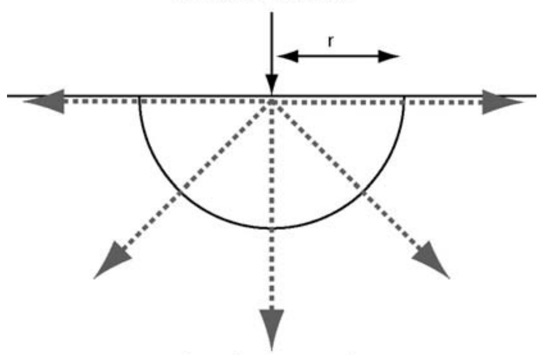
223 B2 Electric current flow in a half-space

B2.1 Single current electrode

- In the lab, the electrical resistivity of a rock sample can be measured by placing flat electrode plates on each side of a rectangular sample. In this geometry the electric current flow is **parallel** and the simple equation derived in B1.1 can be used to compute the resistivity, ρ from the measured resistance (R).

- However this approach is not practical for the measuring the resistivity of the Earth, since we cannot inject current from large plates.

- Consider the electric current flow from a simple electrode (metal spike).



- Consider an electric current, I, flowing from an electrode.

- The air has a very high electrical resistivity, so all current flows in the Earth.

- From symmetry arguments, the current spreads out uniformly in all directions in the Earth.

- It can be shown that the voltage varies with distance from the electrode as:

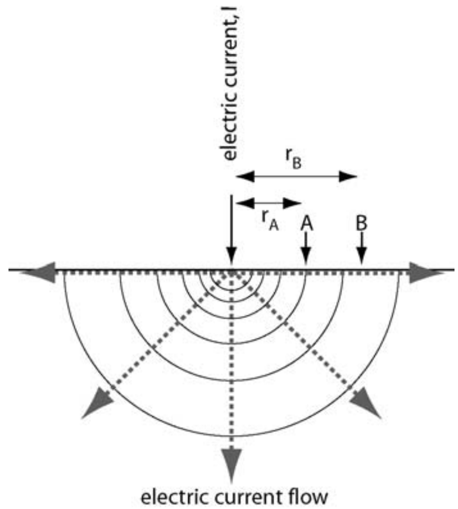
$$V = -\frac{I\rho}{2\pi r}$$

- In this case the voltage is defined as zero when r = ∞.

- This geometry can be used to measure the resistivity of the Earth.

- Two types of electrode are used. **Current electrodes** inject electric current into the Earth. **Potential electrodes** are used to measure voltages.

- The voltage between the two potential electrodes, A and B is $\Delta V_{AB} = V_A - V_B$



- Using the result from the previous page, the voltage difference measured between the potential electrodes A and B is

$$\Delta V_{AB} = \frac{I\rho}{2\pi}\left[\frac{1}{r_A} - \frac{1}{r_B}\right]$$

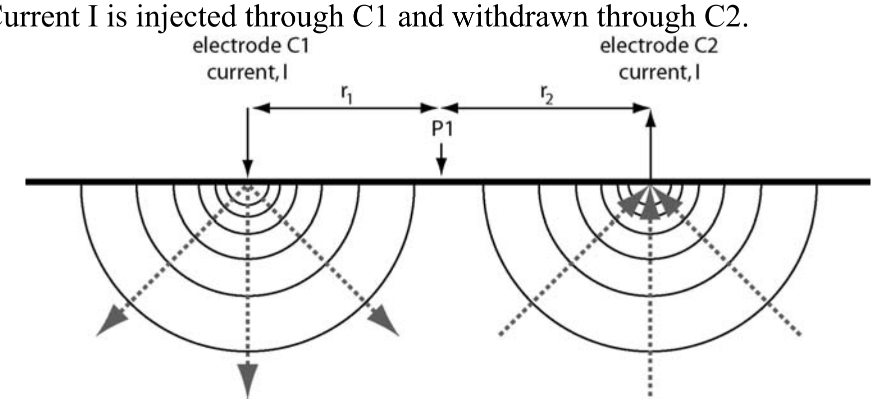
- Rearranging this equation gives $\rho = \dfrac{2\pi\Delta V_{AB}}{I(\frac{1}{r_A} - \frac{1}{r_B})}$

  Note that this is essentially Ohms Law with a geometric factor added. Everything on the right side can be measured, allowing us to compute ρ

- Why is this not a practical way to measure the resistivity of the Earth?

B2.2 Two-current electrodes

- A practical survey requires two current electrodes to make an electric circuit. Current I is injected through C1 and withdrawn through C2.



- To compute the voltage at electrode P1, we can simply add the voltages generated by current from C1 and C2.

$$V_{P1} = \frac{I\rho}{2\pi r_1} - \frac{I\rho}{2\pi r_2} = \frac{I\rho}{2\pi}\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

- Note that V_{P1} will be zero as r_1 or $r_2 \rightarrow \infty$ and when $r_1 = r_2$ (on a plane equidistant between C1 and C2).

