . This type of a first order approximation is good if the identified model is “close” to the true model. For large identification errors, the approximation may not work well. However, it provides a method to design optimal excitation signal for identification. Clearly, if the approximation is not good enough, the optimal input designed may not be “optimal”. Nevertheless, the method presented below provides insight into the frequency components of an optimal excitation signal. Now using, Parseval’s theorem,

$$J^N = \frac{1}{2\pi} \int_{-\pi}^{\pi} \tilde{T}(e^{-i\omega}) F(e^{-i\omega}) \begin{bmatrix} \Phi_{u^*} & 0 \\ 0 & \Phi_e \end{bmatrix} F^T(e^{i\omega}) \tilde{T}^*(e^{i\omega}) d\omega \quad (34)$$

where

$$\tilde{T}(e^{-i\omega}) = \begin{bmatrix} \tilde{G}_f & \tilde{G}_r & \tilde{H} \end{bmatrix} \quad (35)$$

$$F(e^{-i\omega}) = \frac{1}{(1 - G_f G_r)^2} \begin{bmatrix} 1 + G_r & G_r^2 H(1 + G_r) \\ G_f(1 + G_f) & H(1 + G_r(2 - G_f G_r)) \\ 0 & G_r(1 + G_r)(1 - G_f G_r) \end{bmatrix} \quad (36)$$

and  $u^*$  is a “typical” input signal to the process. This is useful in shaping the bias and variance errors. Please note that this is not same as the identification input  $u$ . Therefore, the above objective function for large  $N$  and high model orders can be written as

$$J = E[J^N] = \frac{1}{2\pi} \int_{-\pi}^{\pi} \text{tr}(E((\tilde{T})^* \tilde{T})C(\omega))d\omega \quad (37)$$

where

$$C(\omega) = F(e^{-i\omega}) \begin{bmatrix} \Phi_{u^*} & 0 \\ 0 & \Phi_e \end{bmatrix} F^T(e^{i\omega}) \quad (38)$$

After some straightforward calculations it is easy to see that the diagonal elements of  $C$  are

$$\begin{aligned} |1 - G_f G_r|^4 C_{11} &= |1 + G_r|^2 \Phi_{u^*} + |G_r|^4 |H|^2 |1 + G_r|^2 \Phi_e \\ |1 - G_f G_r|^4 C_{22} &= |G_f|^2 |1 + G_f|^2 \Phi_{u^*} + |H|^2 |1 + G_r(2 - G_f G_r)|^2 \Phi_e \\ |1 - G_f G_r|^4 C_{33} &= |G_r|^2 |1 + G_r|^2 |1 - G_f G_r|^2 \Phi_e \end{aligned}$$

If the bias errors are ignored, then

$$E(\tilde{T}^N (\tilde{T}^N)^*) = \text{cov} \begin{pmatrix} \tilde{G}_f \\ \tilde{G}_r \\ \tilde{H} \end{pmatrix} \quad (39)$$

Note that this is a diagonal covariance matrix. The following result, based on Theorem 13.3 in Ljung (1999), can now be stated.

**Theorem 3.2** Consider the optimization problem

$$\min_{\Phi_u} \{J : \int \alpha \Phi_u \leq 1\} \quad (40)$$

where  $\alpha$  is a normalization constant, then the optimal input is given by

$$\Phi_u^{opt}(\omega) = \mu \sqrt{\Phi_\nu \left[ C_{11} + \frac{|G_r|^2}{|G_f|^2} \left( 1 + \frac{|1 - G_f G_r|^2}{|G_r|^2 |G_f|^2} \right) C_{22} \right]} \quad (41)$$

for some constant  $\mu$ .

