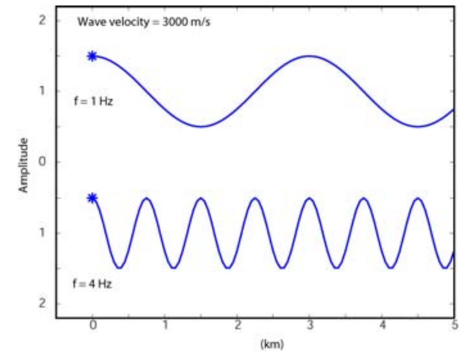
C1 : Basic principles of seismology

C1.1 General introduction to wave phenomena

A wave can be defined as a **periodic disturbance** that transmits energy through a medium, without the **permanent displacement** of the medium.

Also required that energy is converted back and forward between two different types.

Consider the two waves shown in the MATLAB movie **waves.m**



Frequency (f) : The number of cycles a given point moves through in 1 second. Frequency is measured in Hertz (Hz). If the frequency is very low, then it is common to refer to the period (T) of the signal in seconds. $T = 1/f$

Angular frequency (ω): Frequency is the number of rotations per second. The angular frequency is the number of radians per second and given by $\omega=2\pi f$

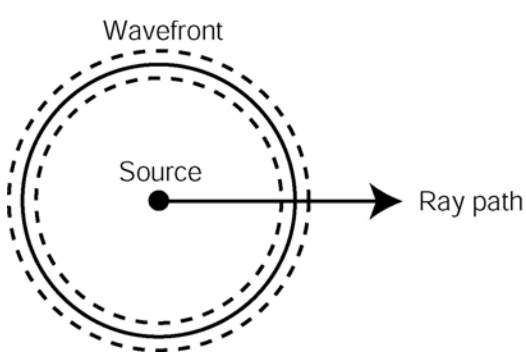
Wavelength (λ) : Distance in metres between two points of the wave having the same phase (*e.g.* two crests or two troughs).

If the waves moves at a velocity, *v*, then $v = f\lambda$

- Is this relationship is correct for the figure above?
- Note that points on the wave move up and down, they are not translated to the right.
- In this case, the velocity is **independent** of frequency. This type of wave behaviour is called **non-dispersive**. If velocity varies with frequency, the wave is said to be **dispersive**

In seismology, we need to understand how waves will travel in the Earth. For example, how fast will they go, which direction, how will amplitude vary with distance etc. In general this requires the solution of some complicated **differential equations**.

In Geophysics 210 we will approach this subject through visualization. Wave propagation can be considered in two ways, by considering either **wavefronts** or **rays**. These are complementary ways of talking about waves:

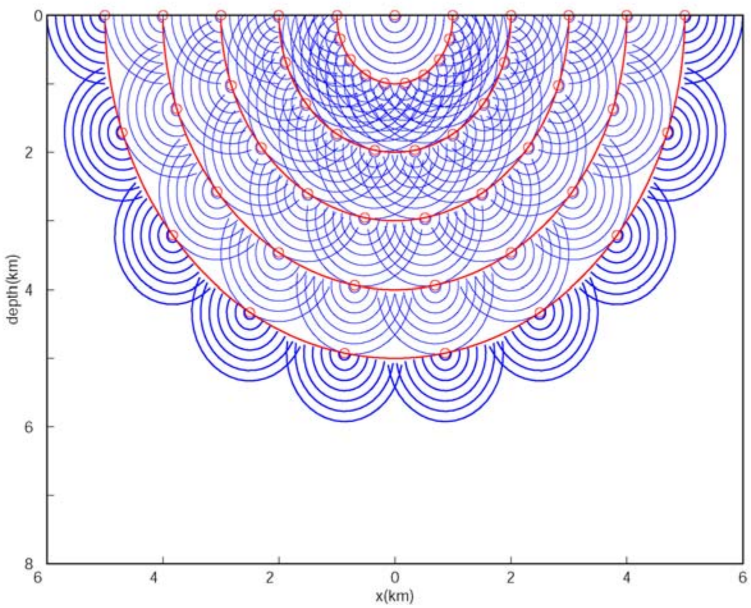


Rays denote the direction in which the wave travels.

Wavefronts are points on the wave with the same phase (e.g. a line along the crest of a wave is a wavefront).

Note that wavefronts and rays are at **right angles** to each other.

One way to visualize wave propagation over time is through **Huyghens Principle**. This states that all points on a wavefront can be considered secondary sources of wavelets. These secondary wavelets propagate outwards and at a time later, the overall wavefront is the envelope of secondary wavelets. Examples for a point source is shown below.



