Math 201 (Fall 2009)
Differential Equations

Assignment #6

1. Find the singular points of each of the following equations and determine whether they are regular or irregular.

(a) $y'' + xy' + y = 0$
(b) $e^x y'' + 2y' - xy = 0$
(c) $x^2 y'' - \lambda^2 y = 0$
(d) $x^2 y'' + y' + y = 0$
(e) $(1 - x^2)y'' + y' + y = 0$
(f) $x^2(1 - x)y'' + (1 - x)y' + y = 0$

2. Find the radius of convergence and interval of convergence of the series.

(a) $\displaystyle\sum_{n=0}^{\infty} \frac{2^{-n}}{n+1}(x-1)^n$
(b) $\displaystyle\sum_{n=0}^{\infty} \frac{(-3)^n x^n}{\sqrt{n+1}}$
(c) $\displaystyle\sum_{n=0}^{\infty} \frac{n(x+3)^n}{3^{n+1}}$

3. Determine a lower bound for the radius of series solutions about each given point x_0 for the given differential equation.

(a) $y'' + 4y' + 6xy = 0; \qquad x_0 = 0, \qquad x_0 = 4$
(b) $(x^2 - 2x - 3)y'' + xy' + 4y = 0; \qquad x_0 = 4, \qquad x_0 = -4, \qquad x_0 = 0$
(c) $(1 + x^3)y'' + 4xy' + y = 0; \qquad x_0 = 0, \qquad x_0 = 2$
(d) $xy'' + y = 0; \qquad x_0 = 1$

4. Find two independent power-series solutions around the origin for each of the following differential equations.

(a) $y'' + xy' + y = 0$
(b) $y'' + x^2 y' + xy = 0$

5. Find the first four nonzero terms in the Taylor polynominal approximation for the initial value problem.

(a) $y' = xy - y^2; \qquad y(0) = 1$
(b) $z'' - x^3 z' + xz^2 = 0; \qquad z(0) = -1, \qquad z'(0) = 1$

6. Solve the differential equation

$$y'' + (x - 1)y' + y = 0$$

in powers of $x-2$.

7. Find a power series representation for $f(x)=\tan^{-1}x$ then use this representation to derive the Leibniz formula for π

$$\frac{\pi}{4}=1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\frac{1}{9}-\cdots$$

8. Consider the initial value problem $y'=\sqrt{1-y^2}, y(0)=0$.

(a) Show that $y=\sin x$ is the solution of this initial value problem.

(b) Look for a solution of the initial value problem in the form of a power series about $x=0$. Find the coefficients up to the term in x^3 in this series.

9. Find the solution of the initial value problem

$$y''-xy'+y=0, \qquad y(0)=1, \qquad y'(0)=0\,.$$