**Proof:** Since the covariance matrix is diagonal, the objective function can be written as

$$J = \frac{1}{2\pi} \int_{-\pi}^{\pi} \text{var}(\hat{G}_f) C_{11} + \text{var}(\hat{G}_r) C_{22} + \text{var}(\hat{H}) C_{22} d\omega \quad (42)$$

As the variance of  $\hat{H}$  does not depend on input, the optimization problem is equivalent to

$$\min_{\Phi_u} \left\{ \frac{1}{2\pi} \int_{-\pi}^{\pi} \text{var}(\hat{G}_f) C_{11} + \text{var}(\hat{G}_r) C_{22} d\omega : \int \alpha \Phi_u \leq 1 \right\} \quad (43)$$

Substituting for the variances of  $\hat{G}_f$  and  $\hat{G}_r$ ,

$$\min_{\Phi_u} \left\{ \frac{1}{2\pi} \int_{-\pi}^{\pi} \frac{\Phi_\nu}{\Phi_u} \left[ C_{11} + \frac{|G_r|^2}{|G_f|^2} \left( 1 + \frac{|1 - G_f G_r|^2}{|G_r|^2 |G_f|^2} \right) C_{22} \right] d\omega : \int \alpha \Phi_u \leq 1 \right\} \quad (44)$$

The final result follows directly from Theorem 13.3 in Ljung (1999).

□

**Remark 3.4** Noting that  $\Phi_{y_r u} = \frac{|G_r|^2 |G_f|^2}{|1 - G_f G_r|^2}$ , the optimal input at any frequency is proportional to the magnitude of the recycle model at that frequency and inversely proportional to the cross-spectrum between recycle signal,  $y_r$  and input. At frequencies where  $\Phi_{y_r u}$  is large, the recycle stream itself provides enough excitation and hence, large excitation is not needed in these frequencies. On the other hand, to obtain a reliable recycle model, good excitation is required at least up to the bandwidth of the recycle model. Consider the case where  $G_f$  has a smaller bandwidth than  $G_r$ . The above optimal input ensures that it has good excitation up to the bandwidth of  $G_r$ . From (30), it is easy to see that the variance of recycle model varies inversely with  $|G_f|^4 \Phi_u$ . Hence, good excitation at least up to the bandwidth of the recycle model ensures that the variance of the recycle model does not deteriorate adversely for frequencies larger than the bandwidth of the forward model.

## 4 Industrial Example

In this application, gray-box identification of dynamic models for the bleaching operation in a BCTMP process at Millar Western, in Whitecourt, AB, Canada was performed. Pulp is made from the cellulose fibres of wood chips. There are two basic ways to make pulp. The most common process reduces wood chips to their individual fibres through strong chemical treatment to produce a type of pulp called kraft. On the other hand, a combination of mild chemicals, heat and mechanical action is used to produce, Bleached Chemi-Thermo-Mechanical Pulp (BCTMP). The pulp produced in this way is also referred to as high-yield pulp, because the manufacturing process produces more pulp per tree than traditional pulping methods. Millar Western's pulp mill at Whitecourt produces pulp of this variety.

The BCTMP mill at Whitecourt, consists of two parallel units which are nearly identical in design. These parallel units are named Line 1 and Line 2 respectively. They are subject to nearly the same stochastic environment. However, the production and operating conditions are different. Line 1 produces pulp which generally has lower brightness targets than Line 2 because it uses more of softwood as the raw material in contrast with Line 2 where the raw material is predominantly of the hardwood variety. This is illustrated in Fig. 4, where the plot has been re-scaled for confidentiality reasons. However, it is desired to have a single model for both lines for reasons of long-term use. The reason for using a single model across two different units is that there might be operational constraints in the future which may require the use of either line to produce any of the currently produced grades and it is not possible from an economic viewpoint to re-identify models whenever there is a change in the operational strategy.

Online brightness sensors have been developed for most pulp bleaching processes. They have also been tried on this particular market BCTMP process. However, mill engineers found them to be difficult to use in practice, because they required multiple calibrations to cover the range of grades produced and were expensive to justify from an economic viewpoint. In addition, these sensors were found to have a weekly maintenance requirement which from an operational perspective, can be expensive. On the other hand, the model-based soft-sensors developed in this project have been providing consistently good predictions and are therefore being preferred for brightness and tensile control.