C1.2 Stress and strain

Having considered some general aspects of wave propagation we now need to consider how waves propagate in Earth. Seismic waves are elastic waves with energy converted from **elastic** to **kinetic** and vice versa. Some definitions:

Elastic deformation : Deformed caused by an applied force. Return to it's original shape when the force is removed.

Stress :

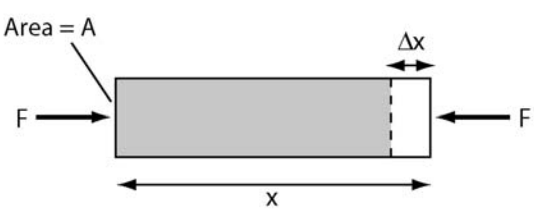
Longitudinal stress (F/A) Force per unit area.
Units = N / m^2

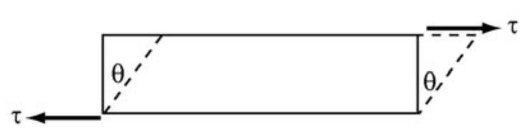
Shear stress (τ)
Applied parallel to the surface

Strain : Normalized measure of deformation of material

Longitudinal strain= $e = \frac{\Delta x}{x}$

Shear strain= $\tan \theta$



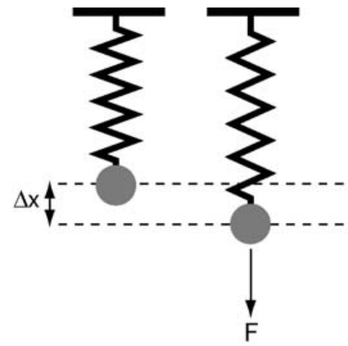


Hooke's Law

Stress and strain can be related through various equations. The simplest is Hooke's Law that describes the extension of a spring. Hooke stated his law in 1678 as:

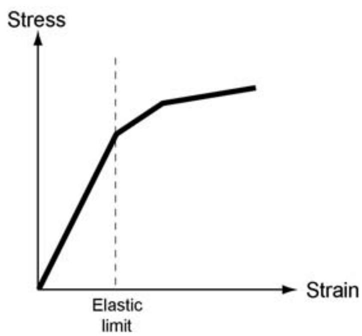
"*As the extension, so the force*"

This is a **linear** relation between stress and strain. For a simple spring $F = k\Delta x$



k = spring constant (measures stiffness of spring)
F = force used to stretch the spring (stress)
Δx = amount of stretch (strain)

k is the ratio of stress / strain. *i.e.* How much force needed to produce a given strain.



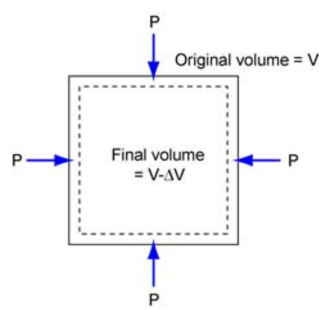
- If elastic the object returns to the original shape when stress removed.
- Only valid up to elastic limit. Elastic limit for rocks = 10^{-4} or less
- Beyond this point, deformation is permanent (plastic deformation)
- Ultimately the rock will reach failure

Need to consider finite size of a rock sample to apply this to seismology. So can define

Longitudinal modulus ψ = longitudinal stress / longitudinal strain

Shear modulus $\mu = \dfrac{\textit{shear stress, } \tau}{\textit{shear strain, } \tan\theta}$

Bulk modulus $K = \dfrac{\textit{volume stress, } P}{\textit{volume strain, } \Delta V/V}$



A simple longitudinal compression will change both the volume and shape of the cylinder. Thus these modulii are linked as $\psi = K + \frac{4}{3}\mu$

Further reading

A much more rigorous analysis can be found in Fowler (2005), Appendix 2.
http://en.wikipedia.org/wiki/Yield_(engineering)
http://en.wikipedia.org/wiki/Hooke's_law

C1.3 Seismic waves in the Earth

Waves in the Earth can be divided into **two** main categories:

(a) Body waves travel through the **bulk** medium.
(b) Surface waves are confined to **interfaces**, primarily the Earth-Air interface.

