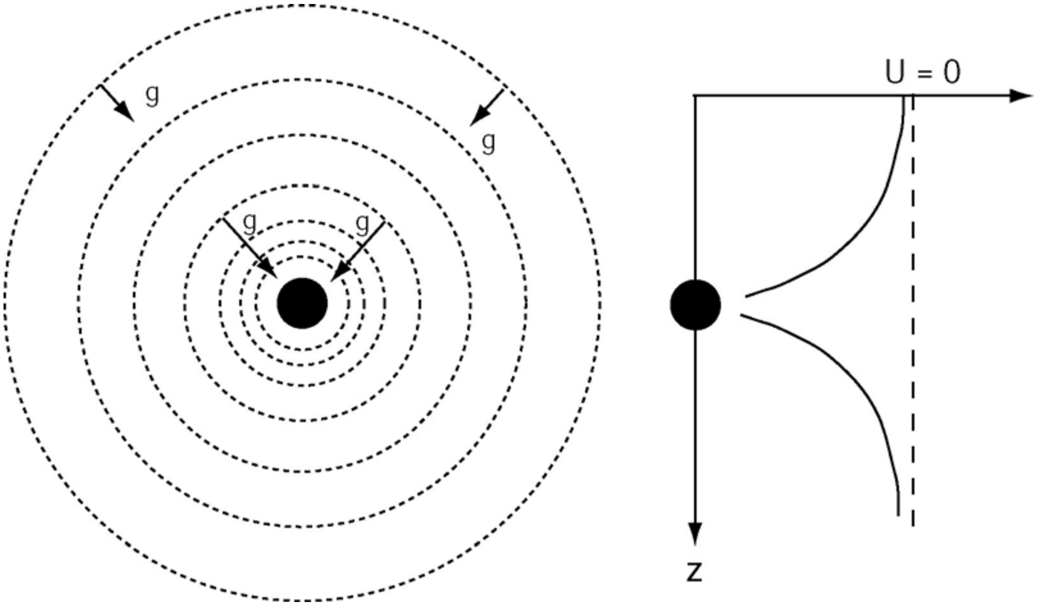
B5 : Gravitational potential and the geoid

B5.1 Gravitational potential energy

We have considered the Earth's gravity field in terms of the acceleration that a test mass would experience (**g**).

An alternative way to visualize the gravity field is by using the **gravitational potential energy (**U**)**. This has some advantages because **g** is a vector while U is a scalar.

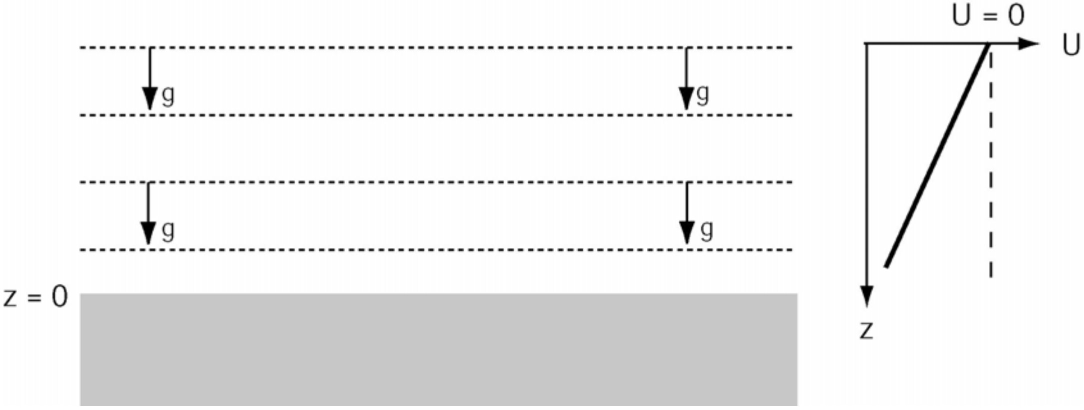
Example 1 : Point mass at z = 0



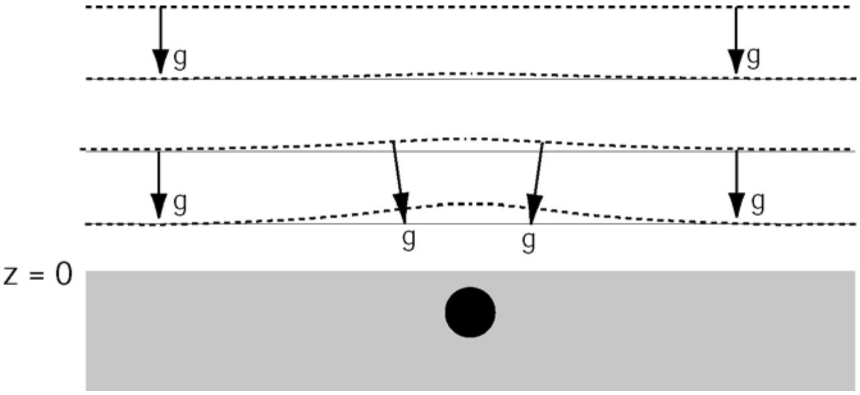
- Each of the circles shown above represents an **equipotential surface**. That means no work is required to move a mass along the surface.
- However, moving away from the mass requires that work is done and converted into gravitational potential energy.
- If you move towards the mass, then some gravitational potential energy is converted into kinetic energy (falling).
- The acceleration (pull) of gravity (**g**) is at right angles to the equipotential surface.
- Rate of change of potential with distance is proportional to gravitational acceleration ($\boldsymbol{g} = -\nabla U$)

