Multi-Product Competition and Informative Advertising

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December 2008

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Abstract

In several product categories, such as cereals, ice cream, and detergents, competing firms market a variety of products to meet the needs of their diverse consumers. Furthermore, these firms inform consumers about their products using advertising. In this paper, we propose a novel framework for studying multi-product competition and informative advertising in a horizontally differentiated market. Our conceptualization of product advertising and brand advertising recognizes that consumer’s attention span is limited, consumer memory is fallible and captures effects such as advertising clutter, advertising confusion, and brand dilution. Using the framework, we analyze how market structure and different types of informative advertising affect a firm’s product-line length.

We find that, when firms release product specific advertising, the number of products offered by a monopolist increases with advertising reach, but the result is reversed in the presence of competition. By contrast, when firms release brand advertising to inform consumers about their products, the length of product-line can be increasing for both the monopolist and duopolists, but under very different conditions. In choosing between product and brand advertising, the monopolist favors brand advertising. The duopolists, on the other hand, prefer product advertising; however, they increase the emphasis placed on brand advertising as the diversity in consumers’ taste increases or consumers’ attention span decreases.

Keywords: Multi-Product Competition, Informative Advertising, Product Line, Game Theory.
1. INTRODUCTION

In many product categories, firms offer a variety of products to meet the diverse needs of consumers. For example, Kellogg’s offers 23 different varieties of cereals while General Mills markets over 10 varieties of cereals. Similarly, Häagen-Dazs produces 29 flavors of ice cream and Ben & Jerry’s sells 39 varieties of its original ice cream. We also see evidence of multi-product competition in product categories such as toothpaste (Crest versus Colgate) and laundry detergent (Proctor & Gamble detergents versus Unilever detergents). An interesting aspect of these horizontally differentiated markets is that the competition among products is often nonlocal, as consumers can walk a few steps down the aisle and purchase any of the competing products. Moreover, though firms offer a wide variety of products, some consumers’ preferred varieties may still be unavailable. In some instances, even though a consumer’s preferred products are available, she may not be informed about them. Typically, these multi-product firms use product and/or brand advertising to inform consumers about their product lines. A critical marketing challenge facing a multi-product firm is: How can it offer multiple products without cannibalizing its own product line but tapping new market segments and gaining sales from its competitor. In this paper, we investigate how informative advertising and market structure may affect a multi-product firm’s product-line decisions.

Focusing on a market where consumers have diverse tastes and firms offer multiple products, we attempt to answer four questions: How does imperfect information and market structure influence multi-product competition? How does product advertising affect a firm’s product-line length? How does brand advertising impact the number of products offered by a firm? and What should be the optimal level of brand advertising? Building on Chen and Riordan (2007) and Amaldoss and He (2008), we propose a novel model of multi-product competition and informative advertising with many interesting features. In our formulation, consumers are distributed on a plane along different taste dimensions (spokes), each product directly competes with every other product in the market (nonlocal competition), the market can be partially covered, and consumers may have limited attention span and fallible recall (cognitive limitations). Our analysis highlights how different types of informative advertising interact with market structure to shape a firm’s product-line decisions.

First, we attempt to highlight some key forces that influence a multi-product firm’s product-line decision in a simple setting. Toward this goal, we examine a model where a firm offers multiple products and its advertising informs consumers about all of its products. In this prelude on multi-product competition, we treat the reach of a firm’s advertising reach as ex-
ogenous to the model though we later endogenize the advertising decision. As we naturally expect, if consumers are perfectly informed about the products offered by a firm, a monopolist will offer more products when consumer valuation increases. However, if consumers are imperfectly informed about a firm’s product line, our analysis shows that the relationship between a monopolist’s product line length and its advertising reach takes an inverted-U shape. When the reach of advertising is low, it is profitable for a monopolist to offer more products and improve its market coverage. But when advertising reach is high, it becomes more profitable for a monopolist to offer fewer products and reduce the cannibalization within its product line.

In the presence of competition, we obtain a different result. The number of products offered by a duopolist declines with advertising reach. This occurs because, although improved advertising reach increases the demand from informed consumers whose preferred products are offered by the firm, it also increases the size of the consumer segment where it must compete against another firm for sales. In order to dampen this competitive effect, duopolists offer fewer products when advertising reach increases. After this initial analysis, we advance to consider a more nuanced form of product advertising.

On average, a consumer is exposed to thousands of advertisements in a day and consumers manage this information overload by not paying attention to a large fraction of the advertisements (e.g., Petrecca 2006, Anderson and de Palma 2006). New technologies are also making it easier for consumers to zap advertisements (Siddarth and Chattopadhyay 1998). Thus, contrary to a standard assumption in models of informative advertising, consumers are not likely to process all the advertisements to which they are exposed. We incorporate limited attention span into our model of product advertising by imposing a constraint on the number of advertisements that consumers can process. Marketing literature also suggests that if consumers process too many advertisements, it impairs their ability to recall the ad messages (Keller 1987, Riebe and Dawes 2006), a phenomenon established in laboratory as well as in field settings (Pieters and Bijmolt 1997). An important consequence of this impaired recall is that it increases a firm’s cost of advertising. In our model, we allow for the possibility that the marginal cost of advertising can increase with the number of product varieties a firm offers. Under product advertising, we find that the number of products offered by a monopolist will not exceed the consumer’s attention span. Note that adding a new product to a firm’s product line reduces the probability of a consumer paying attention to the advertisements of any of the existing products because of the consumer’s limited attention span. As the monopolist internalizes the entire negative externality, it reduces its incentive to offer too many products. Furthermore, we note that the monopolist will offer more products as advertising reach increases provided the
number of products is less than consumer’s attention span and the cost of advertising clutter is below a threshold. In the presence of competition, however, the number of products offered will exceed the attention span of consumers and hence we will observe advertising clutter. We obtain this result because when a duopolist adds a product to its product line, it reduces the probability consumers are aware of each of its products and the competing firm’s products. In a duopoly, because an individual firm does not fully internalize the negative externality induced by adding another product to its product line, we may observe too many products. Again, as advertising reach increases, a duopolist will increase the length of its product line, but only if the resulting advertising confusion is not too costly. Next we examine informative brand advertising.

The literature on brand advertising and memory points to the fact that if too many products are advertised under a brand umbrella, then the linkages between a brand and its numerous products become weaker (Raaijmakers and Shiffrin 1981, Keller 1991 and 1998). Consequently, a firm needs to carefully weigh the advantage of introducing a new product to better serve the needs of its diverse consumers against the prospect of brand dilution. We find that the optimal number of products offered by a monopolist increases with brand advertising provided brand dilution is below a threshold. This finding, however, is reversed in the presence of competition. The optimal number of products offered by duopolists increases with brand advertising only if brand dilution is above a threshold. We obtain this result because brand dilution reduces cannibalization within a firm’s own product line as well as softens competition from the products of the other firm.

Finally, we consider a case where firms have the option to engage in product advertising as well as brand advertising. In this scenario, there is scope for brand dilution, brand reinforcement, and advertising overload. Our analysis suggests that it is more profitable for a monopolist to use only brand advertising, as brand advertising helps the monopolist to better handle consumer’s limited attention span by chunking information into a composite brand advertisement. It is useful to note that although brand advertising creates scope for brand dilution, it also helps the monopolist to offer more products and better address the needs of its diverse consumers. Next, unlike a monopolist, a duopolist uses both product and brand advertising. In contrast to a monopolist, a duopolist does not fully internalize the negative externality induced by its additional product, and hence a duopolist is motivated to offer more products and use product advertising. But if a duopolist only engages in product advertising it will cause information overload, and brand advertising helps to alleviate this problem and better cater to the needs of its customers. Our analysis further shows that the use of brand advertising will increase as
the diversity in consumers’ tastes grows, but decrease as consumer’s attention span increases.

**Related Literature.** Our work is related to the theoretical literature on informative advertising and horizontal product differentiation. In a seminal work on multi-product competition, Klemperer (1992) shows that it is profitable for duopolists to compete head-to-head with identical products rather than offer interspersed products. By offering identical products, competing firms reduce consumers’ incentive to visit multiple stores and resort to comparison shopping. Consequently, contrary to some of our intuitions, price competition is softened when product lines are similar (head-to-head) rather than dissimilar (interlaced). This result was obtained using a circle model of horizontal differentiation where competition is local. Caminal (2006) shows that fewer products will be available in a duopoly compared to a monopolistically competitive market, and yet the number of products available will be more than the social optimum. In these horizontal differentiation models, there is no scope for price discrimination because firms charge the same price for all their products. In vertical differentiation models of multi-product competition, however, firms offer products of varying quality and profit from second-degree price discrimination (Champsaur and Rochet 1989, Gilbert and Matutes 1993, see also Villas-Boas p. 312).

Our paper builds on prior work on informative advertising. Using a circle model, Soberman (2004) shows that price decreases with informative advertising when product differentiation is modest but increases with advertising when product differentiation is high. In a circle model competition is local in that a small change in a firm’s price only affects its neighboring firms, but not the other firms in the market. A drawback of the circle model is that symmetry necessitates incumbent firms to relocate in the product space on the entry of a new firm (see also Grossman and Shapiro 1984, Salop 1979). By incorporating informative advertising into the spokes model of Chen and Riordan (2007), Amaldoss and He (2008) examine how prices of single-product firms are affected by the reach of advertising and improvements in advertising technology. They show that informative advertising can lead to lower prices if consumer valuations are high. However, if consumer valuations are moderate, informative advertising can lead to higher prices. Furthermore, when consumer valuations are high, price increases with diversity in tastes. This result, however, is reversed if consumer valuations are moderate. They note that, while improvements in advertising technology lead to higher levels of advertising when consumer valuation is high, the opposite can be true when consumer valuation is moderate. In contrast to our work, this body of literature studies the behavior of single-product firms.

Our work is closely related to Villas-Boas (2004), which presents several innovative ideas on multi-product competition and communication. Specifically, Villas-Boas studies the effect of
informative advertising on a monopolist’s product line in the context of the firm producing two products located at either end of a Hotelling line. He shows that when informative advertising causes confusion among consumers or when consumer heterogeneity is sufficiently small, a firm may choose to advertise one product only. In a vertically differentiated market, it is more profitable to intensively advertise the lower quality product. Like Villas-Boas (2004), we also study how informative advertising shapes firms’ product-line decisions. But we focus exclusively on a horizontally differentiated market and allow firms to offer a large number of products.

**Contribution.** In many product markets, firms offer multiple products and competition is nonlocal. In this paper, we develop a parsimonious model of multi-product competition that reflects this market reality. In our framework, consumers are distributed on a spokes network and competition is non-localized in that each product is in direct competition with every other product in the market. Furthermore, unlike a circle model, our model allows products and firms to be symmetric without the need to change the location of current products when a new product enters the market. Second, we propose parsimonious models of product advertising and brand advertising that are faithful to key behavioral findings such as limited attention span, brand dilution, and brand reinforcement. Third, we show how different types of informative advertising (product and brand advertising) and market structure (monopoly and duopoly) affect a multi-product firm’s product-line length. To the best of our knowledge, these issues have not been investigated in any of the prior literature on multi-product competition.

The rest of the paper is organized as follows. Section 2 outlines our model of multi-product competition and as a prelude analyzes the effect of imperfect information on a firm’s product-line decision. Section 3 introduces a model of product advertising and multi-product competition and analyzes its implications for a monopoly and duopoly. In Section 4, we study the effect of brand advertising and how firms make the trade-off between product advertising and brand advertising. Finally, Section 5 concludes by summarizing the findings and outlining some directions for further research.

### 2. A PRELUDE

In this section, we present a parsimonious model of multi-product competition where a firm’s advertising informs consumers about all the products in its portfolio. The purpose of this initial analysis is to lay bare the forces that shape a multi-product firm’s product-line decision in a context where consumers are not perfectly informed about the firm’s products. In order to accomplish this goal, we treat the reach of a firm’s advertising as exogenous to the model. After
this prelude, we advance to analyze a more nuanced model of product advertising, where we allow for the possibility that consumers’ attention span can be limited and product advertising can clutter the advertising medium (see Section 3). In this formulation, a firm separately advertises each of its products and the reach of its advertising is endogenous to the model. Using the model, we study how product advertising and multi-product competition affect a firm’s product line. Next, we analyze brand advertising (see Section 4). While brand advertising can inform consumers of products within the brand umbrella, it may dilute the attention given to any individual product. We explore the consequences of this challenge in our model of brand advertising. Finally, we explore the trade-off a firm makes between product advertising and brand advertising. Thus, in a phased manner using models of increasing nuance and complexity, we attempt to unravel how different types of advertising and market structure shape a multi-product firm’s product-line decision. Now we proceed to introduce the spokes model and then incorporate imperfect information and multi-product competition into the framework.