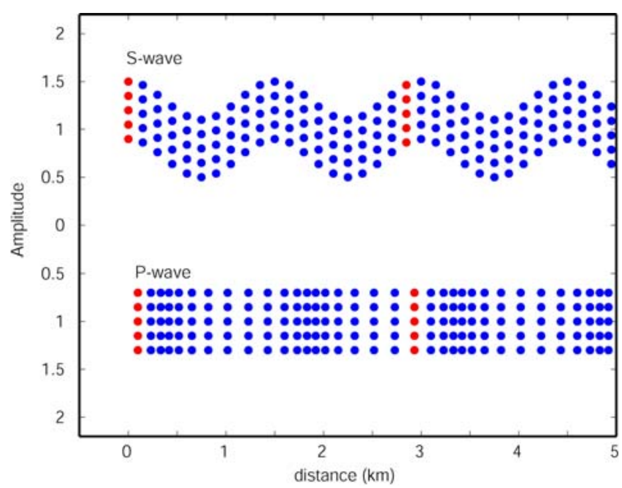
C1.3.1 Body waves

Body waves in the Earth can be divided into two types:

P-waves : Particle motion is in the **same direction** as the wave propagation. They are also called **compressional** or **longitudinal** waves. P = primary

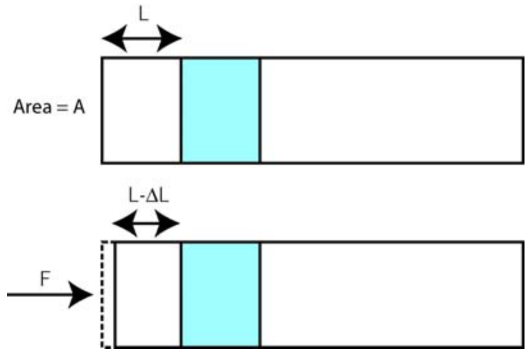
and

S-waves: Particle motion is at **right angles** to the wave propagation. Also called shear waves or transverse waves.



The **velocity** of body waves can be calculated from the **properties of the material**, as outlined below.

Consider a column of rock with cross-sectional area (A).



If a force (*F*) is applied at the left end, this is a *longitudinal stress = F/A*.

The deformation of the leftmost (white) disk can be quantified as the *longitudinal strain* = $\frac{\Delta L}{L}$

Previously defined the **longitudinal modulus** as ψ = *longitudinal stress / longitudinal strain*

This strain produces a force that will cause the shaded section of the rock to accelerate to the right. This lowers the stress to the left, but increases it to the right. This causes the next section of the rock to move and so on. Can show that a **wave motion** will move down the column at a velocity

$$v = \sqrt{\frac{\psi}{\rho}}$$

where ρ is the density of the material. Note that the stiffer the medium (larger ψ) the greater the force on the shaded cylinder, thus acceleration is higher and wave velocity is greater. Similarly, as density increases, the shaded section becomes heavier and it's acceleration (and wave velocity) for a given force will decrease.

In general, the calculation of velocity is more complicated as the deformation will involve both compression and shearing. The bulk modulus and shear modulus must be considered. Thus the P-wave velocity can be written as

$$v_P = \left[\frac{K + \frac{4}{3}\mu}{\rho}\right]^{\frac{1}{2}}$$ and the S-wave velocity as $$v_S = \left[\frac{\mu}{\rho}\right]^{\frac{1}{2}}$$

Note that:

● P-waves always travel faster than S-waves(hence primary and secondary names)

● Two shear wave polarizations exist. Consider a wave travelling horizontally. Particles can move vertically (SV) or horizontally (SH)..

● In a liquid μ=0 while *K* is always non-zero. Thus only P-waves can travel in a liquid, since shear stresses cannot exist. Important for outer core

● These expressions for v_P and v_S do not depend on frequency, thus body waves (both P-waves and S-waves) are **non-dispersive**.

● As the rock cylinder is stretched, it will get longer and thinner. This effect can be quantified through **Poissons ratio**. This is defined as:

$$\sigma = \frac{\text{lateral strain}}{\text{longitudinal strain}}$$

- Individual values of v_P and v_S depend on several modulii and density. This can make it difficult to compare the velocities of similar rocks.

  It can be shown that $v_p/v_s = [2(1-\sigma)/(1-2\sigma)]^{1/2}$

  For typical consolidated crustal rocks, σ ~ 0.25 and v_p/v_s ~ 1.7. An increase in v_p/v_s and/or Poisson's ratio can be indicative of the presence of fluids.

Further reading
More detailed derivation of wave equation in Fowler(2005), chapter 4 and Appendix 2.