- The figure below shows a quantitative evaluation of the electric current flow pattern. Unlabelled lines are contours of constant voltage (also called equipotential lines). Lines with percentage values show current flow lines.

- Electric current flow is at right angles to the equipotential surfaces.

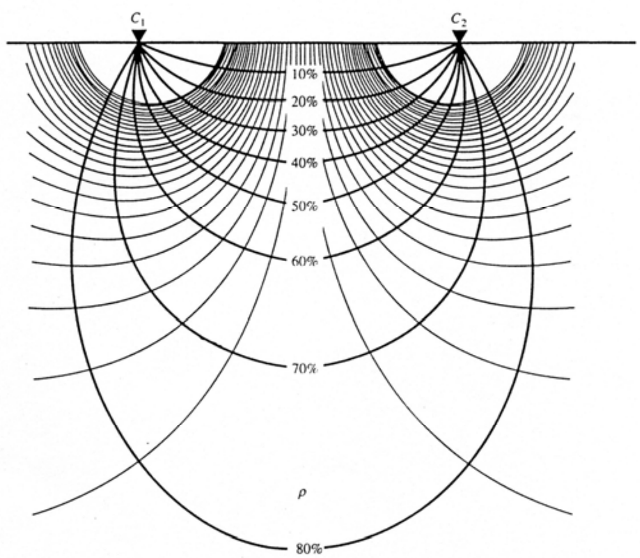


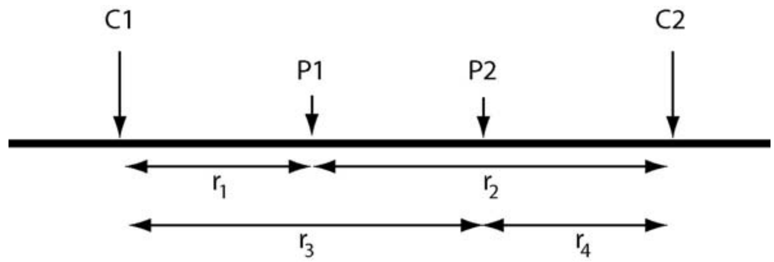
Figure 5-8 Equipotential surfaces and current lines of flow. Labels indicate percent of total current that penetrates to the depth of the line.

- Electric current does not flow directly from one current electrode to the other in a straight line. This is because the charge carriers repel one another.

- Approximately 50% of the electric current flows within a depth *a* of the surface.

- The apparent resistivity can be considered the average resistivity over a volume that is located between the electrodes, and in the depth range from the surface to a depth equal to the *a*-spacing.

- However, to measure a voltage, we need **two** potential electrodes to connect to a voltmeter. Consider the arrangement of electrodes shown below.



- The voltage difference measured between potential electrodes P1 and P2 is

$$\Delta V = V_{P1} - V_{P2} = \frac{I\rho}{2\pi}\left[\left(\frac{1}{r_1} - \frac{1}{r_2}\right) - \left(\frac{1}{r_3} - \frac{1}{r_4}\right) \right] \quad = \frac{I\rho}{2\pi}\left[\frac{1}{r_1} - \frac{1}{r_2} - \frac{1}{r_3} + \frac{1}{r_4} \right]$$

- Now let us make the geometry of the array simple, with the 4 electrodes separated by a distance *a*. Then we have $r_1 = r_4 = a$ and have $r_3 = r_3 = 2a$

$$\Delta V = V_{P1} - V_{P2} = \frac{I\rho}{2\pi}\left[\left(\frac{1}{a} - \frac{1}{2a}\right) - \left(\frac{1}{2a} - \frac{1}{a}\right) \right] = \frac{I\rho}{2\pi a}\left[\left(1 - \frac{1}{2}\right) - \left(\frac{1}{2} - 1\right) \right] \quad = \frac{I\rho}{2\pi a}$$

  This represents a solution to a **forward problem** i.e. for a model of the Earth (resistivity, ρ) we can predict the value of ΔV that will be observed in a geophysical survey.

- Simple rearrangement gives us a solution to the corresponding **inverse problem**.

$$\rho = \frac{2\pi a \Delta V}{I}$$

  This equation shows how the **resistivity of the Earth** can be computed from **field measurements** of *ΔV, I* and the electrode spacing (*a*).

- If the Earth has a uniform structure, with the resistivity equal to ρ at all points, then the measured resistivity value will equal the actual resistivity value of the Earth.

- However, if the resistivity is variable, the resistivity computed will be an **average value** over the region in which the current is flowing. This average resistivity is termed the **apparent resistivity** and defined as

$$\rho_a = \frac{2\pi a \Delta V}{I}$$