**Spokes Model.** Consider a product market where consumers seek $N$ different varieties (e.g., flavors). We model this market as a spokes network on a plane. Consumers preferring a variety are represented by a spoke of length $\frac{1}{2}$. Each consumer on a spoke is identified by the distance, $x \in [0, \frac{1}{2}]$, at which she is located from the origin of the spoke. Note that at the origin of a spoke $x = 0$ whereas at the center of the spokes network $x = \frac{1}{2}$. We assume that consumers are distributed uniformly on the spokes network, and the total mass of consumers on the $N$ spokes is $\frac{N}{2}$. To help appreciate the model structure, we illustrate in Figure 1 a multi-product monopoly where consumers seek nine varieties ($N = 9$) and the monopolist offers five products ($n = 5$).

As in Chen and Riordan (2007), we assume that each consumer considers at most two products. In addition to considering the product located on the spoke in which she lies (local brand), each consumer considers one of the products located in any of the other spokes (nonlocal brand). We assume that the nonlocal brand preferred by a consumer is exogenously fixed a priori. Furthermore, all nonlocal brands are equally likely to be the second preferred product. The assumption that consumers consider at most two brands helps to obtain a pure strategy equilibrium and also reflects the notion that consumers often have a small consideration set (Nedungadi 1990, Hauser and Wernerfelt 1990). Each consumer purchases at most one unit of the product. The base value of all the product varieties is the same and it is denoted by $v$. If the consumer located at $x$ on spoke $l$ is aware of her local brand $l$ and chooses to purchase the product, she will derive the following indirect utility:

$$ U(x, p_l) = v - tx - p_l, $$ (1)
Fig. 1. A multi-product monopoly where consumers seek nine varieties \((N = 9)\) and the monopolist offers five products \((n = 5)\)

where \(t\) is her sensitivity to product characteristics, and \(p_l\) is the price of the product. But if the consumer wants to purchase any other product of which she is informed, namely variety \(m\) on spoke \(m\) such that \(m \neq l\), then the indirect utility from this nonlocal brand will be:

\[
U(x, p_m) = v - t(1 - x) - p_m.
\]  

Note that the consumer located on spoke \(l\) is \(\frac{1}{2} - x\) units of distance away from the center of the spokes network, and brand \(m\) is \(\frac{1}{2}\) unit of distance further away. Thus the total distance between the consumer and nonlocal brand \(m\) is \(\frac{1}{2} - x + \frac{1}{2} = 1 - x\). The consumer will purchase local brand \(l\) if \(U(x, p_l) > U(x, p_m)\). Hence the marginal consumer who is indifferent between the two products is located at a distance \(\frac{1}{2} + \frac{p_m - p_l}{2t}\) from the brand \(l\). The demand for local brand \(l\) is \(\min\{\frac{1}{2} + \frac{p_m - p_l}{2t}, 1\}\), implying that \(\frac{|p_m - p_l|}{t} \leq 1\).

**Imperfect Information.** In the current formulation, advertising informs about product availability and prices. As in previous informative advertising models (e.g., Butters 1977, Grossman and Shapiro 1984, Soberman 2004), we assume that consumers recall the characteristics and prices of the products seen in advertisements while formulating their purchase decisions. However, they do not engage in any costly information search. This assumption does not imply that consumers are unaware of the underlying market structure; it just suggests that consumers rely on advertising to learn about the prices and the products available in the mar-
In this section, we consider a simple form of informative advertising: A firm’s advertising informs consumers about all the products produced by the firm. That is, if the reach of firm \( i \)’s advertising is \( \phi_i \), it implies that consumers become aware of each of the firm’s products with probability \( \phi_i \).

**Multi-product Competition.** Next we analyze a multi-product monopoly and a multi-product duopoly so that we can understand how the nature of competition tempers a firm’s product-line decisions.

**Multi-product Monopoly.** Consider a monopolist who offers \( n \) (\( 2 \leq n \leq N \)) different product varieties. Recall that Figure 1 illustrates a multi-product monopoly where consumers prefer \( N = 9 \) different varieties but the monopolist offers only \( n = 5 \) products. As a consumer’s consideration set includes only two varieties, the number of possible combinations of varieties that can be preferred by consumers is:

\[
\frac{N(N-1)}{2}.
\]  

(3)

The possible combinations of product varieties that the monopolist can satisfy is:

\[
\frac{n(n-1)}{2}.
\]  

(4)

Hence, the fraction of consumers who will have access to the two products in their consideration set and are informed about both of them is given by:

\[
\phi^2 \frac{n(n-1)}{N(N-1)}.
\]  

(5)

The fraction of consumers whose two preferred products are offered by the monopolist but are informed about only one of them is:

\[
2\phi(1-\phi) \frac{n(n-1)}{N(N-1)}.
\]  

(6)

The monopolist may not offer all the varieties preferred by consumers, as \( n \leq N \). Consequently the monopolist may supply only one of the two products preferred by some consumers. The fraction of the market, that has any one of its two preferred products available and is informed about it, is given by:

\[
2\phi \frac{n(N-n)}{N(N-1)}.
\]  

(7)

Assume that the fixed cost of introducing a new product is \( F > 0 \) and the constant marginal cost is \( c = 0 \). As the total fixed cost of introducing \( n \) products is \( nF \) and the total mass of consumers is \( \frac{N}{2} \), the fixed cost per consumer is \( \frac{2nF}{N} \).
Upon aggregating the demand given in equations (5), (6) and (7) and computing the resulting monopolist’s profits we have:

$$\pi_M = \left[ \frac{n (n - 1)}{N(N - 1)} \phi^2 + 2\phi (1 - \phi) \frac{n (n - 1)}{N(N - 1)} + \frac{2\phi n (N - n)}{N(N - 1)} \right] p_M - \frac{2nF}{N},$$

where $p_M$ is the monopolist’s price. In practice, prices can be quickly changed, whereas product line changes are much slower and costly. This implies that when a firm changes its prices it takes its product-line decision as fixed. We capture this aspect of reality by adopting a two-stage game structure: in the first stage the monopolist determines the optimal number of products to offer, and in the second stage it sets its price.

Because the optimal price of a monopolist is $v - t$, it is natural to expect a monopolist to offer more products when consumer valuation increases. However, on examining the effect of advertising reach on the optimal number of products, we obtain the following result.

**Proposition 1** (i) If $v \leq v_1$, the monopolist does not offer any product; (ii) if $v_1 < v < v_2$, then the number of products offered increases with advertising reach; (iii) but, if $v > v_2$, the number of products offered decreases with advertising reach, where $v_1 = \frac{2(F + t\phi)(N-1) + t\phi^2}{\phi^2 + 2\phi(N-1)}$ and $v_2 = \frac{2F}{\phi} + t$.

The first part of the proposition suggests that if consumer valuation is very low, a monopolist will not offer any product. However, if consumer valuation grows further, it may become optimal for a monopolist to increase the number of its products as the reach of its advertising increases. As the third part of the proposition notes, if consumer valuation grows sufficiently large, it is profitable for a monopolist to offer fewer products as advertising reach increases.

In essence, Proposition 1 posits an inverted-U shaped relationship between a monopolist’s product-line length and advertising reach. This finding may run counter to some of our intuition: a monopolist will offer more products as consumer valuation increases. On probing further we note that this intuition is valid only if all consumers are informed of the products, that is $\phi = 1$.\(^1\) This observation raises the possibility that the inverted-U relation between valuation and product-line length could be moderated by the level of advertising reach. To better grasp the moderating role of advertising reach, note that when a monopolist offers more products, it increases the fraction of consumers who can potentially purchase at least one of the two varieties in their consideration sets (see equations 5–7). The resulting increased market coverage

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\(^1\)We find that when $\phi = 1$, $n^* = \frac{1}{2(v-t)} [2N(v-t-F) - (v-t-2F)]$ and $\frac{\partial n^*}{\partial v} = \frac{F}{(v-t)} (N-1) > 0$ if $v > \frac{2F(N-1)}{2N-1} + t$. The monopolist does not offer any product if $v \leq \frac{2F(N-1)}{2N-1} + t$. See Appendix A for more details on the perfect information case.
is particularly helpful when the reach of its advertising is relatively low, that is $\phi < \frac{2F}{v-t}$. There is also a counterbalancing force at work here. Because a consumer can afford to purchase any of the two varieties in her consideration set, and because it is costly to introduce an additional product, it is important for a monopolist to reduce the cannibalization within its own product line. Note that $\frac{(v-t)\phi}{2}$ is the expected revenue from selling a product variety to the local consumers in a spoke. If this revenue is more than the fixed cost of introducing a new product, then the monopolist may be motivated to introduce additional products. But introducing additional products increases the prospect of cannibalization within the monopolist’s product line. Consequently, if $\phi > \frac{2F}{v-t}$, a monopolist offers fewer products as advertising reach increases.

Next, we examine how the presence of competition may affect a firm’s product-line length.

Multi-product Duopoly. To understand how competition can affect equilibrium behavior, consider a duopoly where firms $i$ and $j$ supply $k_i$ and $k_j$ products, respectively. Figure 2 shows a multi-product duopoly where consumers prefer $N = 8$ different varieties although each duopolist offers $k_i = k_j = 3$ products. As in the previous model, each firm simultaneously chooses the number of products to offer in the first stage of the game. In the second stage, each firm sets a price for its product, namely $p_i$ and $p_j$.

Focusing attention on firm $i$, we note that the demand for its products can emanate from different segments of consumers. Firm $i$ could cater to informed consumers whose two preferred brands are produced by it. The size of this segment is given by:

$$\phi_i^2 k_i (k_i - 1) N (N - 1).$$  \hspace{1cm} (9)

Firm $i$ could also sell its product to consumers whose two preferred products are made by it although consumers are aware of only one of the products. The demand from such consumers is:

$$2\phi_i (1 - \phi_i) \frac{k_i (k_i - 1)}{N (N - 1)}. \hspace{1cm} (10)$$

Consumers who have only one of firm $i$’s products in their consideration set but are aware of the product could also contribute to its sales. For some of these consumers, their other preferred variety may be offered by firm $j$, and the demand from this group is given by:

$$2\phi_i (1 - \phi_j) \frac{k_i k_j}{N (N - 1)}. \hspace{1cm} (11)$$

For certain other consumers, their other preferred variety may not be offered by any firm.

\footnote{In this symmetric game, as the firms are symmetric, $k_i = k_j = \frac{n}{2}$ in equilibrium. Further, as all the products are symmetric, $p_i = p_j$ in equilibrium.}
Fig. 2. A multi-product duopoly where consumers seek eight varieties \((N = 8)\) and each duopolist offers three products\((k_i = k_j = 3)\)

![Diagram of a multi-product duopoly with consumers seeking eight varieties and each duopolist offering three products.]

Notes: Firm \(i\) occupies the spokes in solid lines, while firm \(j\) occupies the spokes in dotted lines. The two spokes in solid gray lines are not served.

The demand for firm \(i\)'s product from such consumers is:

\[
\frac{2\phi_i k_i}{N} \left( 1 - \frac{k_i + k_j - 1}{N - 1} \right). \tag{12}
\]

Finally, the consideration set of some consumers includes one product from each of the two firms and they are informed about both the products. In this case, the demand for firm \(i\)'s product depends on the prices of the two competing firms and it is given by:

\[
\frac{2\phi_i \phi_j k_i k_j}{N (N - 1)} \left( \frac{1}{2} + \frac{p_j - p_i}{2t} \right). \tag{13}
\]

We assume that \(v - t - p_i > 0\) so that all consumers could afford to purchase the product and there is scope for competition between the two firms. Now the profits of firm \(i\) are as follows:

\[
\pi_i = \left[ \frac{\phi_i^2 k_i (k_i - 1)}{N(N - 1)} + 2\phi_i (1 - \phi_i) \frac{k_i (k_i - 1)}{N(N - 1)} + 2\phi_i (1 - \phi_j) \frac{k_i k_j}{N(N - 1)} \right] p_i - \frac{2k_i F}{N}. \tag{14}
\]

We have the following result on how a duopolist’s advertising reach affects the length of its product line.