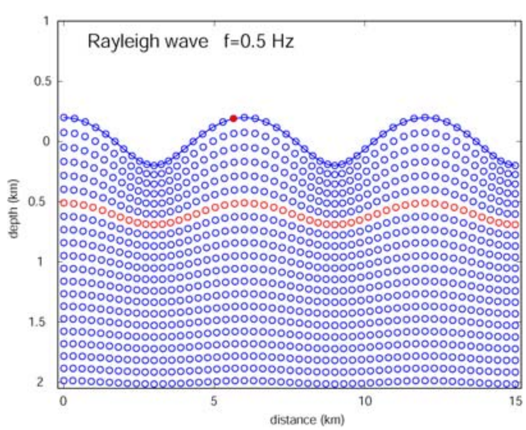
More animations can be found at
http://web.ics.purdue.edu/~braile/edumod/waves/WaveDemo.htm

C1.3.2 Surface waves

Surface waves are localized at the Earth's surface and can be divided into two types.
See Fowler (2005) Figure 4.4

Rayleigh Waves (L_R)

- Occur on the surface of any object. e.g ripples on a lake.
- Particle motion is in a retrograde ellipse. Combination of and vertical polarized S-waves.
- Large earthquakes can generate surface waves that travel around the globe. They can be large in amplitude and cause a lot of damage during earthquakes.
- In exploration seismology, **ground roll** is a Rayleigh wave that travels across the geophone array.
- Movie clip of surface waves after an underground nuclear explosion in Alaska
- The velocity of a Rayleigh wave does not vary with frequency when travelling in a uniform medium and it is slower than an S-wave. In a layered Earth the velocity of a Rayleigh wave varies with frequency (it is **dispersive**) and can be used to infer velocity variation with depth. Example in Fowler (2005) Figure 4.5 and 4.6



Love waves (L_Q)

- have a horizontal particle motion analogous to SH -waves.
- Love waves only exist if the Earth is layered and
- are always dispersive.

C1.4 Typical seismic velocities for Earth materials

Typical values for P-wave velocities in km s^{-1} include:

Sand (dry)	0.2-1.0
Wet sand	1.5-2.0
Clay	1.0-2.5
Tertiary sandstone	2.0-2.5
Cambrian quartzite	5.5-6.0
Cretaceous chalk	2.0-2.5
Carboniferous limestone	5.0-5.5
Salt	4.5-5.0
Granite	5.5-6.0
Gabbro	6.5-7.0
Ultramafics	7.5-8.5
Air	0.3
Water	1.4-1.5
Ice	3.4
Petroleum	1.3-1.4

● why does v_p apparently increase with density? e.g. for the sequence granite-ultramafics.

The equation $v_P = \left[\frac{K + \frac{4}{3}\mu}{\rho}\right]^{\frac{1}{2}}$ suggests that v_p should **decrease** as density increases.

● Birch's Law (Fowler Figure 4.2). Linear relationship of seismic velocity and density.

Variation of seismic velocity with depth

With increasing depth, compaction increase the density of a rock through reduction of pore space. The rigidity of the rock also increases with depth. The net effect is that velocity will increase with depth, even if the lithology does not change.

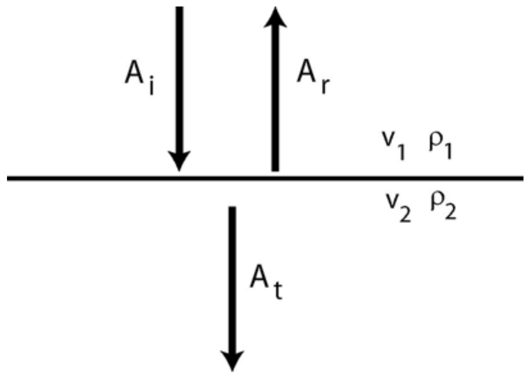
C1.5 Propagation of seismic waves

As a seismic wave travels through the Earth, several factors will change the **direction** and **amplitude** of the waves. When detected at the surface, an understanding of these factors can tell us about sub-surface structure.

C1.5.1 Reflection coefficients at normal incidence

Consider a seismic wave that is travelling **vertically** downwards, as shown in the figure below. If the wave reaches an interface, some of the energy will be transmitted, and some will be reflected. Note that the waves travels from medium 1 into medium 2.

