Predator-Prey and Host Parasitoid Dynamics: A Bifurcation Theory Approach

PART I: Sensitivity of the General Rosenzweig–MacArthur Model to the Mathematical Form of the Functional Response

The Rosenzweig–MacArthur predator-prey model has been shown to be sensitive to the mathematical form used to model the predator response function even if the forms have the same basic shape: zero at zero, monotone increasing, concave down, and saturating. We revisit this model to help explain this sensitivity in the case of Holling type II: Monod, Ivlev, and hyperbolic trigonometric response functions. We consider the local and global dynamics and determine the possible bifurcations with respect to variation of the carrying capacity of the prey, a measure of the enrichment of the environment. We give analytic expressions that determine the criticality of the Andronov-Hopf bifurcation, and prove that although all three forms can have supercritical Hopf bifurcation, only the hyperbolic trigonometric form can give rise to subcritical Hopf bifurcation, saddle-node bifurcation of periodic orbits, and multiple limit cycles, providing a counterexample to a conjecture of Kooij and Zegeling (1996) and a result in a paper by Attili and Mallak (2006). We revisit the ranking of responses, according to their potential to destabilize dynamics, and show that given data, not only the choice of functional form, but the choice of number and position of data points influences predicted dynamics.

PART II: Pest Control by Generalist Parasitoids

Magal, Cosner, and Ruan (Math. Med. Biol. 25,1-20; 2008) studied both spatial and non-spatial host-parasitoid models motivated by the need for biological control of horse-chestnut leafminers that have spread through Europe. In the non-spatial model, they considered control by predation of leafminers by a generalist parasitoid population with functional response modeled using a Holling type II (Monod) form. They showed that there can be at most six equilibrium points, and discussed their local stability. We revisit their model in the non-spatial case, and identify cases missed in their investigation and the ramifications for possible pest control strategies. Both the local stability of equilibria and global properties are considered. A bifurcation theoretical approach is used. We provide analytical expressions for fold and Hopf bifurcations. Numerical results show very interesting dynamics, e.g., multiple coexisting limit cycles, homoclinic orbits, codimension one bifurcations including: Hopf, fold, transcritical, cyclic-fold, and homoclinic bifurcations, as well as codimension two bifurcations including: Bautin and Bogdanov-Takens bifurcations.