C3.2 Velocity-depth models

C3.2.1 Forward and inverse problems in geophysics

- So far we have considered the **forward problem** for seismic waves travelling in the Earth. This is a **prediction of travel times based on a model** with a number of layers with differing velocity.

  Velocity model for Earth > travel times

  The forward problem is generally **unique**, which means there is only one solution (travel) time for a given Earth model.

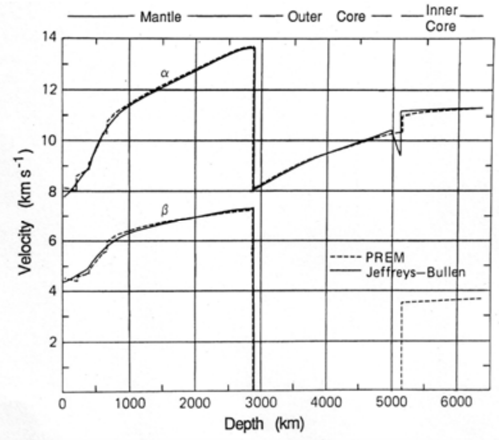
- To determine the velocity inside the Earth we must solve the **inverse problem**.

  Travel times > Velocity model for Earth

  The inverse problem is generally **non-unique**, which means that there may be multiple solutions (Earth models) that can account for the same data (travel times).

C3.2.2 Deriving velocity from travel times

- An inversion is the solution of the inverse problem.
- The first inversion of teleseismic travel times was described by Wiechert and Herglotz (1907)
- Can use an automated inversion, or a trial-and-error approach to find a velocity model that can account for the observed travel times.



- Fowler 8-1: This shows two widely used models for the velocity of the Earth as a function of radius.
- α = P-wave velocity; β = S-wave velocity
- Jeffreys and Bullen (1939)
- PREM = Preferred Reference Earth Model (Dziewonski and Anderson, PEPI, 1981)

- Density variations were shown in B8
- $\alpha = \left[\frac{K + \frac{4}{3}\mu}{\rho} \right]^{\frac{1}{2}}$ $\qquad\qquad$ $\beta = \left[\frac{\mu}{\rho} \right]^{\frac{1}{2}}$
- α(r) and β(r) can be derived from seismic travel times, where r is distance from the centre of the Earth.
- To determine Earth structure we need to find K(r), μ(r) and ρ(r)
- With two equations and three unknowns, we cannot do this!
- A third equation can be derived from knowledge of how compression (compaction) of rock with depth changes the density. This is called a **self compression model**.

  Can derive the **Adams-Williamson equation** (Fowler page 332-333) which states

  $$\frac{d\rho}{dr} = -\frac{GM_r \rho(r)}{r^2 \phi}$$

  where $\phi = \alpha^2 - \frac{4}{3}\beta^2$ is termed the seismic parameter and M_r is the mass contained within the radius r

- The travel time data, combined with other data can be used to determine density, elastic modulii etc as a function of depth.
- With some additional constraints (e.g. total mass of Earth) the parameters K(r), μ(r) and ρ(r) can be derived. Example in Fowler Figure 8.5