For each medium, the **impedance** is defined as the product of density (ρ) and seismic velocity (v), Z = ρv



The reflection coefficient, R, is defined as

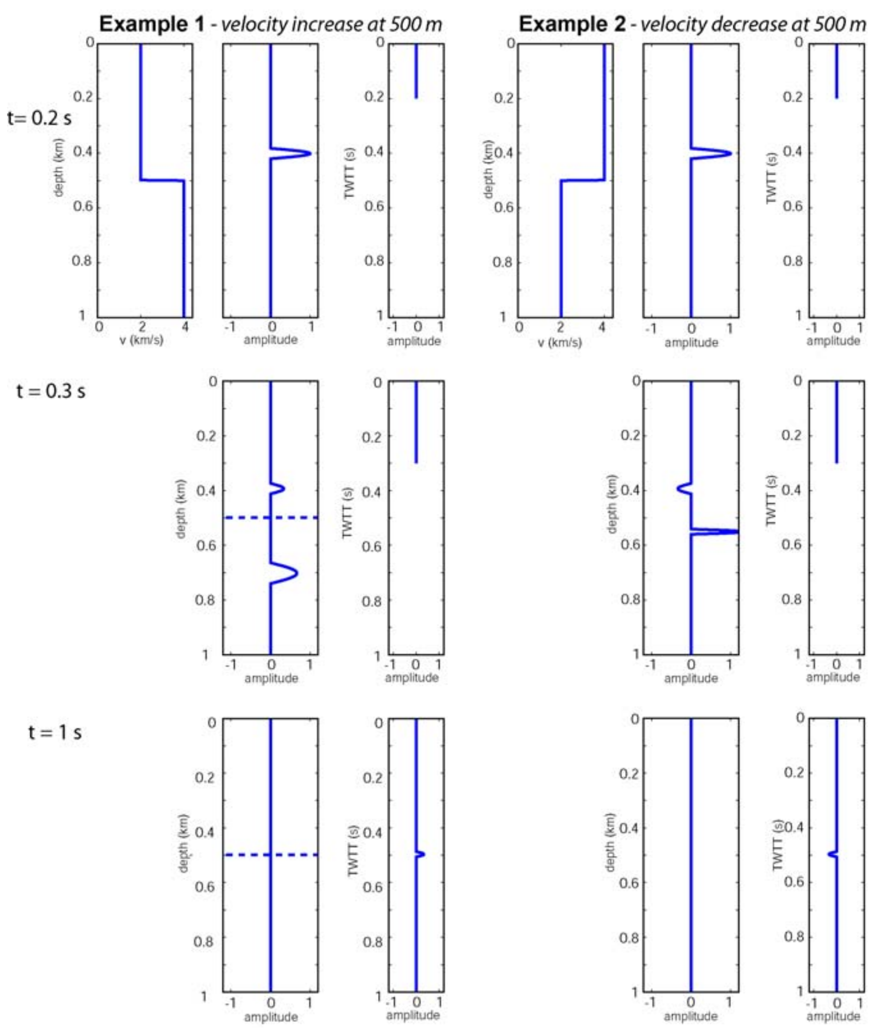
$$\frac{A_r}{A_i} = R = \frac{v_2\rho_2 - v_1\rho_1}{v_2\rho_2 + v_1\rho_1} = \frac{Z_2 - Z_1}{Z_2 + Z_1}$$

and the transmission coefficient, T, is defined as

$$\frac{A_t}{A_i} = T = \frac{2v_1\rho_1}{v_2\rho_2 + v_1\rho_1} = \frac{2Z_1}{Z_2 + Z_1}$$

The **reflection coefficient** is a measure of the change in impedance across the interface. These equations are called the **Zoeppritz equations**. If the wave is incident at an angle they become more complicated.

Note that reflection co-efficients are expressed in terms of **energy**, not amplitude.



Example 1 : The MATLAB script reflect_v1.m generates a movie showing how a seismic pulse propagates in the Earth. For simplicity, density is constant and only velocity varies with depth. In this example there is an increase in seismic velocity (and impedance) at 500 m. Note that the reflection has the same polarity as the down going pulse.

What are the values of *R* and *T*? Is the largest amplitude in the reflected or transmitted wave?

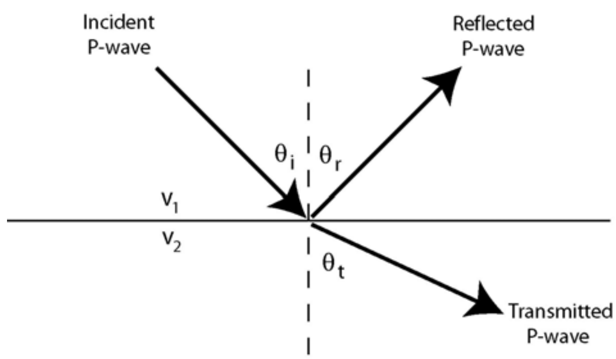
Example 2 : Same geometry as Example 1, but a decrease in velocity (impedance) occurs at 500 m depth.