Example 2 : g constant as elevation increases. $\boldsymbol{g} = -\nabla U$

Earth surface at z=0



Example 3 : Point mass located just below the Earth's surface

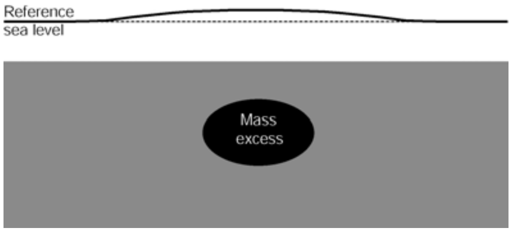


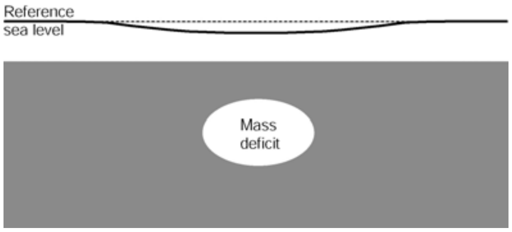
- Examples 1 and 2 can be combined to represent a mass buried in the Earth
- Note that this has the effect of bending the equipotential surfaces upwards.
- **g** always orthogonal to equipotential surfaces

Equipotentials and the sea surface

- Water will flow downhill until it has reached the lowest possible level, since it is not rigid. Thus on a calm day, when ocean currents and weather are ignored, the sea surface is an **equipotential surface**. Large mounds of water do not persist in the open ocean!
- Suppose that there is some excess mass (high density) buried under the seafloor. This will distort the equipotential surface so that it bulges **upwards** and results in a small rise in sea level above the mass excess.

- Can also think of this as the mass excess pulling water towards itself.





Similarly, if there is a mass deficit (low density) then a slight **depression** will form in the sea surface.

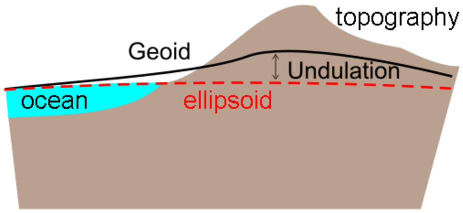
Question : Would this be a good place for water skiing?

Hint - How is the pull of gravity related to the sea surface?

5.2 The Geoid

- The **geoid** is defined as the **equipotential surface that coincides with mean sea level**. If the Earth had uniform density and no topography, then the geoid and International Reference Ellipsoid (IRE) would be identical.
- However, non uniform density and topography result in the geoid being relatively bumpy.
- In the **oceans**, the geoid is defined by the sea-level and can be measured with radar altimetry.
- On **land** the geoid corresponds to the level that a hypothetical ocean would have. Location must be computed from gravity measurements (land based and satellite data). See details of GRACE described below.

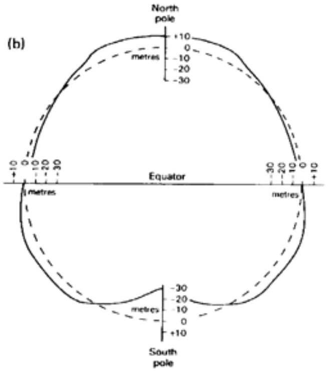
- Differences between geoid and IRE are called **geoid undulations**.

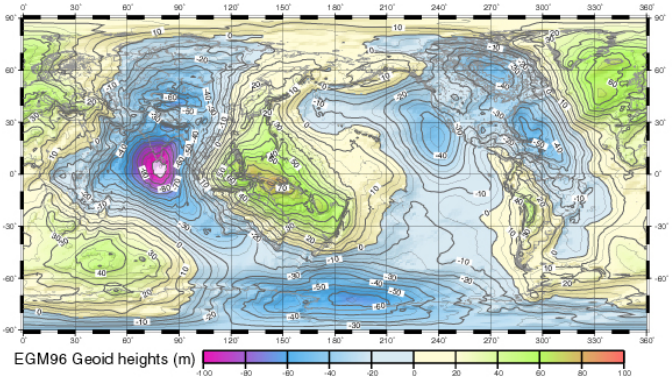


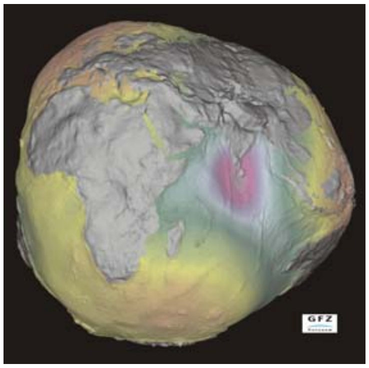
An interactive program that calculates the height of the Geoid:
http://www.ngs.noaa.gov/cgi-bin/GEOID_STUFF/geoid99_prompt1.prl