**Proposition 2** In the presence of competition, the number of products offered by a firm decreases with advertising reach.
As noted in the discussion of Proposition 1, in the absence of competition we observe an inverted-U relationship between product-line length and advertising reach. This result was a consequence of the monopolist trying to increase its market coverage and at the same time attenuating cannibalization of its products. Proposition 2 shows that we obtain a qualitatively different result in the presence of competition. To understand the intuition for this finding, note that in a duopoly model several forces are at play: price effect, competition effect, market expansion effect and cannibalization effect.

We find that the equilibrium price is given by:

\[ p_i = \frac{1}{\phi_j k_j} [2t (N - 1) - t\phi_i (k_i - 1)] - t. \]  

(15)

This implies that price decreases when advertising reach increases. Then turning attention to the effect of advertising reach on demand, we note that for a given number of product \( k_i \), as the reach of firm \( i \)'s advertising increases, it raises the demand from informed consumers whose two preferred products are offered by the firm (see equation 9). But, in equilibrium, the size of this segment will remain lower than that of the group of informed consumers whose consideration set has one product from each of the two competing firms (see equation 13). Thus, when advertising reach increases, it heightens competition in the market. In an attempt to dampen the resulting negative impact on profits, firms offer fewer products when advertising reach increases. In the current analysis, we have assumed that a firm’s advertising informs consumers about all the products in its product line. A polar opposite form of advertising is one where a firm separately advertises each product in its product line. We next analyze this form of product advertising and also endogenize firms’ advertising decisions.

3. A MODEL OF PRODUCT ADVERTISING

When firms separately advertise each of their products, they may release multiple advertisements and potentially clutter the advertising media. In reality, on average a consumer is exposed to 3,000 to 5,000 advertisements a day, and this information overload can exceed consumers’ limited attention span (Petrecca 2006). Consumers handle this information overload by screening out a large fraction of the advertisements (e.g., Anderson and de Palma 2006, Van Zandt 2004). Consumers even have access to devices that zap advertisements (Siddarth and Chattopadhyay 1998). Thus, contrary to the standard assumption in models of advertising,

\[^{3}\text{It is useful to consider the case when advertising reach increases all the way to 1, that is all consumers are perfectly informed. Even in this case, the number of products offered by duopolists is more than than that offered by the monopolist. For more details on this claim, see Appendix A.}\]
consumers may not process all the advertisements to which they are exposed (e.g., Butters 1977, Grossman and Shapiro 1984, Robert and Stahl 1993, Soberman 2004). In light of this behavior, we assume that consumers process only \( \tau > 2 \) advertisements in a window of time and that a firm advertises \( n \) different products. The probability, that the advertisement of any given product is processed, is given by \( \frac{\tau}{n} \) if \( n > \tau \). But if the number of products advertised is less than \( \tau \), consumers have the attention span to process all the advertisements. This implies that, if a firm releases \( n \) product-specific advertisements each with a reach of \( \omega \in (0,1) \), the probability that a consumer processes the advertisement of a given product is \( \omega \) if \( n \leq \tau \), but reduces to \( \frac{\tau \omega}{n} \) if \( n > \tau \). Using this formulation of product advertising, we next explore how market structure may affect a firm’s product-line decision.

Multi-product Monopoly First, let us consider the case where the monopolist offers \( n > \tau \) products. The monopolist may be able to supply the two preferred products of some consumers, who may also process the advertisements of their preferred products. The fraction of such consumers in the market is:

\[
\frac{n(n-1)}{N(N-1)} \left( \frac{\tau \omega}{n} \right)^2.
\]

(16)

In the case of some consumers, though the monopolist supplies both their preferred products, they are informed about only one of the two products. The size of this segment is given by:

\[
\frac{2n \tau \omega}{n} \left( 1 - \frac{\tau \omega}{n} \right) \frac{n(n-1)}{N(N-1)}.
\]

(17)

Finally, the fraction of consumers who have only one preferred brand available and have seen its advertisement is:

\[
\frac{2n \omega (N-n)}{n N(N-1)}.
\]

(18)

Let \( A(n, \omega) \) denote the cost of informing \( \omega \) fraction of the consumers about all of its \( n \) products. We assume that the advertising cost increases at an increasing rate with the level of reach and the number of products advertised. That is, \( A(n,0) = 0 \), \( A_n = \frac{\partial A}{\partial n} > 0 \), \( A_{nn} = \frac{\partial^2 A}{\partial n^2} > 0 \), \( A_\omega = \frac{\partial A}{\partial \omega} > 0 \), and \( A_{\omega \omega} = \frac{\partial^2 A}{\partial \omega^2} > 0 \), implying that the advertising cost function is convex in the number of product varieties and advertising reach.

A common finding in advertising research is that if consumers are exposed to too many advertisements in a product category, their ability to recall the ad messages is reduced (e.g., Riebe and Dawes 2006). This happens because the multiple advertisements create overlapping ad memory traces and thereby weaken the linkages between the products in a category and the corresponding ad memory traces. For instance, Keller (1987) shows that the recall of product messages is higher when the number of competing advertisements is one rather than two (see...
also Burke and Srull 1988). On analyzing the recall data for 2,677 television commercials from 39,000 consumers over a 17 year period, Pieters and Bijmolt (1997) note that competitive ad clutter has a negative impact on ad recall. In the context of our model, these findings imply that the marginal cost of advertising a product can increase if advertising reach increases. That is, \( A_{wn} > 0 \). It is also possible that a monopolist can benefit from economies of scale if it advertises more products, implying that \( A_{wn} < 0 \).

The monopolist’s profits, when it offers \( n > \tau \) products, can be written as:

\[
\pi_M = \left[ \frac{(n-1) (\tau \omega)^2}{Nn (N-1)} + \frac{2\tau \omega (n - \tau \omega) (n-1)}{Nn (N-1)} + \frac{2\tau \omega (N-n)}{N (N-1)} \right] p_M - \frac{2nF}{N} - A(n, \omega). \tag{19}
\]

If the monopolist supplies \( n < \tau \) products, then product advertising will not congest consumer’s attention span, and the probability of a consumer processing a given product’s advertisement will remain \( \omega \). The corresponding monopolist’s profits are:

\[
\pi_M = \left[ \frac{n (n-1)}{N (N-1)} \omega^2 + 2\omega (1 - \omega) \frac{n(n-1)}{N (N-1)} + 2\omega \frac{n(N-n)}{N (N-1)} \right] p_M - \frac{2nF}{N} - A(n, \omega). \tag{20}
\]

We assume that the monopolist first formulates its product strategy in terms of number of products to offer and its advertising reach, and then sets its price. This decision structure reflects the notion that it is easier for a firm to change the prices of its products and that a firm treats other marketing mix decisions as fixed while setting prices. Solving the game backward, we first determine the equilibrium prices and then compute the equilibrium number of products. We have the following result about product advertising.

**Proposition 3** Under product advertising, (i) the number of products offered by a monopolist will not exceed consumer’s attention span, \( \tau \); (ii) the product-line length will be less than \( \tau \) if \( \frac{2(v-t)\omega-2\tau}{N(N-1)} > A_n \); and (iii) when \( n^* < \tau \), the number of products offered increases with the reach of advertising \( \omega \), if \( A_{n\omega} < \frac{2(v-t)(N-1)}{N(N-1)}(2n-1)\omega \).

Note that a monopolist’s price remains \( v-t \) irrespective of the number of products it supplies. So these results are a consequence of the effect of consumers’ attention span and advertising reach on the demand for a monopolist’s products. To see the intuition for the first part of the proposition, note that if the number of products offered by the monopolist is more than consumer’s attention span, consumers will process the advertisements of only \( \tau \) products and not notice the advertisements of other products. Consequently, on introducing the \( \tau + 1^{th} \) product, the monopolist reduces the probability that consumers see the advertisement of each of its products. This externality damps the demand for all its products. As the entire negative
externality needs to be internalized by the monopolist, it is not motivated to supply more than \( \tau \) products. Furthermore, offering more products increases the marginal cost of advertising as \( A_n > 0 \). Precisely, on differentiating the monopolist’s profits (see equation 19) with respect to \( n \), we have:

\[
\frac{\partial \pi_M}{\partial n} = -\frac{2F n^2 (N-1) + \tau^2 \omega^2 (v-t)}{N n^2 (N-1)} - A_n < 0.
\]  

(21)

This implies that it is not profitable for a monopolist to offer more than \( \tau \) products. So a monopolist may only supply \( 0 < n \leq \tau \) products. Of course, if the fixed cost of introducing a product is sufficiently high, then the monopolist will not supply any product. The second part of the proposition specifies the condition when we have an interior solution for \( n^* \). Notice that the monopolist’s profits given in equation (20) are a concave function of \( n \). Moreover,

\[
\frac{\partial \pi_M}{\partial n} = 2 \frac{[(v-t) \omega - F] (N-1) - (v-t) (2n-1) \omega^2}{N (N-1)} - A_n.
\]  

(22)

This suggests that under the condition specified in the second part of the proposition, the monopolist will supply \( 0 < n^* < \tau \) products.

To grasp the intuition for the third part of the proposition, recall that \( A_n \) gives the marginal cost of advertising and \( A_{n\omega} \) indicates how this marginal cost increases with advertising reach. Thus the size of \( A_{n\omega} \) indicates how much advertising clutter impairs consumer recall and in turn increases advertising costs. The third part of the proposition shows that the optimal number of products offered by a monopolist increases with advertising reach provided \( A_{n\omega} \) is below a threshold. In our basic model of informative advertising, we showed that the number of products offered by a monopolist has an inverted-U relationship with advertising reach. Here again we obtain a similar pattern of results. However, as the cost of product advertising depends on both the number of products advertised and reach, the condition specified in the third part of the proposition is related to the size of \( A_{n\omega} \).

Multi-product Duopoly The two competing firms, \( i \) and \( j \), offer \( k_i \) and \( k_j \) products at prices \( p_i \) and \( p_j \), respectively. In the first stage of the game, each firm simultaneously determines its product strategy; namely the number of products to offer and the reach of its advertising. In the second stage, firms simultaneously set their prices.

The demand for firm \( i \)'s products can come from five different segments of consumers. One segment comprises of consumers whose two preferred products are supplied by firm \( i \) and members of this segment are also informed about the products. If \( \omega_i \) is the reach of firm \( i \)'s advertising and \( \tau \) is consumer’s attention span, the probability a consumer in this segment sees both their preferred products is \( \left( \frac{\tau \omega_i}{k_i + k_j} \right)^2 \). Note that we have \( k_i + k_j \) in the denominator of this
expression as it represents the total number of advertisements released by the two competing firms. The demand from this segment for firm $i$’s products is given by:

$$\left( \frac{\tau \omega_i}{k_i + k_j} \right)^2 \frac{k_i (k_i - 1)}{N (N - 1)}.$$  \hfill (23)

The second group of consumers also have both their preferred products supplied by firm $i$, but they are aware of only one of them. The demand from this segment for firm $i$’s product is:

$$\frac{2\tau \omega_i}{k_i + k_j} \left( 1 - \frac{\tau \omega_j}{k_i + k_j} \right) \frac{k_i (k_i - 1)}{N (N - 1)}.$$  \hfill (24)

The third group of consumers prefer one product from each of the two competing firms. Although both the products are available, they are aware of only the product supplied by firm $i$, and hence the demand from this group is given by:

$$\frac{2\tau \omega_i k_i}{(k_i + k_j) N} \left( 1 - \frac{k_i + k_j - 1}{N - 1} \right).$$  \hfill (25)

In the case of the fourth group of consumers, only one of their two preferred products is available and it is supplied by firm $i$. The demand from this group is:

$$\frac{2\tau \omega_i k_i}{(k_i + k_j) N} \left( 1 - \frac{k_i + k_j - 1}{N - 1} \right).$$  \hfill (26)

The fifth group of consumers prefer one product from each of the two firms and they are aware of both the products. The demand for firm $i$’s products from this segment is:

$$\frac{\tau^2 \omega_i \omega_j}{(k_i + k_j)^2 N (N - 1)} \left( \frac{1}{2} + \frac{p_j - p_i}{2t} \right).$$  \hfill (27)