The reflected pulse has the opposite (negative) polarity to the down going pulse. Is this what the Zoeppritz equations predict?

What are the values of *R* and *T*? Is the largest amplitude in the reflected or transmitted wave?

C1.5.2 Reflection and refraction at non-normal incidence

In a more general case, the seismic wave will be incident on an interface at some angle of incidence, θ_i. Note that the angle of incidence is measured from the normal to the ray. Snell's Law was developed for optics, but can equally be applied to the seismic case. Consider a P-wave that strikes the interface shown below. In this case $v_2 > v_1$



Reflected and refracted P-waves are generated from the incident P-wave. For the reflected P-wave, $\sin\theta_i = \sin\theta_r$ which requires $\theta_r = \theta_i$. Snell's Law states that the ray parameter, *p*, for the incident and refracted waves will be constant.

$$p = \frac{\sin\theta_i}{v_1} = \frac{\sin\theta_t}{v_2} = \frac{\sin\theta_r}{v_1}$$

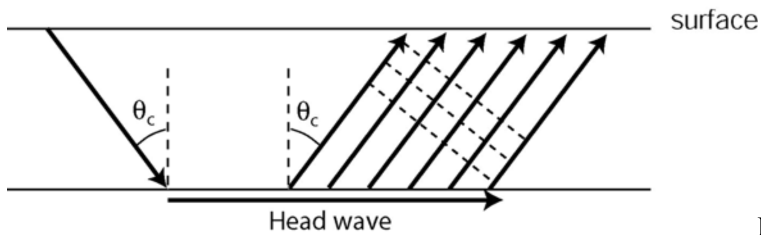
Thus with a **velocity increase** ($v_2 > v_1$) the ray is **refracted away** from the normal.

If the **velocity decreases** ($v_2 < v_1$) then the ray **is refracted towards** the normal.

Note that if $v_2 > v_1$ then there will be a value of θ_i which results in $\sin\theta_t = 1$. This gives a value of $\theta_t = 90^\circ$ and the refracted waves travels horizontally. In this configuration $\theta_i = \theta_c$ and is called **the critical angle**.

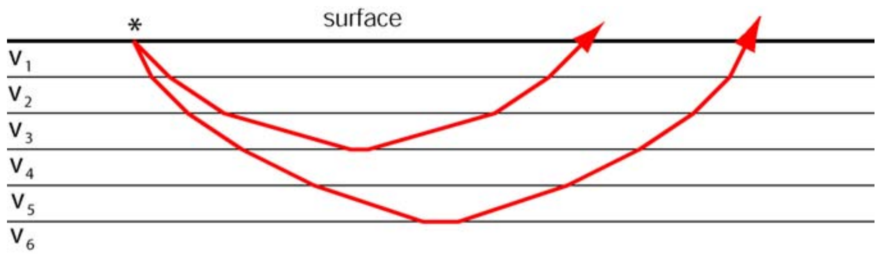
$$\frac{\sin 90^o}{v_2} = \frac{\sin\theta_c}{v_1} \quad \text{and by rearranging we find that} \quad \theta_c = \sin^{-1}\left(\frac{v_1}{v_2}\right)$$

The wave travelling horizontally is called a **head wave**. For a head wave to develop, we must have $v_2 > v_1$. Using Huyghen's Principle, it can be shown that the head wave will generate upward propagating wave at an angle θ_c to the normal. When these waves reach the geophones they are called refracted arrivals. When $\theta_i > \theta_c$ the wave is totally reflected.



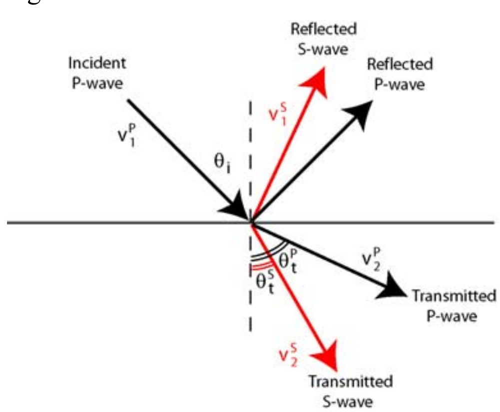
MATLAB: **raytrace_v3.m**

A monotonic increase in seismic velocity with depth is commonly observed. Why?



This will result in seismic waves that turn at depth and return to the surface.