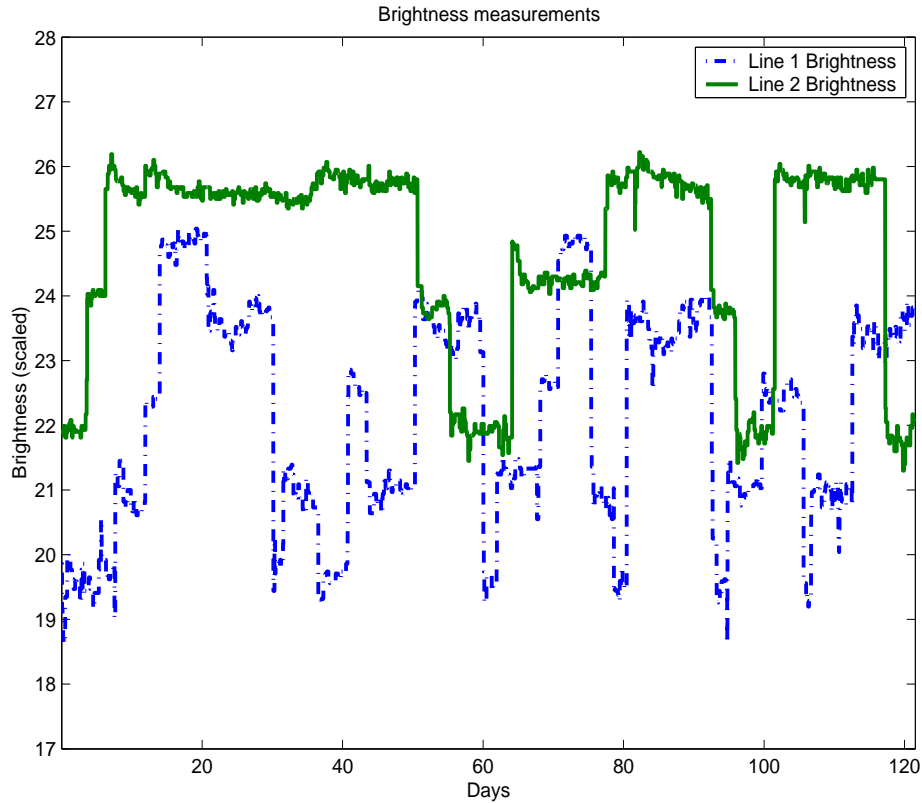


Figure 4: Line 1 and Line 2 Brightness measurements

A simplified version of this schematic is presented in Fig. 5. Each unit has four manipulated inputs, two measured disturbance variables and two outputs. The manipulated inputs are chemical add-rates (Peroxide and Caustic) to the two towers. There are two additional measurements (Aspen and Freeness) which are classified as measured disturbances. The outputs are the irregularly measured quality variables *viz.*, Brightness and Tensile strength of pulp. The forward path dynamics of the process can be captured by delay dominant low-order models. The presence of the recycle stream alters the dynamics of the process significantly. Hence it is necessary to take this into account while building a model for the process. Consequently, the average fraction of chemical being recycled must be captured in model.

The observed dynamics were used to characterize this process as a time delay dominant recycle process. The following observations justify this hypothesis. The inputs are sampled every 10 minutes. The process consists of 2 towers (P1 and P2 tower). The chemical add-rates are manipulated upstream of each tower. It is observed that on changing the P1 add-rates, the output variables change 5 hours and 20 minutes later. On changing the P2 add-rates, the output variables change 3 hours and 40 minutes later. It is also observed that the direct change in the brightness and tensile is almost instantaneous, the change occurs with a couple of 10 minute sampling intervals, after the delays mentioned in the previous point. This characterizes the fast dynamics of the process. In addition to the direct change in the brightness and tensile there is a change because of the chemicals recycled to the front-end of the bleach plant.

