MATH 201 Problem Set 10 (CH. 10.2-10.5)

1.Find the values of λ for which the given problem has a nontrivial solution. Also determine the corresponding nontrivial solution.

$$y'' + \lambda y = 0, \quad 0 < x < \frac{\pi}{2}$$
$$y'(0) = 0, \quad y'(\frac{\pi}{2}) = 0$$

2. Show that for a nontrivial solution

$$y'' + \lambda y = 0, \quad 0 < x < \pi$$
$$y(0) - y'(0) = 0, \quad y(\pi) = 0$$

The eigenvalues λ_n *satisfy* $\tan(\sqrt{\lambda_n}\,\pi) + \sqrt{\lambda_n} = 0$

3..Solve the heat flow problem

$$\frac{\partial u}{\partial t} = 3\frac{\partial^2 u}{\partial x^2}, \quad 0 < x < \pi, \quad t > 0$$
$$u(0,t) = u(\pi,t) = 0, \quad t > 0,$$
$$u(x,0) = \sin 4x + 3\sin 6x - \sin 10x, \qquad 0 < x < \pi$$

4.Solve the wave equation

$$\frac{\partial^2 u}{\partial t^2} = 9\frac{\partial^2 u}{\partial x^2}, \quad 0 < x < \pi, \quad t > 0$$
$$u(0,t) = u(\pi,t) = 0, \quad t > 0,$$
$$u(x,0) = 6\sin 2x + 2\sin 6x, \qquad 0 < x < \pi$$
$$\frac{\partial u}{\partial t} = 11\sin 9x - 14\sin 15x, \qquad 0 < x < \pi$$

5.Compute the Fourier series for the given function f on the specific interval.

(a) $f(x) = |x|, -\pi < x < \pi$

(b) $f(x) = \begin{cases} x+\pi, & -\pi < x < 0 \\ x, & 0 < x < \pi \end{cases}$

(c) $f(x) = e^x, -\pi < x < \pi$

6.Compute the Fourier sine series for the given function.

(a) $f(x) = x^2, \quad 0 < x < \pi$

(b) $f(x) = \pi - x, \quad 0 < x < \pi$

(c) $f(x) = e^x, \quad 0 < x < 1$

7.Compute the Fourier cosine series for the given function.

(a) $f(x) = 1 + x, \quad 0 < x < \pi$

(b) $f(x) = e^x, \quad 0 < x < 1$

(c) $f(x) = \sin x, \quad 0 < x < \pi$

8. Find a formal solution to the given initial-boundary value problem.

(a)

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}, \quad 0 < x < \pi, \quad t > 0$$
$$u(0,t) = u(\pi,t) = 0, \quad t > 0,$$
$$u(x,0) = x^2, \quad 0 < x < \pi$$

(b)

$$\frac{\partial u}{\partial t} = 7\frac{\partial^2 u}{\partial x^2}, \quad 0 < x < \pi, \quad t > 0$$
$$\frac{\partial u}{\partial x}(0,t) = \frac{\partial u}{\partial x}(\pi,t) = 0, \quad t > 0,$$
$$u(x,0) = 1 - \sin x, \quad 0 < x < \pi$$

(c)

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}, \quad 0 < x < \pi, \quad t > 0$$
$$u(0,t) = 0, \quad u(\pi,t) = 3\pi, \quad t > 0,$$
$$u(x,0) = 0, \quad 0 < x < \pi$$

(d)

$$\frac{\partial u}{\partial t} = 3\frac{\partial^2 u}{\partial x^2} + x, \quad 0 < x < \pi, \quad t > 0$$
$$u(0,t) = u(\pi,t) = 0, \quad t > 0,$$
$$u(x,0) = \sin x, \quad 0 < x < \pi$$

(e)

$$\frac{\partial u}{\partial t} = 4\frac{\partial^2 u}{\partial x^2}, \quad 0 < x < \pi, \quad t > 0$$
$$\frac{\partial u}{\partial t}(0,t) = 0, \qquad u(\pi,t) = 0, \quad t > 0,$$
$$u(x,0) = f(x), \quad 0 < x < \pi$$