On aggregating the demand from all the five groups of consumers, and computing the profits of firm $i$, we have:

$$\pi_i = \left[ \frac{\tau \omega_i}{k_i + k_j} \frac{k_i (k_i - 1)}{N (N - 1)} + \frac{\tau \omega_j}{k_i + k_j} \left( 1 - \frac{\tau \omega_i}{k_i + k_j} \right) \frac{k_i (k_i - 1)}{N (N - 1)} + \frac{\tau \omega_i}{k_i + k_j} \left( 1 - \frac{\tau \omega_j}{k_i + k_j} \right) \frac{k_i k_j}{N (N - 1)} \right] p_i - \frac{2k_i F}{N} - A(k_i, \omega_i),$$  \hfill (28)

where $F > 0$ is the fixed cost of introducing a product and $A(k_i, \omega_i)$ is the cost of achieving an advertising reach of $\omega_i$ for each of the $k_i$ products. We assume that the advertising cost increases at an increasing rate in that $A(k_i, 0) = 0, A_{k_i} = \frac{\partial A}{\partial k_i} > 0, A_{k_i k_i} = \frac{\partial^2 A}{\partial k_i^2} > 0, A_{\omega_i} = \frac{\partial A}{\partial \omega_i} > 0,$ and $A_{\omega_i \omega_i} = \frac{\partial^2 A}{\partial \omega_i^2} > 0$. Competitive advertising clutter is likely to be a more significant issue in the case of a multi-product duopoly, and hence we may have $A_{\omega_i k_i} > 0$. However, if the economies of scale due to advertising were so significant to override the clutter effect, we may have $A_{\omega_i k_i} < 0$. We assume that $v - t - p_i > 0$ so that there is some competition between the two firms. We have the following result on equilibrium behavior.
Proposition 4 Under product advertising, (i) the number of products offered by the two duopolists will exceed consumer’s attention span, \( \tau \), and (ii) the number of products offered by a duopolist increases with advertising reach if

\[
A_{k_i \omega_i} < \frac{t \left[ 8k_i (N - 1)^2 + \tau^2 \omega_i^2 (5k_i - 3) - 2\tau \omega_i (7k_i - 2)(N - 1) \right]}{2N (N - 1) k_i^2}.
\]

(29)

These results are different from the findings presented in Proposition 3. To appreciate the forces behind the current proposition, consider the case where firm \( i \) increases its product offering such that the total number of products in the market exceeds \( \tau \). By doing so, firm \( i \) may be able to offer the preferred products of some more consumers. However, as the number of advertisements exceeds consumer’s attention span, consumers will not notice some of the product advertisements. Consequently, the probability that consumers are aware of the products of firm \( i \) reduces, and it also has a negative impact on the awareness level of firm \( j \)’s products.

In a duopoly, as an individual firm does not fully internalize the negative externality induced by its decision to offer an additional product, we may observe advertising clutter. Our analysis shows that the duopolist’s optimal number of products \( k^* \) has an interior solution when the following conditions hold:

\[
A_{k_i} < \frac{4k_i (N - 1) \left[ 2t (N - 1) - Fk_i \right] - 2t \tau \omega_i (4k_i - 1)(N - 1) + t\tau^2 \omega_i^2 (2k_i - 1)}{2N (N - 1) k_i^2},
\]

\[
A_{k_i k_i} > \frac{t\tau \omega_i}{8N (N - 1) k_i^4} \left[ \tau \omega_i (4k_i - 1) - 8k_i (N - 1) \right].
\]

(30)

These conditions suggest that we have an equilibrium \( k^* \) if the marginal cost of advertising is not too high and the cost function is sufficiently convex.

In the basic model, we noted that in the presence of competition, the number of products offered by a firm declines as advertising reach increases. But now we obtain a different result. To appreciate the result, note that the equilibrium price is

\[
p_i = \frac{2t (N - 1) (k_i + k_j) - t \tau \omega_i (k_i - 1)}{\tau \omega_j k_j} - t,
\]

(31)
suggesting that price reduces, as advertising reach increases, in the case of product advertising as well. Yet the number of products supplied by a duopolist increases with advertising reach. The reason for this finding is that consumer’s limited attention span attenuates price competition among the competing firms. It is easy to see in equation (31) that if the size of \( \tau \) reduces, equilibrium prices will increase. Furthermore, if consumers are less likely to be confused by the competing advertisements such that \( A_{\omega_i k_i} \) is below the threshold specified in the proposition, then it may become profitable for a firm to offer more products as advertising reach increases.
Of course, if a firm enjoys economies of scale such that $A_{\omega, k_i} < 0$, then the threshold stated in the proposition will be readily satisfied. It is useful to note that the threshold is a convex function of $\tau$, implying that it increases at an increasing rate as $\tau$ increases. This suggests that if consumers process fewer advertisements (that is, smaller $\tau$), it will be profitable for a firm to increase its product line only if consumers get less confused by ad clutter (that is, the upper bound on $A_{\omega, k_i}$ gets reduced). Having considered product advertising, we next turn attention to brand advertising.

4. A MODEL OF BRAND ADVERTISING

We first outline a model of brand advertising and then discuss the trade-off that firms make between brand advertising and product advertising.

**Brand advertising.** Consider a consumer who thinks of the brand Swatch when she contemplates watches. Similarly, when she thinks of cereals, thoughts about Kellogg’s brand come to her. Which specific product will come to this consumer’s mind when she thinks of Swatch or Kellogg’s? The answer to this question depends on the strength of associations between the brand and its numerous products. In fact, if too many products are offered under a brand name, the linkages between a brand and its products become weak. We refer to this reduction in the accessibility of a brand’s individual products as *brand dilution*.\(^4\) There is a need to balance this potential for brand dilution against a firm’s desire to add another new product under its brand umbrella.

Informative brand advertising comes in different formats. To fix ideas, consider the simple case where a firm uses a composite advertisement to inform consumers about all its products. As in the basic model of advertising, denote the reach of firm $i$’s advertising by $\phi$. We capture the notion of brand dilution by letting the probability that consumers are informed about a product be $\frac{\phi}{f(n)}$, where $n$ is the number of products under a firm’s brand umbrella. We assume that $f(1) = 1$, implying that brand dilution is not an issue if a firm supplies one product. Moreover, $\frac{\partial f(n)}{\partial n} > 0$, suggesting that brand dilution increases at the number of products increases. Note that we can recover the results of the basic model if $f(n) = 1$. Using this formulation, we next examine how the likelihood of brand dilution may influence the number of products offered by

\(^4\)There is an extensive body of literature suggesting that consumers organize their memory as a network of nodes connected by links (e.g., Raaijmakers and Shiffrin 1981, Cowley and Mitchell 2003) This spreading-activation model has been helpful in studying brand associations (e.g., Keller 1991, 1998). There is also related literature on trademark infringement that studies how counterfeits and related products may lead to trademark dilution (see Pullig, Simmons and Netemeyer 2006).
a firm.

Multi-product Monopoly. To facilitate exposition, let $\Phi \equiv \frac{\phi}{f(n)}$. The demand for firm $i$’s products is similar to the demand pattern discussed in the basic model (except that we need to replace $\phi$ with $\Phi$ in equations 5–7). Hence, the monopolist’s profits are given by (compare with equation 8):

$$\pi_M = \left[ \frac{n(n-1)}{N(N-1)} \Phi^2 + 2\Phi(1-\Phi) \frac{n(n-1)}{N(N-1)} + \frac{2\Phi n(N-n)}{N(N-1)} \right] p_M - \frac{2nF}{N} - A(\phi), \quad (32)$$

where $A(\phi)$ is the cost of informing $\phi$ fraction of the consumers about the monopolist’s products. We assume that $\frac{\partial A(\phi)}{\partial \phi} > 0$ and $\frac{\partial A}{\partial \phi} > 0$, implying that the advertising cost function is convex.

As the monopolist uses a composite advertisement to inform consumers about all its products, the cost of advertising reach need not depend on the number of products advertised. Hence, we make the simplifying assumption that $\frac{\partial A}{\partial n} = 0$.

The monopolist first determines the optimal number of product varieties and the level of advertising reach, and then sets prices. We have the following result about equilibrium behavior.

**Proposition 5** The monopolist’s optimal number of product varieties increases with brand advertising when $\frac{\partial f(n)}{\partial n} < \frac{(f(n))^2((N-1)-\phi f(n))(2n-1)}{nf(n)(N-1)2(\phi n-1)}$.

If the monopolist adds another product to its portfolio, it increases its ability to offer the preferred products of some more consumers. At the same time, the additional product dilutes the accessibility of each of its products at the rate of $\frac{\partial f(n)}{\partial n}$. Consequently, the monopolist needs to balance these two countervailing forces. The proposition suggests that a monopolist will offer an additional product only if the rate of brand dilution is below a threshold. Next we investigate how brand dilution affects the product line of competing firms.

Multi-product Duopoly. For ease of exposition, let $\Phi_i \equiv \frac{\phi_i}{f(k_i)}$. The segmentwise demand for firm $i$’s products closely parallels the expressions presented in the basic model, except that we need to replace $\phi_i$ in equations (9)–(13) with $\Phi_i$. Now the profits of firm $i$ can be expressed as follows (compare to equation 14):

$$\pi_i = \left[ \frac{\Phi_i^2 k_i(k_i-1)}{N(N-1)} + 2\Phi_i(1-\Phi_i) \frac{k_i(k_i-1)}{N(N-1)} + 2\Phi_i(1-\Phi_j) \frac{k_i k_j}{N(N-1)} \right] p_i - \frac{2k_i F}{N} - A(\phi_i). \quad (33)$$

In the first stage of the game, firms choose the number of products to offer and the level of advertising reach. In the second stage, firms choose their prices. We have the following result on firm behavior.
Proposition 6 The duopolist’s optimal number of product varieties increases with the reach of brand advertising when 
\[
\frac{\partial f(k_i)}{\partial k_i} > \frac{[6(N-1)-\Phi_i(8k_i-5)]f(k_i)}{2(3k_i-2)(N-1)-\Phi_i(8k_i^2-10k_i+3)}.
\]

Recall that in the basic model, the duopolist’s optimal number of products always decreased if advertising increased. Proposition 6 shows that brand advertising could lead to a different result. It is also useful to note that this duopoly result is qualitatively different from the monopoly result presented in Proposition 5. To grasp the intuition for this result, notice that on solving for the price-setting subgame we obtain
\[
p_i^* = \frac{1}{\Phi_j k_j^*} [2t (N - 1) - t\Phi_i (k_i - 1)] - t. \tag{34}
\]

Now it is easy to see that, if \( \Phi_i \) increases, the equilibrium price will decline. Hence, if a firm increases the number of products it supplies while keeping its advertising reach constant, then because of the resulting brand dilution the value of \( \Phi_i \) reduces, and this helps the equilibrium price to increase. In essence, if the rate of brand dilution is above a critical threshold, it mitigates the cannibalization and competitive effects of brand advertising and motivates a firm to increase the number of products it supplies. However, as clarified in the basic model, in the absence of such mitigation, the cannibalization and competitive effects have a negative influence on the length of a firm’s product line. Moreover, the value of the threshold specified in the proposition reduces as \( N \) increases. This suggests that, if the heterogeneity in consumers’ taste increases, firms will have a greater incentive to offer more products, keeping all other variables constant. Next we discuss how much a firm will rely on brand advertising instead of product advertising.

The Trade-off between Brand Advertising and Product Advertising. To understand this trade-off, we consider a situation where firms can engage in product and brand advertising. That is, the firm can advertise each of its products in a separate product advertisement and it also has the option of advertising its products multiple times in a composite brand advertisement. The key decision facing the firm is: what fraction of its advertising reach should come from brand advertising?

In this scenario, as a firm can release its composite brand advertising multiple times, a new force comes into play. Research has shown that consumers organize their knowledge about products and brands as a network of nodes connected by links (Raaijmakers and Shiffrin 1981, Keller 1991 and 1998). Hence, when a consumer sees any of the brand advertisements, it may activate her memory about the brand and reinforce the memory trace of the brand. We refer to this strengthening of the brand’s memory trace as a reinforcement effect. There is also a potential downside to having multiple products associated with a brand name. The
multiple associations may impede the likelihood of accessing any of the associated products. Consequently, there is scope for such brand dilution in the current setting. In the case of product advertising, as we discussed earlier, consumers can face information overload, leading to congestion of their limited attention span. One way by which consumers handle such an information overload is by paying attention to only a small fraction of the advertisements to which they are exposed. Next we analyze how these different effects influence the trade-off between brand advertising and product advertising.