Initially black-box routines were used to develop models, without explicitly taking the recycle effect into account. These models did not separate the forward dynamics from the recycle dynamics. In

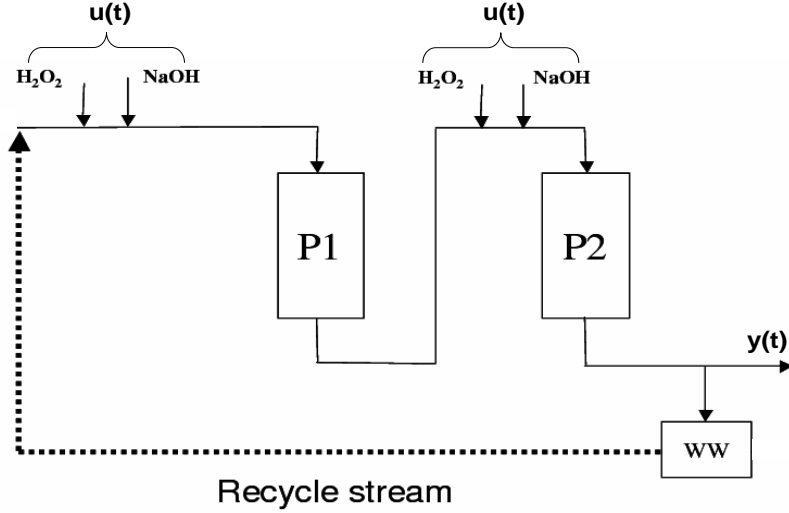


Figure 5: Simplified flow sheet of bleaching operation in a BCTMP mill

addition, because black-box routines were used, an approximation of the overall dynamics of the process was obtained. Even though the predictions are good, the model is not reliable because the step-responses do not match with the expected fast-dynamics and characteristic staircase shape associated with the recycle effect.

### Online Predictions

The results of online prediction of Line 2 Brightness and Line 1 Tensile using the proposed method are shown in Figs. 6 and 7 for a period of 12 months. The values on the y-axis have been masked for confidentiality reasons. The predictions for Line 1 Brightness and Line 2 Tensile are similar to the results shown and these figures are not shown in the interest of conserving space. All these predictions are infinite horizon predictions. It is clear from the high values of the correlation coefficient and the low RMSE values (shown in the respective figures) that the predictions are quite satisfactory. In addition, the model dynamics conform to what is known from prior knowledge. Full details about the implementation and other issues such as irregular sampling involved in this problem are dealt in Raghavan *et al.* (2005). The final FIR models identified for predicting Brightness and Tensile using this procedure are summarized below. For confidentiality reasons the coefficients have been re-scaled and the intercept term is masked.  $y_1(t)$  is the brightness and  $y_2(t)$  is the tensile strength<sup>2</sup>.  $u_1, u_2, u_3, u_4$  are the four inputs corresponding to peroxide and caustic chemical add-rates to the two towers.

$$\begin{aligned}
 y_1(t) &= R(q)\{u_1(t-32) + 5.6u_2(t-22) + 6.4(u_3(t-32) + u_4(t-22)) - 0.2u_1(t-32)(u_3(t-32) \\
 &\quad + u_4(t-22)) - 1.2u_2(t-22) \times (u_3(t-32) + u_4(t-22))\} + 0.2u_5(t) + 0.01u_6(t) + \alpha_1 \\
 y_2(t) &= R(q)\{-130u_1(t-32) + 30u_2(t-22) + 480u_3(t-32) + 430u_4(t-22) + u_1(t-32)u_6(t) \\
 &\quad - 0.4u_2(t-22)u_6(t) - 190u_1(t-32)u_3(t-32) - 300u_2(t-22)u_4(t-22) + 76u_1(t-32)u_3(t-32)^2 \\
 &\quad + 110u_2(t-22)u_4(t-22)^2\} - 18u_5(t) - 4u_6(t) + \alpha_2
 \end{aligned}$$

where  $R(q) = (1 + a_R q^{-32} + a_R^2 q^{-64} + a_R^3 q^{-96} + a_R^4 q^{-128})$  and  $a_R = 0.17$ .