In general we must also consider that the incident P-wave will generate a reflected S-wave and a refracted S-wave through a process called **mode conversion**. See Fowler(2005) Figure 4.35. Again Snell's Law can be used to calculate the angles of reflection and refraction.



$$v_2^P > v_2^S \quad \text{and} \quad v_2^P > v_1^P$$

$$p = \frac{\sin\theta_i}{v_1} = \frac{\sin\theta_t^P}{v_2^P} = \frac{\sin\theta_t^S}{v_2^S}$$

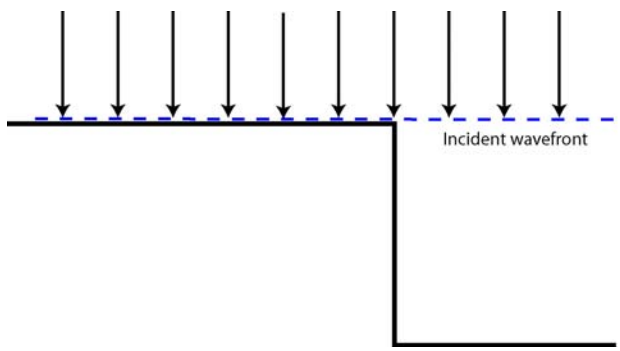
The amplitudes of the transmitted and reflected waves can be calculated from Zoeppritz's equations, which have a more complicated form than those listed in C1.5.1 The amplitude of the reflected wave varies with angle, and is sensitive to the change in impedance across the boundary. Example in Fowler (2005) Figure 4.37.

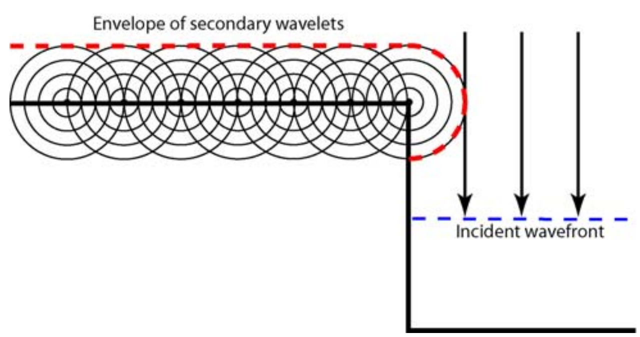
This is the physical basis of the **amplitude versus offset** (AVO) technique.

C1.5.3 Diffraction

Seismic energy can sometimes travel in regions where ray theory (Snell's Law) does not predict that it will go. This is typified by diffraction which occurs when a wave strikes an object that is significantly smaller than a wavelength. A diffractor radiates seismic energy in all directions.

When a wave strikes a corner, Huyghens Principle shows that the corner will generate waves that propagate in all directions. We will see in real seismic reflection data that these waves can have significant amplitudes and are detected over a wide area at the surface.





C1.5.4 Factors that cause the amplitude to change as wave propagates

(a) Geometrical spreading: Imagine a wave travelling outward from a point source. If the wave has travelled a distance r , then the wavefront covers an area $A = 4\pi r^2$ .At this point the wave has an amplitude of X. The energy is a wave is proportional to X^2 . Thus

Total energy = $E = X^2 4\pi r^2$

Conservation of energy requires that this quantity remain **constant** as *r* increases. Thus

$X_1^2 4\pi r_1^2 = X_2^2 4\pi r_2^2$ and

$X^2 \alpha \frac{1}{r^2}$ and $X \alpha \frac{1}{r}$

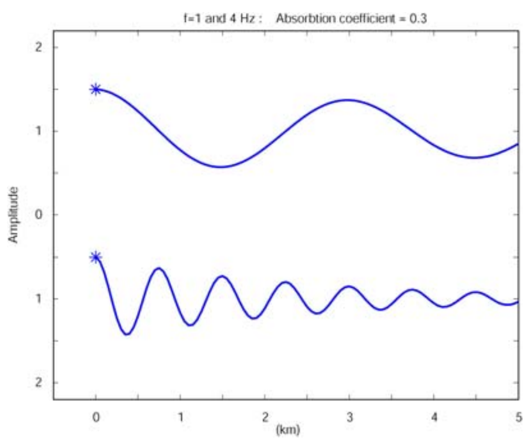
As a seismic wave travels in the Earth the amplitude will decrease as *1/r*, even if no energy is lost. This phenomena is known as **spherical divergence** or **geometric spreading**.