Multi-product Monopoly. Consider a monopolist who offers \( n \) products and reaches \( \omega \) fraction of the market through its advertising. Let \( \delta \) fraction of its advertising reach come from brand advertising. As before, we assume that consumers process only \( \tau > 2 \) advertisements in a window of time. Now the probability that a consumer is informed about the products of the monopolist through brand advertising is given by \( g(n, \tau)\delta\omega \), where \( 0 \leq g(n, \tau) \leq 1 \) captures the brand reinforcement and dilution effects. If the reinforcement effect dominates the dilution effect, then \( \frac{\partial g(\cdot)}{\partial n} > 0 \). Otherwise, we will observe \( \frac{\partial g(\cdot)}{\partial n} < 0 \). If consumers’ attention span increases, it improves the reinforcement effect of brand advertising, and hence we have \( \frac{\partial g(\tau, n)}{\partial \tau} > 0 \). When \( n \leq \tau \), consumer attention span is not a binding constraint, and we have \( g(n, \tau) = 1 \).

For a specific example of \( g(n, \tau) \), consider the case where the reach of brand advertising is \( \delta\omega \) and the composite advertisement is released \( n \) times. In this case, the probability that a consumer is informed about the firm’s brand is \( 1 - (1 - \delta\omega)^\tau \). This expression captures the notion of brand reinforcement. To account for potential brand dilution, the expression can be scaled down by \( \frac{1}{f(n)} \). Hence, in this case \( g(\tau, n)\delta\omega = \frac{1}{f(n)} \left[ 1 - (1 - \delta\omega)^\tau \right] \). Furthermore, the effect of brand reinforcement will dominate potential brand dilution if \( \frac{f'}{f(n)^2} \left[ (1 - \delta\omega)^\tau - 1 \right] > 0 \).

Now as the firm achieves \( 1 - \delta \) fraction of its reach through product advertising, the probability that a consumer is aware of the monopolist’s product through product advertising is \( \tau (1 - \delta)\omega \). Thus, the overall probability of a consumer being informed about the firm’s products is given by:

\[
g(\tau, n)\delta\omega + \frac{\tau (1 - \delta)\omega}{n},
\]
where \( n > \tau \). But when \( n \leq \tau \), we have \( g(\tau, n)\delta\omega + (1 - \delta)\omega = \omega \). In this case, the firm can rely only on product advertising. Moreover, as \( \tau \) is a variable, it is important to understand the optimal level of brand advertising when \( n > \tau \).

For ease of exposition, let \( \Upsilon \equiv g(\tau, n)\delta\omega + \frac{\tau (1 - \delta)\omega}{n} \). Now the monopolist’s profit function can be written as:

\[
\pi_M = \left[ \frac{n (n - 1) \Upsilon^2}{N (N - 1)} + \frac{2\Upsilon (1 - \Upsilon) n (n - 1)}{N (N - 1)} + \frac{2\Upsilon n (N - n)}{N (N - 1)} \right] p_M - \frac{2nF}{N} - A(n, \omega).
\]
On examining the optimal level of brand advertising that a monopolist may engage in, we have the following result.

**Proposition 7** It is more profitable for a monopolist to use only brand advertising to promote its products.

The intuition for this finding is simple. As we discussed earlier in the case of product advertising, consumers’ attention span is limited, which restrains a monopolist from releasing many product advertisements. This restraint is a direct consequence of the additional product advertisements exceeding consumers’ attention span and reducing the probability of consumers becoming aware of any of its products. Brand advertising helps the firm to better manage the attention span issue by chunking all the product information in a composite brand advertisement. Though the composite advertisement raises the prospect of brand dilution, it may help the monopolist supply more products and thereby better cover the diverse needs of the market. Hence, as indicated in the proposition, the monopolist engages only in brand advertising. However, we obtain a different result in the presence of competition.

**Multi-product Duopoly.** In a duopoly, as noted earlier, each firm does not fully internalize the negative effects of the information overload caused by its advertising and hence remains motivated to offer more products. To understand how the presence of competition tempers the optimal level of brand advertising, we focus attention on the case where \( k_i + k_j > \tau \). In this case, we can view \( k_i + k_j - \tau \) as a measure of the information overload induced by advertising. If a duopolist uses brand advertising to achieve \( \delta \) fraction of its advertising reach, the probability that a customer is aware of its products through product and brand advertising is:

\[
0 \leq g_i (\tau, k_i, k_j) \delta_i \omega_i + \frac{\tau (1 - \delta_i) \omega_i}{k_i + k_j} \leq 1. \tag{37}
\]

For example, if the reach of brand advertising of firm \( i \) is \( \delta_i \omega_i \) and the composite advertisement is released \( k_i \) times, then the probability that a consumer is informed about the firm’s brand is \( 1 - (1 - \delta_i \omega_i)^{2k_i} \). Hence, in this case \( g_i (\tau, k_i, k_j) \delta_i \omega_i = \frac{1}{f(k_i)} \left[ 1 - (1 - \delta_i \omega_i)^{2k_i} \right] \). Furthermore, the effect of brand reinforcement will dominate potential brand dilution if

\[
\frac{f_i'(1)}{f_i^2} \left[ (1 - \delta_i \omega_i)^{2k_i} - 1 \right] - \frac{\tau k_i f_i}{f_i^2 (k_i + k_j)^2} \ln (1 - \delta_i \omega_i) (1 - \delta_i \omega_i)^{2k_i} > 0, \tag{38}
\]

where \( f_i \equiv f(k_i) \) and \( f_i' \equiv \frac{\partial f(k_i)}{\partial k_i} \).

To simplify notation, let \( \Upsilon_i \equiv g_i (\tau, k_i, k_j) \delta_i \omega_i + \frac{\tau (1 - \delta_i) \omega_i}{k_i + k_j} \). Now we can express the profits of firm \( i \) as follows:

\[
\pi_i = \left[ \Upsilon_i^2 \frac{k_i (k_i - 1)}{N(N-1)} + 2 \Upsilon_i (1 - \Upsilon_i) \frac{k_i (k_i - 1)}{N(N-1)} + 2 \Upsilon_i (1 - \Upsilon_i) \frac{k_i k_j}{N(N-1)} \right] p_i - \frac{2k_i F}{N} - A(k_i, \omega_i). \tag{39}
\]
Proposition 8  
i) In equilibrium, a duopolist uses both brand advertising and product advertising; ii) the optimal level of brand advertising increases as consumers’ tastes become more diverse; but (iii) the optimal level of brand advertising decreases with $\tau$ when $k_i + k_j - \tau > \frac{1-g_i(\tau, k_i, k_j)}{\partial g_i(\tau, k_i, k_j)/\partial \tau}$.

Unlike a monopolist, a duopolist does not fully internalize the negative externalities induced by its advertising. Consequently, a duopolist is more motivated to supply the preferred products of more customers and also engage in product advertising. But only engaging in product advertising will significantly overload consumers’ limited attention span. This congestion, however, can be attenuated by engaging in brand advertising, wherein all the products are advertised through a composite advertisement. The use of such brand advertising increases in situations where consumers’ tastes are very diverse. When the diversity in taste ($N$) increases, a firm needs to offer more products ($k_i$); and one way to achieve the target advertising reach and also offer more products is by increasing the level of brand advertising ($\delta_i$).

Specifically, we find that the optimal level of brand advertising, namely $\delta_i^\ast$ is given by:

$$
\delta_i^\ast = \frac{2 (k_i + k_j) (N - 1) - \tau \omega_i (3k_i - 2)}{(k_i + k_j) \omega_i [2 (k_i - 1) g_i (\tau, k_i, k_j) + k_i g_j (\tau, k_j, k_i)] - \tau \omega_i (3k_i - 2)}.
$$

(40)

(41)

It is easy to see that this optimal level increases with $N$. To understand the intuition behind this finding, note that as consumers’ tastes become more diverse, the optimal number of products offered by a firm will increase. Also, an increase in the number of products supplied by a firm will motivate the firm to engage in more brand advertising. Precisely, we find that a duopolist’s optimal level of brand advertising, namely $\delta_i^\ast$, is increasing in the number of its product varieties, namely $k_i$, when

$$
\frac{\partial g_i(\tau, k_i, k_j)}{\partial k_i} > -\frac{1}{2 (k_i - 1)} \left\{ \frac{[6(k_i+k_j)^2(N-1)-\tau \omega_i (3k_i-2)^2]g_i(\tau,k_i,k_j)-2\tau(3k_j+2)(N-1)}{(k_i+k_j)[2(k_i+k_j)(N-1)-\tau \omega_i (3k_i-2)]} + k_i \frac{\partial g_j(\tau,k_j,k_i)}{\partial k_i} \right\}.
$$

Note that when the competing firm $j$ increases the number of products it supplies, it induces a negative externality on the effectiveness of advertising. The size of $\frac{\partial g_j(\tau,k_j,k_i)}{\partial k_i}$ provides an indication of this negative externality. We find that the optimal level of brand advertising
decreases with the number of products offered by the competing firm, when

$$\frac{\partial g_i(\tau, k_i, k_j)}{\partial k_j} < \frac{1}{2(k_i - 1)} \left\{ \frac{[6(k_i+k_j)(N-1)-\tau\omega_i(3k_i-2)^2](\tau k_i k_j - 2\tau(3k_j+2)(N-1))}{(k_i+k_j)[(N-1)-\tau\omega_i(3k_i-2)]} + k_i \frac{\partial g_i(\tau, k_i, k_j)}{\partial k_i} \right\}. \quad (42)$$

Recall that brand advertising allows scope for both reinforcement and dilution effects. Now how much the overall effectiveness of brand advertising is affected by an additional product from firm $j$ is given by $\frac{\partial g_i(\tau, k_i, k_j)}{\partial k_i}$. If this net effect is not reduced too much because of brand dilution, then it is profitable for a duopolist to add another product to its portfolio and increase the emphasis placed on brand advertising.

The last part of the proposition suggests that a firm may decrease its use of brand advertising when consumers’ attention span increases. As $g_i(\tau, k_i, k_j)$ captures both the reinforcement and dilution effects of brand advertising, $\frac{\partial g_i(\tau, k_i, k_j)}{\partial \tau}$ shows how much the effectiveness of brand advertising improves with $\tau$. This improvement is a direct consequence of the reinforcement effect. On the other hand, $1 - g_i(\tau, k_i, k_j)$ can be viewed as a measure of the ineffectiveness of brand advertising. So $\frac{1-g_i(\tau, k_i, k_j)}{\partial g_i(\tau, k_i, k_j)/\partial \tau}$ indicates how much improving consumer’s attention span can reduce the ineffectiveness of brand advertising. If this reduction is less than $k_i + k_j - \tau$, which can be interpreted as information overload, then firms should reduce the emphasis placed on brand advertising.

5. CONCLUSIONS

The purpose of this paper was to understand how informative advertising influences multi-product competition in horizontally differentiated markets. Toward this goal, we developed a basic model of informative advertising, and then extended the model to consider product advertising and brand advertising. Our analysis offers some insights into how the different types of informative advertising and market structure influence the number of products offered by a firm.

1. What is the effect of imperfect information on multi-product duopoly competition? If consumers’ were perfectly informed about a firm’s products, then a monopolist will offer more products as consumer’s valuation increases. This makes intuitive sense as the price charged by a monopolist increases with consumer valuation. Using a basic model of informative advertising, we show that if consumers are imperfectly informed, a monopolist’s product-line length has an inverted-U shaped relationship with consumer valuation. Furthermore, this relationship is tempered by the level of advertising reach. When advertising reach is low, it is profitable for a firm to add more products to its portfolio.
and satisfy the needs of more consumers. However, when advertising reach is higher, a monopolist reduces the number of products in its portfolio in an attempt to decrease cannibalization within its own product line. In the presence of competition, however, a firm always reduces the length of its product line as advertising reach increases. This is because higher advertising reach increases head-on competition between the duopolists and reduces equilibrium price.

2. **How does product advertising affect the length of a firm’s product line?** Empirical research on advertising suggests that consumers’ attention span is limited and consumers tend to handle information overload by screening out a large fraction of advertisements. Another common empirical finding is that competitive ad clutter reduces advertisement recall and thereby increases advertising cost. We incorporated these empirical regularities in a model of product advertising. On analyzing the model, we find that the number of products advertised by a monopolist will not violate consumer’s cognitive constraint. In the presence of competition, however, the total number of products advertised by duopolists will exceed consumer’s attention span, thereby inducing advertising clutter. Furthermore, the number of products offered may increase with advertising reach, if the size of $A_{\omega,k_i}$ is below a threshold. This finding runs counter to the results obtained in our basic model. The intuition for the current result is that limited consumer attention span helps to soften price competition. Furthermore, if consumers are less prone to be confused by the competing advertisements such that $A_{\omega,k_i}$ is below a threshold, it is indeed profitable for a firm to offer more products as advertising reach increases.

