Math 201 (Fall 2009)
Differential Equations

Assignment #3

1. Find the particular solution of the following differential equation by variation of parameter

(a) $y'' + y = \csc t$

(b) $t^2y'' + ty' - y = t\ln t, \qquad t > 0$

2. Let g be a continue function and y be defined by

$$y(t) = \frac{1}{2}\int_0^t \sin 2(t-\tau)\, g(\tau) d\tau \tag{$*$}$$

(a) Apply the Leibniz formula (the generalization of the fundamental theorem of calculus) to show that the function y defined by $(*)$ satisfies the initial value problem

$$y'' + 4y = g(t), \qquad y(0) = 0, \qquad y'(0) = 0$$

(b) Use the variation of parameters to solve the initial value problem and show that the solution is given by $(*)$.

3. Find all values of α for which all solutions of

$$t^2y'' + \alpha ty' + \frac{5}{2}y = 0$$

approach zero as $t \to 0$.

4. Determine the general solution of the given differential equation that is valid in any interval not including the singular point.

(a) $(t+1)^2y'' + 3(t+1)y' + 0.75y = 0$

(b) $t^2y'' + 3ty' + 5y = 0$

(c) $t^2y'' - 5ty' + 9y = 0$

5. Find a particular solution of

$$y'' - y' - 6y = e^{-t}$$

first by undetermined coefficients and then by variation of parameters.

6. Solving the following Cauchy-Euler equations by using the substitution $t = e^x$, $Y(x) = y(t) = y(e^x)$ to change them to constant coefficient equation.

(a) $t^2y'' + ty' - 9y = 0$

(b) $t^2y'' + 3ty' + y = t + \frac{1}{t}$

(c) $t^3y''' + 4t^2y'' - 5ty' - 15y = t^4$

7. Let y_1 be a given nontrivial solution of the associated solution. Find a second linearly independent solution using reduction of order.

(a) $t^2y'' - 3ty' + 4y = 0, \qquad y_1(t) = t^2$

(b) $ty'' - (2t+1)y' + (t+1)y = 0, \qquad y_1(t) = e^t$

(c) $x^2y'' + xy' + (x^2 - \frac{1}{4})y = 0, \qquad y_1(x) = x^{-1/2}\sin x,$ (Bessel equation)

8. A weight of 49g is suspended from a spring of modulus $\frac{5}{2}$ g/cm (corresponding to stiffness).The coefficient of friction in the system is estimated to be $\frac{1}{10}$ g/(cm/sec). At $t = 0$, the weight is pulled down 6 cm from its equilibrium position and released from that point with an upward velocity of 20 cm/sec. Find the subsequent displacement of the weight as a function of time. When does the weight pass through its equilibrium position? Take $g = 980\text{cm/sec}^2$

9. The position of a certain spring-mass system satisfies the initial value problem

$$\frac{3}{2}y'' + ky = 0, \qquad y(0) = 2, \qquad y'(0) = v\,.$$

If the period and amplitude of the resulting motion are observed to be π and 3, respectively, determine the values of k and v.

10. (Abel formula) If y_1 and y_2 are solutions of the differential equation

$$y'' + p(t)y' + q(t)y = 0$$

where p and q are continuous functions, then the Wronskian $W[y_1, y_2]$ is given by Abel formula

$$W[y_1, y_2] = C\exp\left(-\int p(t)dt\right)$$

(a) Show that the Wronskian W satisfies the first order differential equation

$$W' + pW = 0$$

(b) Solve the separable equation in (a).

(c) Apply Abel formula to the following differential equation

$$2t^2y'' + 3ty' - y = 0, \qquad t > 0, \qquad y_1 = t^{1/2}, \qquad y_2 = t^{-1}$$