(b) Attenuation : As a wave passes through the Earth, the particle motion causes the material to be distorted and the wave energy is converted in heat. This results in an additional loss of energy, that is described by an exponential decay:

$$X = X_o e^{-kr}$$

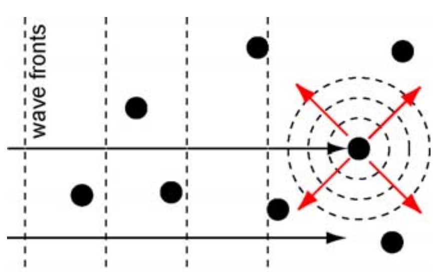
Where e = 2.718, X_o is the amplitude at $r=0$ and k is a constant. If k is small, the attenuation will be small, as k increases, the attenuation becomes stronger. In a distance $1/k$ the amplitude falls from X_o to $X_o \frac{1}{e}$.

Another common definition is the absorption coefficient, α, expressed in **decibels per wavelength**. This is based on the observation that the energy lost is dependent on the number of oscillations per second produced by the wave. Thus high frequencies will attenuation more quickly than low frequencies. This is illustrated in the MATLAB script **waves_attenuation.m.**



- A consequence of frequency-dependent attenuation is that the shape of a seismic pulse can change as it propagates through the Earth.
- This occurs because, as the pulse travels the short wavelength signals attenuate more quickly.
- The long wavelengths dominate, giving the pulse a smoother shape and longer duration.

(c) Scattering



Suppose a medium is inhomogeneous and contains some grains with a different seismic velocity to the host rock. Seismic waves will be diffracted / scattered from these grains and energy will be lost from the coherent wavefronts and turned into random seismic energy. The net result is that energy will be lost.

Footnote : Decibels

A seismic wave changes in amplitude from A_1 to A_2 as it travels from point 1 to point 2. The corresponding intensity changes from I_1 to I_2. Note that $I_1 = A_1^2$. This change in decibels can be expressed as :

$$dB = 10\log_{10}\left(\frac{I_2}{I_1}\right) = 20\log_{10}\left(\frac{A_2}{A_1}\right)$$

C1.6 Seismic energy sources

Seismic exploration is an **active technique**. In contrast to gravity studies, a signal must be generated. A range of techniques can be used, depending on the depth of study.

C1.6.1 Commercial seismic exploration

More details in http://www-geo.phys.ualberta.ca/~unsworth/UA-classes/224/notes224.html

- Shallow exploration on land : hammer on a plate, weight drops, specialized guns
- Offshore : air guns, explosives
- Deeper studies on land : vibroseis, conventional explosives, nuclear explosions

Drilling shot holes in Tibet, 1994

1000 kg shot

C1.6.2 Earthquakes

See section C2.

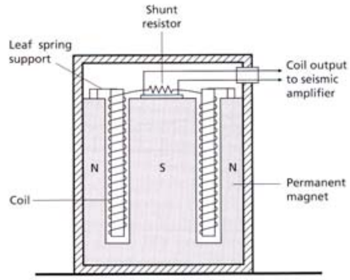
C1.6.3 Frequency content of seismic sources

Earthquake surface waves	0.1-0.01	Hz
Earthquake body waves	10-0.1	Hz
Vibroseis	10-100	Hz
Air guns	10-100	Hz
Explosives	10-300	Hz

C1.7 Seismic detectors

C1.7.1 Electromagnetic detection (geophones)

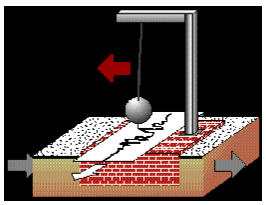
On land, the surface moves as a P-wave or S-wave arrives. Generally reflected signals arrive at steep angles of incidence. Thus P-waves produce surface motion that is dominantly vertical. Geophones measure ground motion by converting motion into electrical signals. Most geophones measure a single component (vertical), but multiple component ones are sometimes used.



Geophones are manufactured to detect a particular frequency band. This should match the seismic source being used in a particular survey.

C1.7.2 Mechanical seismometer

Measure lower frequencies than geophones. Use a stationary mass. Measures motion of the Earth relative to the mass. Can measure vertical or horizontal motion.



http://www.thetech.org/exhibits/online/quakes/seismo/

For earthquake studies a more permanent installation is usually required. Three components are usually recorded and the sensor is tuned to detect lower frequencies. Often the seismometer is placed in a shallow vault to minimize wind and other forms of noise.

Seismometers, can also be deployed in the deep ocean (Ocean bottom seismometers – OBS) and are dropped to the seafloor from a ship. Coupling with the seafloor allows 3 components of motion to be recorded (i.e. P-waves and S-waves can be detected).

Dalhousie University OBS on deck

Scripps Institution of Oceanography OBS

C1.7.3 Accelerometers

C1.7.4 Hydrophones

Only sensitive to pressure changes so only P-waves detected. Used in marine surveys.