Geoid elevation in Edmonton = ______ m

The geoid was increasingly well defined by satellite data collected from 1957 onwards. Long wavelength features are shown below.

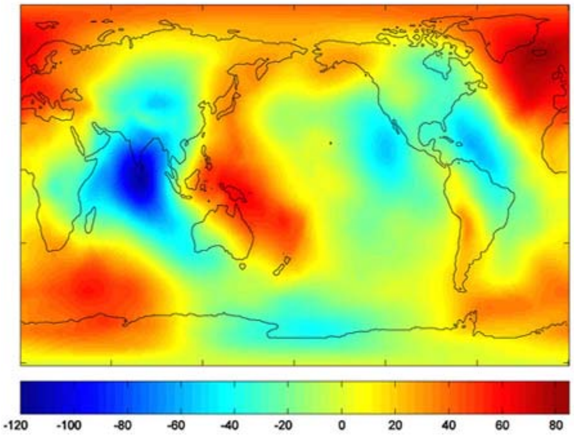






- Undulations are 100 m above and below the IRE. Major hole in the Indian Ocean and a bulge over Indonesia and Australia.
- Geoid highs are caused by excess mass.
- Geoid lows caused by mass deficit

A more detailed view of the geoid has come from the GRACE project (Gravity Recovery and Climate Experiment). This has been underway since 2002 and uses microwave measurements of the distance between two satellites to measure the gravity field with very high precision. (http://www.csr.utexas.edu/grace/)

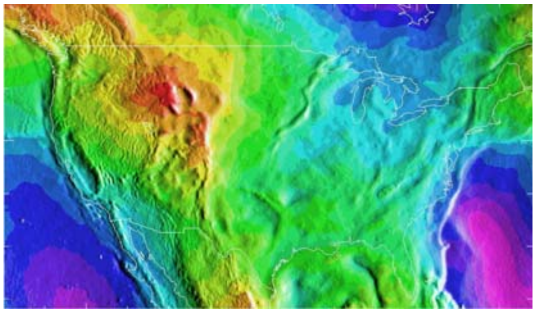


Geoid from GRACE data (m)

5.3 Why is the geoid important?

5.3.1 Information about density structure in the interior of the Earth

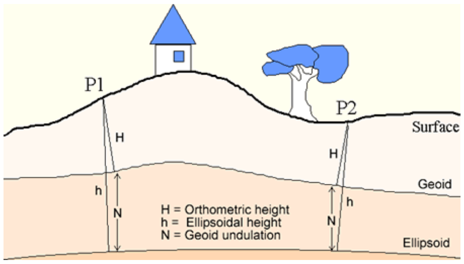
- The geoid contains a lot of information about the internal structure of the Earth.
- Figure below shows the geoid superimposed on a topography map. Red: -7.7 m Magenta = -52.8 m.



- The **long spatial wavelengths** of geoid anomalies indicate that they originate in **deep density contrasts** that are associated with mantle convection.
- Observed geoid variations can be explained if density contrasts from subduction zones persist to mid-mantle depths (Hager, 1984).

5.3.2 Surveying

- The geoid is always orthogonal to the local acceleration of gravity (**g**).
- Thus elevations measured with levelling (surveying) will be relative to the geoid and are expressed as heights above sea level.
- In contrast surveys that use GPS satellites will give elevations relative to the IRE.
- To reconcile the two measurements of elevation, the difference between the IRE and geoid (N) must be known.



H = height above geoid (orthometric height)

h = height above IRE (ellipsoidal height)

N = geoid height (undulation)

To illustrate this difference, consider a ship sailing across the Indian Ocean. GPS measurements will indicate that it drops into a 70 m deep hole. Conventional surveying tells us that it stays at sea level the whole time.

More reading : http://en.wikipedia.org/wiki/Geoid

References

Hager, B., Subducted slabs and the Geoid: constraints on mantle rheology, *J. Geophys. Research*, **89**, 6003-6015, 1984.

Han, S.C., C.K. Shum, M. Bevis, C.Ji, C.Y. Kuo, Crustal dilation observed by GRACE after the 2004 Sumatra-Andaman earthquake, *Science*, 313, 658-662, 2006.

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