<sup>2</sup>Please note that the final model is nonlinear and recycle streams are just one aspect of a bigger problem that has been solved in Raghavan *et al.* (2005)

### Brightness Predictions

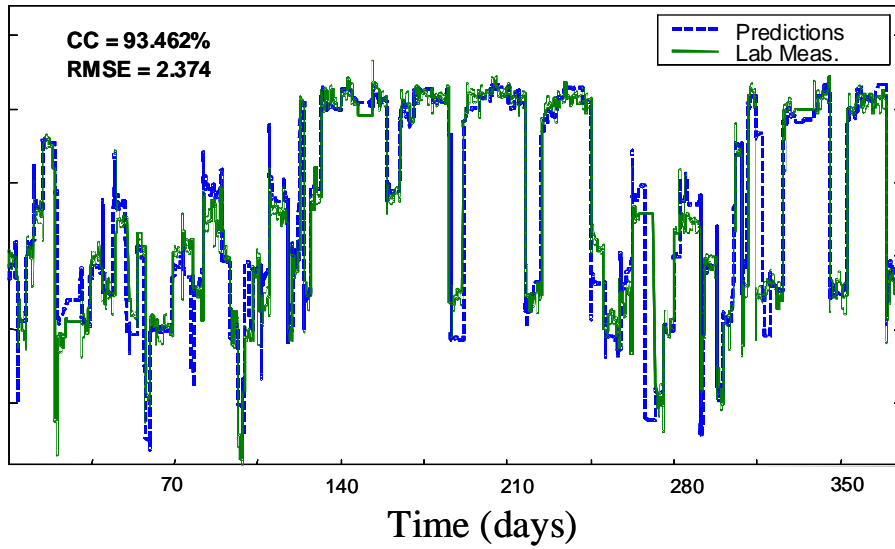


Figure 6: Line 2 Brightness Predictions

### Tensile Predictions

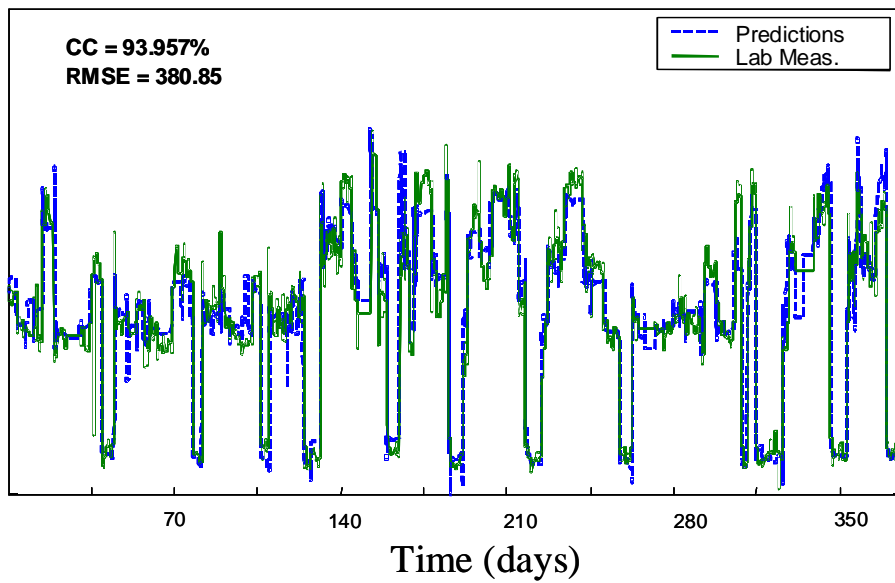


Figure 7: Line 1 Tensile Predictions

## 5 Conclusions

It is shown that consistent estimates of both forward and recycle models for delay dominant recycle systems can be obtained by using a lagged output as one of the inputs. Expressions for the variance of estimated transfer functions are provided. Finally, an optimal input that provides the best mean-squared error of the estimated models is also presented. The results obtained in this paper are validated through an industrial example.

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