3. **What is the effect of brand advertising on a firm’s product line?** To explore this issue, we examined a simple form of brand advertising where a firm releases a composite advertisement for all the products within its brand umbrella. A common problem with brand advertising is that, if too many products are promoted under a brand umbrella, it could potentially lead to brand dilution — a reduction in the cognitive accessibility of individual products. However, the additional products may help a firm to offer the preferred products of more consumers. In a monopoly, if the level of brand dilution is below a threshold, it is profitable for a firm to offer more products as advertising reach increases. The results, however, are reversed in a duopoly. Here, if the level of brand dilution is above a threshold, the duopolists will be motivated to offer more products as advertising reach increases. The intuition behind this finding is that brand dilution helps to attenuate cannibalization and soften price competition.
4. What is the optimal level of brand advertising? Our analysis shows that it is more profitable for a monopolist to engage in brand advertising, as it helps to better handle the cognitive constraints of consumers. A duopolist, however, may use both brand and product advertising. Unlike a monopolist, a duopolist does not fully internalize the effects of advertising clutter induced by it, and is therefore motivated to offer more products and engage in product advertising. We find that the optimal level of brand advertising increases, if consumer’s attention span decreases or if consumers’ tastes becomes more diverse.

Limitations and Directions for Further Research. There are several avenues for further research. While we focused on a horizontally differentiated market, future research can explore how different types of advertising influence the product-line decisions of multi-product firms in a vertically differentiated market (see also Villas-Boas 2004 and Champsaur and Rochet 1989). Several emerging advertising technologies help firms to better target their communication and potentially reduce cannibalization within its product line and also save costs. Future research can study how targeted advertising affects multi-product competition. Also, in developing our model we assumed that products and firms are symmetric to gain analytical tractability. Relaxing this assumption and examining its implications in a more restrictive setting is another avenue for further research. There is also a need to challenge our model’s predictions with field and experimental data (see Bayus and Putsis 1999 and Draganska and Jain 2005, Amdaloss and Rapoport 2005). Finally, incorporating behavioral regularities into normative models and understanding their implications for firm behavior remains a fruitful area for marketing research (see Syam et al. 2005, Ho et al. 2006, Amdaloss and Jain 2008 and Ho and Su forthcoming).
APPENDIX A: TECHNICAL NOTES

Multi-product monopoly with perfectly informed consumers. The focus of our paper is on informative advertising (imperfect information) and multi-product competition. In this section, we present an analysis of the case where consumers are perfectly informed so that it is easier to see how multi-product competition affects firm’s behavior. We shall see that the results of this analysis are consistent with what we would expect in oligopolistic competition. It is useful to contrast these results with those of the basic model presented in Section 2.

As discussed in section 2, each consumer’s consideration set includes two varieties. Hence, the number of possible combinations of varieties that can be preferred by consumers is:

\[
\frac{N(N-1)}{2}.
\]

(A–1)

The potential combinations of product varieties that the monopolist can offer is:

\[
\frac{n(n-1)}{2}.
\]

(A–2)

Hence, it follows that the fraction of consumers who have access to two varieties, fraction of consumers who have access to one variety, and fraction of consumers whose two preferred products are unavailable are:

\[
\alpha \equiv \frac{n(n-1)}{N(N-1)},
\]

\[
\beta \equiv \frac{2n(N-n)}{N(N-1)},
\]

\[
\gamma \equiv \frac{(N-n)(N-n-1)}{N(N-1)},
\]

(A–3)

respectively. Note that \(\alpha + \beta + \gamma = 1\).

As in our basic model, assume that the fixed cost of introducing a new product is \(F > 0\) and the constant marginal cost is \(c = 0\). Further, as the total fixed cost of introducing \(n\) products is \(nF\) and the total mass of consumers is \(\frac{N}{2}\), the fixed costs per consumer is \(\frac{2nF}{N}\). Next we examine a multi-product monopoly and then study a multi-product duopoly.

The monopolist’s profits are as follows:

\[
(\alpha + \beta) p_M - \frac{2nF}{N}.
\]

(A–4)

It is easy to see that the monopolist would set its prices such that \(p_M = v - t\). Taking derivative of equation (A–4) with respect to \(n\) and setting it to 0, and solving for optimal number of product varieties we obtain

\[
n^* = \frac{1}{2(v-t)} [2N(v-t-F) - (v-t-2F)].
\]

(A–5)
It follows that no product is offered if \( v \leq \frac{2F(N-1)}{2N-1} + t \). Observe that

\[
\frac{\partial n^*}{\partial v} = \frac{F}{(v-t)^2} (N-1) > 0,
\tag{A-6}
\]

and

\[
\lim_{v \to \infty} n^* = N - \frac{1}{2}.
\tag{A-7}
\]

The preceding analysis shows that the monopolist’s optimal number of product varieties will not be large enough to satisfy the tastes all its diverse consumers, that is \( n^* < N \).

**Multi-product duopoly with perfectly informed consumers** Consider two competing firms indexed by \( i, j = 1, 2, i \neq j \). In the first stage, the two symmetric firms simultaneously choose the number of product varieties they supply, namely \( k_i \) and \( k_j \). In the second stage, the firms simultaneously set prices for all product varieties after observing \( k_i \) and \( k_j \). In the symmetric subgame perfect equilibrium, \( k_i = k_j = \frac{N}{2} \) and all product varieties are sold at the same price.

Now note that in the second stage of the game, given \( k_i \) and \( k_j \), the fraction of consumers whose (two) preferred brands are both supplied by firm \( i \) is \( \frac{k_i(k_i-1)}{N(N-1)} \), the fraction of consumers whose preferred brands are provided by firms \( i \) and \( j \) (one each) is \( \frac{2k_i k_j}{N(N-1)} \), the fraction of consumers who have only one preferred brand available and it is supplied by firm \( i \) is \( \frac{2k_i}{N} \left(1 - \frac{k_i+k_j-1}{N-1}\right) \). Note that since firm \( i \)’s products can be either the first or second preferred brand, the latter two fractions have the multiplier 2. To see that these probabilities are indeed correctly specified, note that

\[
\begin{align*}
\frac{k_i (k_i - 1)}{N(N-1)} + \frac{2k_i k_j}{N(N-1)} &+ \frac{2k_i}{N} \left(1 - \frac{k_i + k_j - 1}{N-1}\right) \\
&+ \frac{k_j (k_j - 1)}{N(N-1)} + \frac{2k_j}{N} \left(1 - \frac{k_i + k_j - 1}{N-1}\right) \\
&+ \frac{(N-k_i-k_j)(N-k_i-k_j-1)}{N(N-1)} = 1. 
\end{align*}
\tag{A-8}
\]

Now the profits of firm \( i \) are as follows:

\[
\pi_i = \left[ \frac{k_i (k_i - 1)}{N(N-1)} + \frac{2k_i}{N} \left(1 - \frac{k_i + k_j - 1}{N-1}\right) + \frac{2k_i k_j}{N(N-1)} \left(1 + \frac{p_j - p_i}{2t}\right) \right] p_i - \frac{2k_i F}{N}. \tag{A-9}
\]

Using backward induction, we first determine the optimal prices taking the number of product varieties as given. We have:

\[
p_i = \frac{t}{k_j} (2N - k_i - k_j - 1). \tag{A-10}
\]
It is useful to note that the assumption \( v - p_i > 1 \) does not imply \( v \) is arbitrarily large. It merely ensures that firms engage in “normal” oligopolistic competition. Thus, in the situation where \( N \) is arbitrarily large but \( k_i \) is finite, \( p_i \) is not arbitrarily large, it converges to \( v \) instead. It follows that the duopolist’s optimal product prices are strictly lower than those of the monopolist.

In the first stage, firm \( i \) chooses the optimal number of product varieties anticipating \( p_i \). Maximizing equation (A–9) with respect to \( k_i \), we obtain

\[
\begin{align*}
k_i^* &= \frac{1}{12t} [2F(N - 1) + 5t(2N - 1) - \Theta],
\end{align*}
\]  

(A–11)

where

\[
\begin{align*}
\Theta &\equiv \sqrt{4F^2N(N - 2) + 20FNt(2N - 3) + 4Nt^2(N - 1) + (2F + t)^2 + 16Ft}.
\end{align*}
\]  

(A–12)

Substituting \( k_i^* \) into equation (A–10), we have

\[
\begin{align*}
p_i^* &= \frac{t}{k_i^*} \left(2N - 2k_i^* - 1\right) \\
&= \frac{1}{2(2N - 1)} \left[2F(N - 1) + t(2N - 1) + \Theta\right].
\end{align*}
\]  

(A–13)

The fact that \( k_i^* \) is at most \( \frac{N}{2} \) in symmetric equilibrium implies \( F \geq \frac{(N - 2)t}{2N} \) and \( p_i \geq \frac{2(N - 1)t}{N} \). In other words,

\[
k_i^* \left\{ \begin{array}{ll}
= \frac{N}{2} & \text{if } F \leq \frac{(N - 2)t}{2N} \\
< \frac{N}{2} & \text{if } F > \frac{(N - 2)t}{2N}
\end{array} \right.
\]  

(A–14)

It can be shown that

\[
\begin{align*}
\frac{\partial k_i^*}{\partial F} &= -\frac{(N - 1)}{6t} \left[ \frac{2F(N - 1) + 5t(2N - 1)}{\Theta} - 1 \right] < 0, \\
\frac{\partial k_i^*}{\partial t} &= \frac{F(N - 1)}{6t^2} \left[ \frac{2F(N - 1) + 5t(2N - 1)}{\Theta} - 1 \right] > 0, \\
\frac{\partial p_i^*}{\partial N} &= \frac{F}{(2N - 1)^2} \left[ \frac{2F(N - 1) + 5t(2N - 1)}{\Theta} + 1 \right] > 0, \\
\frac{\partial p_i^*}{\partial t} &= \frac{10F(N - 1) + t(2N - 1)}{2\Theta} + \frac{1}{2} > 0, \\
\frac{\partial p_i^*}{\partial k_i^*} &= -\frac{2Nt}{(k_i^*)^2} < 0,
\end{align*}
\]  

(A–15)

which are all intuitive.

**Claim 1** The number of product varieties offered by the duopolists is greater than that of the monopolist, i.e., \( 2k_i^* > n^* \).
Proof. \( F \leq \frac{(v-t)(2N-1)}{2(N-1)} \) (monopoly) and \( F \geq \frac{(N-2)t}{2N} \) (duopoly) imply \( \frac{(N-2)t}{2N} \leq F \leq \frac{(v-t)(2N-1)}{2(N-1)} \).

When \( F = \frac{(N-2)t}{2N} \), \( 2k_i^* = N \) and recall that \( \lim_{v \to \infty} n^* = N - \frac{1}{2} \). It follows that the number of product varieties offered by the duopolists is greater than that of the monopolist at the lower bound of \( F \).

When \( F = \frac{(v-t)(2N-1)}{2(N-1)} \), \( n^* = 0 \) but \( k_i^* = 1 \)

\[
k_i^* = \frac{1}{12t} \left[ (2N - 1)(4t + v) - \sqrt{(2N - 1)^2 (-8t^2 + 8tv + v^2)} \right] > 0, \quad (A-16)
\]

because

\[
(2N - 1)^2 (-8t^2 + 8tv + v^2) - [(2N - 1)(4t + v)]^2 = -24t^2 (2N - 1)^2 < 0. \quad (A-17)
\]

Hence, the number of product varieties offered by the duopolists is greater than that of the monopolist at the upper bound of \( F \).

Furthermore,

\[
\frac{\partial n^*}{\partial F} = -\frac{1}{(v-t)} (N-1) < 0,
\]

\[
\frac{\partial k_i^*}{\partial F} = -\frac{(N-1)}{6t} \left[ \frac{2F(N-1) + 5t(2N-1)}{\Theta} - 1 \right] < 0. \quad (A-18)
\]

which imply that \( n^* \) and \( k_i^* \) are continuous and monotone in \( F \).

Therefore, the number of product varieties offered by the duopolists is greater than that of the monopolist \( \forall F \in \left[ \frac{(N-2)t}{2N}, \frac{(v-t)(2N-1)}{2(N-1)} \right] \). \( \blacksquare \)

**APPENDIX B: PROOFS**

**Proof of Proposition 1**

Proof. As shown in equation (8), the monopolist’s profits are:

\[
\pi_M = \left[ \frac{n}{N} \left( \frac{N}{N-1} \phi^2 + 2\phi (1-\phi) \frac{n}{N} \left( \frac{N}{N-1} \right) + \frac{2\phi n (N-n)}{N} \right) p_M - \frac{2nF}{N} \right]. \quad (B-1)
\]

For a given level of advertising reach, the monopolist first determines the optimal number of product varieties, and then sets its prices. The monopolist’s price is \( v - t \). Now solving the

\(^5\Theta \) is given in equation (A–12).
monopolist’s profit maximization problem with respect to \( n \), we obtain

\[
n^* = \frac{1}{2\phi^2(v-t)} \left\{ 2(N-1) [(v-t) \phi - F] + (v-t) \phi^2 \right\}.
\] (B-2)

It follows that no product is offered if \( v \leq v_1 \equiv \frac{2(F+t\phi)(N-1)+t\phi^2}{\phi^2+2\phi(N-1)} \). Observe that

\[
\frac{\partial n^*}{\partial \phi} = \frac{N-1}{\phi^3(v-t)} [2F - (v-t) \phi]
\geq \frac{N-1}{\phi^3(v-t)} [2F - (v-t) \phi]
= \frac{1}{\phi^2} (N + \phi - 1)
> 0,
\] (B-3)

where

\[
F \equiv \frac{(v-t) \phi^2}{2(N-1)} + (v-t) \phi.
\] (B-4)

which is the upper bound of \( F \). It is straightforward to see that for a given level of advertising reach, \( \frac{\partial n^*}{\partial \phi} > 0 \) when \( v_1 < v < v_2 \), where \( v_2 \equiv \frac{2F}{\phi} + t \). But \( \frac{\partial n^*}{\partial \phi} < 0 \), when \( v > v_2 \). ■

Proof of Proposition 2

Proof. In the case of duopolistic competition, maximizing the duopolist’s profit (see equation 14) with respect to \( k_i \), we obtain

\[
k^*_i = \frac{1}{12t\phi_i^2} [2F(N-1) + 5t\phi_i (2N-2 + \phi_i) - \Lambda].
\] (B-5)

As \( k^*_i \) is at most \( \frac{N}{2} \), it implies that in a symmetric equilibrium:

\[
2F(N-1) + 5t\phi_i (2N-2 + \phi_i) - \Lambda \leq 6Nt\phi_i^2.
\] (B-6)

Further as \( k^*_i \) is at least 1, this implies that in a symmetric equilibrium:

\[
2F(N-1) + 5t\phi_i (2N-2 + \phi_i) - \Lambda \geq 12t\phi_i^2,
\] (B-7)

where

\[
\Lambda \equiv \sqrt{4F^2 + 20FNt\phi_i (2N + \phi_i - 4) + 20Ft\phi_i (2 - \phi_i) + t^2\phi_i^2 (2 - \phi_i)^2 + 4Nt^2\phi_i^2 (N + \phi_i - 2) + 4F^2N (N-2)}.
\] (B-8)

It follows that

\[
\frac{t(2 - \phi_i)}{2N} [4(N-1) - \phi_i (3N-2)] \leq F \leq 2t(N-1) - 3t\phi_i + \frac{t\phi_i^2}{N-1}.
\] (B-9)
On differentiating \( k_i^* \) (see equation B–5) respect to advertising reach, we have

\[
\frac{\partial k_i^*}{\partial \phi_i} = \frac{(N - 1)}{6t\phi_i^4} \left[ \frac{4F^2(N-1)+2Nt\phi_i(15F+t\phi_i)-t^2\phi_i^2(2-\phi_i)-10Ft\phi_i(3-\phi_i)}{A} - (2F + 5t\phi_i) \right].
\]  

(B–10)

Setting \( \frac{\partial k_i^*}{\partial \phi_i} = 0 \), we obtain

\[
\hat{\phi}_i = \frac{1}{t} \left[ \sqrt{t^2 (N - 1)^2 + 4F^2 - 20Ft (N - 1) - (N - 1) t} \right].
\]  

(B–11)

For \( \hat{\phi}_i \geq 0 \), we must have \( 4F^2 - 20Ft (N - 1) \geq 0 \), implying that \( F \geq 5t (N - 1) \). On comparing this value with the upper bound of \( F \) derived above, we find that:

\[
5t (N - 1) - \left[ 2t (N - 1) - 3t\phi_i + \frac{t\phi_i^2}{N - 1} \right] = \frac{t}{N - 1} \left[ 3 (N - 1)^2 + 3\phi_i (N - 1) - \phi_i^2 \right] > 0.
\]  

(B–12)

This suggests that \( \hat{\phi}_i \) does not exist. Note that \( \frac{\partial k_i^*}{\partial \phi_i} \) is monotonic in \( \phi_i \). Now given the monotonicity of \( \frac{\partial k_i^*}{\partial \phi_i} \), it is easy to see that \( \frac{\partial k_i^*}{\partial \phi_i} \leq 0 \). A numerical analysis can further illustrate the relationship. 

\[\square\]

**Proof of Proposition 3**

**Proof.** If the monopolist’s optimal number of products \( n^* \leq \tau \), then the monopolist’s profits are given by

\[
\pi_M = \left[ \frac{n(n-1)}{N(N-1)} \omega^2 + 2\omega(1-\omega) \frac{n(n-1)}{N(N-1)} + 2\omega \frac{n(N-n)}{N(N-1)} \right] p_M = \frac{2nF}{N} - A(n, \omega). \]  

(B–13)

It can be shown that

\[
\frac{\partial \pi_M}{\partial n} = 2 \left[ (v-t)\omega - F \right] \frac{(N-1)-(v-t)(2n-1)\omega^2}{N(N-1)} - A_n,
\]

\[
\frac{\partial^2 \pi_M}{\partial n^2} = -2 \frac{(v-t)\omega^2}{N(N-1)} - A_{nn} < 0.
\]  

(B–14)

Hence, an interior solution exists, if \( \frac{2[(v-t)\omega-F(N-1)-(v-t)(2n-1)\omega^2}{N(N-1)} > A_n \). Otherwise, the monopolist’s optimal number of products \( n^* = \tau \) when \( \pi_M|_{n=\tau} \geq 0 \). We do not consider the trivial case in which the monopolist’s fixed cost is so high that \( n^* = 0 \).

Now we examine how \( \omega \) impacts \( n \). Recall that in optimality,

\[
\frac{\partial \pi_M}{\partial n} = 2 \left[ (v-t)\omega - F \right] \frac{(N-1)-(v-t)(2n-1)\omega^2}{N(N-1)} - A_n = 0,
\]  

(B–15)
On totally differentiating the above expression, we obtain
\[ - \left[ \frac{2 (v - t) \omega^2}{N (N - 1)} + A_{nn} \right] dn + \left\{ \frac{2 (v - t)}{N (N - 1)} [N - 1 - \omega (2n - 1)] - A_{n\omega} \right\} d\omega = 0. \] (B–16)

Hence, we have
\[ \frac{dn}{d\omega} = \frac{2 (v - t) [N - 1 - \omega (2n - 1)] - N (N - 1) A_{n\omega}}{2 (v - t) \omega^2 + N (N - 1) A_{nn}}. \] (B–17)

It follows that \( \frac{dn}{d\omega} > 0 \), if
\[ A_{n\omega} < \frac{2 (v - t) [N - 1 - (2n - 1) \omega]}{N (N - 1)}. \] (B–18)

If the monopolist’s optimal number of products \( n^* > \tau \), then the monopolist’s profits are given by equation (19). It can be shown that
\[ \frac{\partial \pi_M}{\partial n} = - \frac{2 F n^2 (N - 1) + \tau^2 \omega^2 (v - t)}{N n^2 (N - 1)} - A_n < 0, \]
\[ \frac{\partial^2 \pi_M}{\partial n^2} = \frac{2 \tau^2 \omega^2 (v - t)}{N (N - 1) n^3} - A_{nn}. \] (B–19)

It follows that the monopolist’s optimal number of products \( n^* = \tau \) since \( \frac{\partial \pi_M}{\partial n} < 0 \). □

**Proof of Proposition 4**

**Proof.** In the first stage, firm \( i \) chooses the optimal number of product varieties \( k_i \) and the level of advertising reach \( \omega_i \) anticipating the second stage prices. In a symmetric equilibrium,
\[ \frac{\partial \pi_i}{\partial k_i} = \frac{4 k_i (N - 1) [2 t (N - 1) - F k_i] - 2 t \tau \omega_i (4 k_i - 1) (N - 1) + t \tau^2 \omega_i^2 (2 k_i - 1)}{2 N (N - 1) k_i^2} - A_{k_i}, \]
\[ \frac{\partial \pi_i}{\partial \omega_i} = \frac{t [16 k_i^2 (N - 1)^2 - 8 \tau \omega_i k_i (3 k_i - 2) (N - 1) + \tau^2 \omega_i^2 (8 k_i^2 - 10 k_i + 3)]}{4 N (N - 1) \omega_i k_i^2} - A_{\omega_i}, \]
\[ \frac{\partial^2 \pi_i}{\partial k_i^2} = \frac{- t \tau \omega_i}{8 N (N - 1) k_i^4} [8 k_i (N - 1) - \tau \omega_i (4 k_i - 1)] - A_{k_i k_i}. \] (B–20)

For an interior solution of \( k^* \) to exist, we need to have \( \frac{\partial \pi_i}{\partial k_i} > 0 \) and \( \frac{\partial^2 \pi_i}{\partial k_i^2} < 0 \).

Next, we examine how \( \omega_i \) impacts \( k_i \). Note that in optimality,
\[ \frac{\partial \pi_i}{\partial k_i} = \frac{4 k_i (N - 1) (2 t (N - 1) - F k_i) - 2 t \tau \omega_i (4 k_i - 1) (N - 1) + t \tau^2 \omega_i^2 (2 k_i - 1)}{2 N (N - 1) k_i^2} - A_{k_i} = 0. \] (B–21)

Totally differentiating the above expression and noting that firms are symmetric, we obtain
\[ - \left\{ \frac{t \tau \omega_i [8 k_i (N - 1) - \tau \omega_i (4 k_i - 1)]}{8 N (N - 1) k_i^4} + A_{k_i k_i} \right\} dk_i + \left\{ \frac{t [8 k_i (N - 1)^2 + \tau^2 \omega_i^2 (5 k_i - 3) - 2 \tau \omega_i (7 k_i - 2) (N - 1)]}{2 N (N - 1) \omega_i k_i^2} - A_{k_i \omega_i} \right\} d\omega_i = 0. \] (B–22)
We, therefore, have
\[
\frac{dk_i}{d\omega_i} = \frac{4t k_i^2 [8k_i (N - 1)^2 + \tau^2 \omega_i^2 (5k_i - 3) - 2\tau \omega_i (7k_i - 2) (N - 1)] - 8N (N - 1) k_i A_{k_i,\omega_i}}{t\tau \omega_i [8k_i (N - 1) - \tau \omega_i (4k_i - 1)] + 8N (N - 1) k_i^4 A_{k_i k_i}^4}.
\] (B–23)

Since \(t\tau \omega_i [8k_i (N - 1) - \tau \omega_i (4k_i - 1)] + 8N (N - 1) k_i^4 A_{k_i k_i} > 0\) by assumption, it follows that \(d k_i / d \omega_i > 0\) if
\[
A_{k_i, \omega_i} < \frac{t [8k_i (N - 1)^2 + \tau^2 \omega_i^2 (5k_i - 3) - 2\tau \omega_i (7k_i - 2) (N - 1)]}{2N (N - 1) k_i^2}.
\] (B–24)

**Proof of Proposition 5**

**Proof.** In optimality, the monopolist’s profit (see equation 32) should be invariant to brand advertising. It follows that
\[
\frac{\partial \pi_M}{\partial \phi} = \frac{2n (v - t)}{N (N - 1) f(n)} (N - 1 - \Phi (n - 1)) - A_{\phi}
\]
\[
= 0.
\] (B–25)

When we totally differentiate \(\frac{\partial \pi_M}{\partial \phi}\), we obtain
\[
\left\{ (f(n))^2 (N - 1) - \phi f(n) (2n - 1) - nf(n) [(N - 1) - 2\Phi (n - 1)] \frac{\partial f(n)}{\partial n} \right\} \, dn = 0.
\] (B–26)

Hence, it follows that
\[
\frac{dn}{d\phi} = \frac{2n (n - 1) (v - t) f(n) + N (N - 1) (f(n))^3 A_{\phi}}{2 (v - t) \left\{ (f(n))^2 (N - 1) - \phi f(n) (2n - 1) - nf(n) [(N - 1) - 2\Phi (n - 1)] \frac{\partial f(n)}{\partial n} \right\}}
\]
\[
> 0,
\] (B–27)

if
\[
\frac{\partial f(n)}{\partial n} < \frac{(f(n))^2 (N - 1) - \phi f(n) (2n - 1)}{nf(n) [(N - 1) - 2\Phi (n - 1)]}.
\] (B–28)

Note that
\[
\frac{\partial^2 f(n)}{\partial n \partial N} = \frac{\phi (f(n))^2}{n [f(n) - 2\phi + 2n\phi - N f(n)]^2}
\]
\[
\geq 0.
\] (B–29)
Proof of Proposition 6

Proof. In optimality, the duopolist’s profits (see equation 33) should be invariant to brand advertising. Accordingly, we have

\[
\frac{\partial \pi_i}{\partial \phi_i} = t \left\{ \frac{4 (N - 1)^2 + 8 \Phi_i (N - 1) + 3 \Phi_i^2 - 2 \Phi_i k_i [6 (N - 1) - \Phi_i (4k_i - 5)]}{N (N - 1) \phi_i} \right\} \\
- A_{\phi_i} \\
= 0.
\] (B-30)

By totally differentiating \(\frac{\partial \pi_i}{\partial \phi_i}\), we obtain

\[
\left\{ - \frac{4 (N - 1)^2 - \Phi_i^2 (8k_i^2 - 10k_i + 3)}{N (N - 1) \phi_i^2} t - A_{\phi_i} \right\} \frac{d\phi_i}{d\phi_i} + \\
\frac{2t}{N (N - 1) (f (k_i))^2} \left\{ \frac{[2 (3k_i - 2) (N - 1) - \Phi_i (8k_i^2 - 10k_i + 3)] \frac{\partial f(k_i)}{\partial k_i} - 6 (N - 1) - \Phi_i (8k_i - 5) f (k_i)}{2 (3k_i - 2) (N - 1) - \Phi_i (8k_i^2 - 10k_i + 3)} \right\} dk_i \\
= 0. \] (B-31)

It follows that

\[
\frac{dk_i}{d\phi_i} = \frac{\left\{ [4 (N - 1)^2 - \Phi_i^2 (8k_i^2 - 10k_i + 3)] t + N (N - 1) \phi_i^2 A_{\phi_i} \right\} (f (k_i))^2}{2 \phi_i^2 \left\{ [2 (3k_i - 2) (N - 1) - \Phi_i (8k_i^2 - 10k_i + 3)] \frac{\partial f(k_i)}{\partial k_i} - 6 (N - 1) - \Phi_i (8k_i - 5) f (k_i) \right\}}.
\] (B-32)

In the numerator of the above expression,

\[
\begin{align*}
4 (N - 1)^2 - \Phi_i^2 (8k_i^2 - 10k_i + 3) & \geq 4 (N - 1)^2 - \Phi_i^2 (2N^2 - 5N + 3) \\
& \geq 2N^2 - 3N + 1 \\
& > 0.
\end{align*}
\] (B-33)

As the numerator is positive, the sign of \(\frac{dk_i}{d\phi_i}\) is determined by the sign its denominator. Hence, it follows that \(\frac{dk_i}{d\phi_i} > 0\), if

\[
\frac{\partial f(k_i)}{\partial k_i} > \frac{[6 (N - 1) - \Phi_i (8k_i - 5)] f (k_i)}{2 (3k_i - 2) (N - 1) - \Phi_i (8k_i^2 - 10k_i + 3)}.
\] (B-34)

Note that

\[
\frac{\partial^2 f(k_i)}{\partial k_i \partial N} = - \frac{2 \phi_i (f(k_i))^2 (k_i - 1)}{[8 \phi_i k_i^2 - 10 \phi_i k_i + 3 \phi_i - 2 f(k_i) (3k_i - 2) (N - 1)]^2} \leq 0.
\] (B-35)
Proof of Proposition 7

Proof. Multi-product Monopoly

As \( n > \tau \) and \( g(\tau, n) \delta \omega + \frac{\tau(1-\delta)\omega}{n} < 1 \), and \( \Upsilon \equiv g(\tau, n) \delta \omega + \frac{\tau(1-\delta)\omega}{n} \), the monopolist’s profit function is given by

\[
\pi_M = \left[ \frac{n(n-1)\Upsilon^2}{N(N-1)} + \frac{2\Upsilon(1-\Upsilon)n(n-1)}{N(N-1)} + \frac{2\Upsilon n(N-n)}{N(N-1)} \right] p_M - \frac{2nF}{N} - A(n, \omega). \tag{B–36}
\]

Note that \( p_M^* = v - t \). Substituting \( p_M^* \) into the profit function and maximizing \( \pi_M \) with respect to \( \delta \) yields

\[
\delta^* = \frac{n(N-1) - \tau \omega(n-1)}{[n\omega g(\tau, n) - \tau \omega](n-1)}. \tag{B–37}
\]

In the denominator of \( \delta^* \), note that

\[
[n\omega g(\tau, n) - \tau \omega](n-1) < (n-\tau)(n-1) \omega.
\]

When we subtract this expression from the numerator of \( \delta^* \), we obtain

\[
n(N-1) - \tau \omega(n-1) - (n-\tau)(n-1) \omega = n[N-1-\omega(n-1)] > 0.
\]

This implies that \( \delta^* = 1 \). □

Proof of Proposition 8

Proof. Multi-product Duopoly

If \( k_i + k_j > \tau \) and \( g_i(\tau, k_i, k_j) \delta_i \omega_i + \frac{\tau(1-\delta_i)\omega_i}{k_i+k_j} < 1 \), and \( \Upsilon_i \equiv g_i(\tau, k_i, k_j) \delta_i \omega_i + \frac{\tau(1-\delta_i)\omega_i}{k_i+k_j} \), the duopolist’s profits are given by

\[
\pi_i = \left[ \frac{\Upsilon_i^2 k_i(k_i-1)}{N(N-1)} + 2\Upsilon_i(1-\Upsilon_i) \frac{k_i(k_i-1)}{N(N-1)} + 2\Upsilon_i(1-\Upsilon_j) \frac{k_i k_j}{N(N-1)} \right] p_i - \frac{2k_i F}{N} - A(k_i, \omega_i). \tag{B–40}
\]

On solving the pricing subgame, we find that

\[
p_i^* = \frac{1}{\Upsilon_j k_j} \left[ 2t(N-1) - t \Upsilon_i (k_i - 1) \right] - t. \tag{B–41}
\]

Substituting \( p_i^* \) into the profit function and maximizing \( \pi_i \) with respect to \( \delta_i \) yields

\[
\delta_i = \frac{2(k_i + k_j)(N-1) - \tau \omega_i(3k_i - 2)}{(k_i + k_j) \omega_i [2(k_i - 1) g_i(\tau, k_i, k_j) + k_i g_j(\tau, k_j, k_i)] - \tau \omega_i (3k_i - 2)}. \tag{B–42}
\]
in the symmetric equilibrium. Clearly, $\partial \delta^*_i / \partial k_i$ can be simplified to

$$
\frac{\partial \delta^*_i}{\partial k_i} = \frac{1}{\omega_i [(3k_i - 2) ((k_i + j) g_i (\tau, k_i, k_j) - \tau)]^2}.
$$

Noting that in the symmetric equilibrium $g_i (\tau, k_i, k_j) = g_j (\tau, k_j, k_i)$, $\frac{\partial \delta^*_i}{\partial k_i}$ can be simplified to

$$
\frac{\partial \delta^*_i}{\partial k_i} = \frac{1}{\omega_i [(3k_i - 2) ((k_i + j) g_i (\tau, k_i, k_j) - \tau)]^2}.
$$

It follows that $\frac{\partial \delta^*_i}{\partial k_i} > 0$ when

$$
\frac{\partial g_i (\tau, k_i, k_j)}{\partial k_i} > -\frac{1}{2 (k_i - 1)} \left\{ \frac{6 [(k_i + j)^2 (N - 1) - \tau \omega_i (3k_i - 2)] g_i (\tau, k_i, k_j) - 2 (3k_i - 2)(N - 1)}{(k_i + j)^2 [2 (k_i + j)(N - 1) - \tau \omega_i (3k_i - 2)]} \right\}.
$$

Again note that in a symmetric equilibrium, $g_i (\tau, k_i, k_j) = g_j (\tau, k_j, k_i)$. Hence $\frac{\partial \delta^*_i}{\partial k_j}$ can be simplified to

$$
\frac{\partial \delta^*_i}{\partial k_j} = \frac{1}{\omega_i [(3k_i - 2) ((k_i + j) g_i (\tau, k_i, k_j) - \tau)]^2}.
$$
It follows that \( \frac{\partial \delta_i^*}{\partial k_j} > 0 \) if

\[
\frac{\partial g_i (\tau, k_i, k_j)}{\partial k_j} < - \frac{1}{2(k_i - 1)} \left\{ \frac{\tau (3k_i - 2)(2(N - 1) - \omega_i (3k_i - 2) g_i (\tau, k_i, k_j))}{(k_i + k_j) (2(k_i + k_j) (N - 1) - \tau \omega_i (3k_i - 2))} + k_i \frac{\partial g_i (\tau, k_i, k_j)}{\partial k_j} \right\}.
\]  

(B–48)

\[
\frac{\partial \delta_i^*}{\partial \tau} = - \frac{k_i + k_j}{\{ (k_i + k_j) \omega_i [2(k_i - 1) g_i (\tau, k_i, k_j) + k_i g_j (\tau, k_i, k_j)] - \tau \omega_i (3k_i - 2) \}^2} \cdot \left\{ 2(k_i + k_j) (N - 1) \left[ 2(k_i - 1) \frac{\partial g_i (\tau, k_i, k_j)}{\partial \tau} + k_i \frac{\partial g_j (\tau, k_i, k_j)}{\partial \tau} \right] - 2(3k_i - 2)(N - 1) \right\}
\]

\[
-2 \omega_i \left[ \tau \frac{\partial g_i (\tau, k_i, k_j)}{\partial \tau} - g_i (\tau, k_i, k_j) \right] (3k_i^2 - 5k_i + 2)
\]

\[
- \omega_i k_i \left[ \tau \frac{\partial g_j (\tau, k_i, k_j)}{\partial \tau} - g_j (\tau, k_i, k_j) \right] (3k_i - 2)
\]

\[
\equiv - \frac{k_i + k_j}{\{ (k_i + k_j) \omega_i [2(k_i - 1) g_i (\tau, k_i, k_j) + k_i g_j (\tau, k_i, k_j)] - \tau \omega_i (3k_i - 2) \}^2} \cdot \Xi.
\]

By symmetry,

\[
\Xi = 2(3k_i - 2)(N - 1) \left[ (k_i + k_j) \frac{\partial g_i (\tau, k_i, k_j)}{\partial \tau} - 1 \right]
\]

\[
- \omega_i (3k_i - 2)^2 \left[ \tau \frac{\partial g_i (\tau, k_i, k_j)}{\partial \tau} - g_i (\tau, k_i, k_j) \right]
\]

\[
\geq 2(3k_i - 2)(N - 1) \left[ (k_i + k_j) \frac{\partial g_i (\tau, k_i, k_j)}{\partial \tau} - 1 \right]
\]

\[
- (3k_i - 2)^2 \left[ \tau \frac{\partial g_i (\tau, k_i, k_j)}{\partial \tau} - g_i (\tau, k_i, k_j) \right]
\]

\[
\geq (3k_i - 2)^2 \left[ (k_i + k_j - \tau) \frac{\partial g_i (\tau, k_i, k_j)}{\partial \tau} - [1 - g_i (\tau, k_i, k_j)] \right].
\]

It follows that \( \frac{\partial \delta_i^*}{\partial \tau} < 0 \) when \( k_i + k_j - \tau > \frac{1 - g_i (\tau, k_i, k_j)}{\frac{\partial g_i (\tau, k_i, k_j)}{\partial \tau}} \).
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