PIMS Workshop on Stochastic Nonlinear Dynamics

University of Alberta

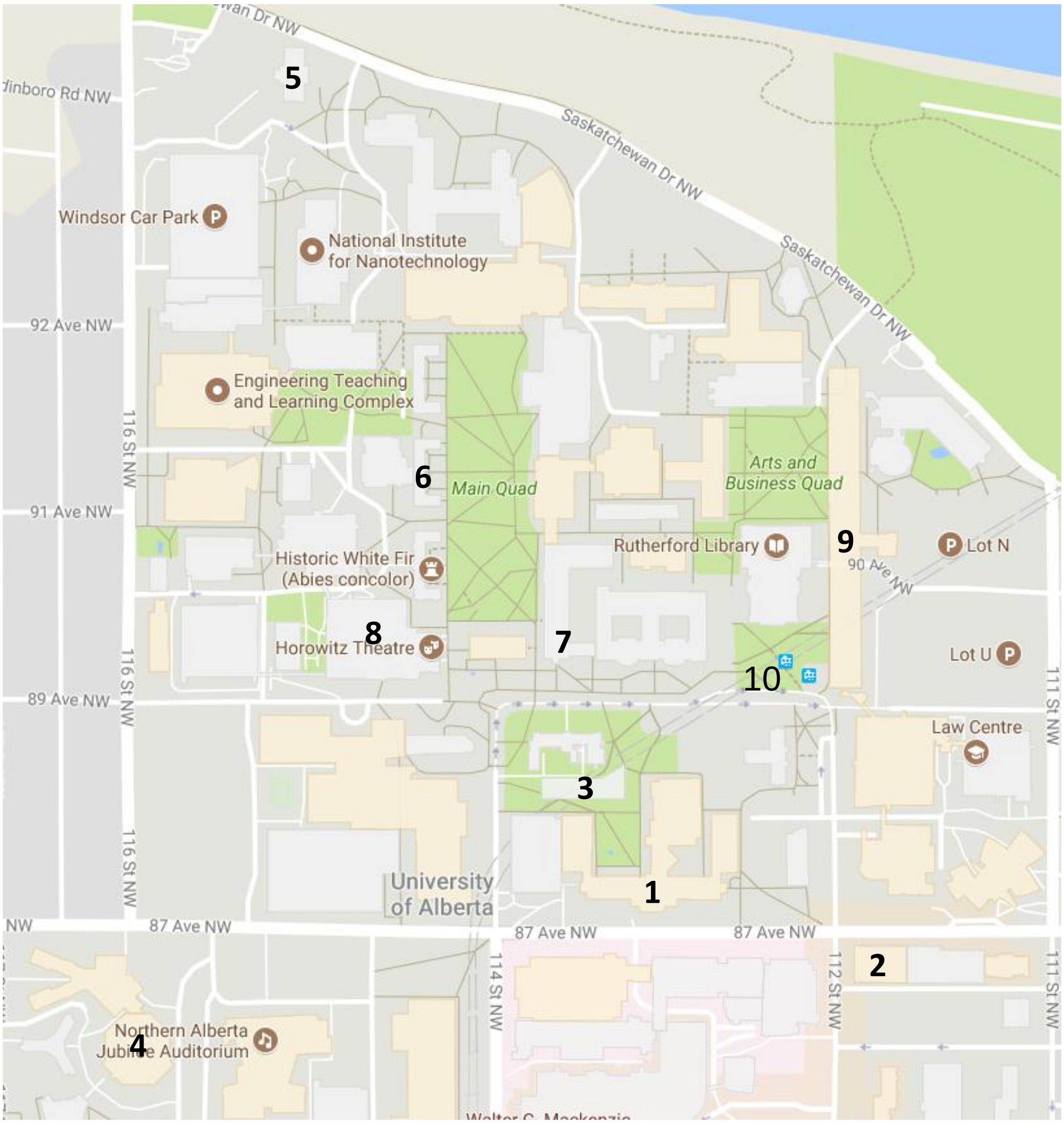
August 12-14, 2017

Schedule

Room: SAB 326 (South Academic Building)

	August 12	**August 13**	**August 14**
Chair	Michael Li	Rachel Kuske	Hong Qian
9:00-9:50	Rachel Kuske	Shui-Nee Chow	Lai-sang Young
9:50-10:40	Yingfei Yi	Michael Li	Shirou Wang
10:40-11:10	**Coffee Break**		
11:10-12:00	Kevin Lin	Arno Berger	Wenxian Shen
12:00-14:00	**Lunch Break**		
Chair	Yingfei Yi	Social Activities/Networking	Arno Berger
14:00-14:50	Hong Qian		Yao Li
14:50-15:40	Gerardo Barrera Vargas		Zhongwei Shen
15:40-16:10	**Coffee Break**		**Coffee Break**
16:10-17:00	Wen Huang		Felix Ye
18:00		Banquet Lister Centre	

CAMPUS MAP



1. Education Centre (ED)
2. Campus Tower (CT)
3. St. Joseph's Residence
4. Lister Centre (LH)
5. Faculty Club
6. Athabasca Hall (ATH)
7. South Academic Building (SAB)
8. Students' Union Building (SUB)
9. HUB Mall and Residence (HUB)
10. University LRT Station

Abstracts

Cut off phenomenon for small perturbations of hyperbolic dynamics

Gerardo Barrera Vargas
University of Alberta
Canada

Consider an ordinary differential equation with a fixed point that is a global attractor. Without loss of generality, assume that the fixed point is the origin. Under general conditions, at times goes any solution of this equation approaches the fixed point exponentially fast. Now add a small random perturbation to this equation. It is well known that, again under very general conditions, as times goes the solution of this stochastic equation converges to an equilibrium distribution that is well approximated by a Gaussian random variable of variance proportional to the strength of the perturbation. General theory of stochastic processes allows to show that this convergence, for each fixed perturbation, is again exponentially fast. We show that the convergence is actually abrupt: in a time windows of small size compared to the natural time scale of the process, the distance to equilibrium drops from its maximal possible value to near zero, and only after this time window the convergence is exponentially fast. This is what is known as the cut-off phenomenon in the context of Markov chain of increasing complexity. Under a proper time scaling, we are able to prove convergence of the distance to equilibrium to a universal function, a fact known as profile cut-off. This is a joint work with Milton Jara.

Digit distributions in dynamics

Arno Berger
University of Alberta
Canada

Numerical data generated by dynamical processes often exhibit a peculiar logarithmic distribution of significant digits, a phenomenon commonly referred to a Benford's Law. With a view on both deterministic and stochastic systems, this talk will describe some of the known mechanisms causing the phenomenon. Several recent results and intriguing open questions will be presented.

Schrodinger equations on a graph - Nelson's approach

Shui-Nee Chow
Georgia Institute of Technology
USA

Following Nelson's derivation, Schrodinger equation can be viewed as a Hamiltonian system on the space of probabilities with a 2-Wasserstein metric. We consider similar matters on finite graphs. We derive a Schrodinger equation on a graph from the discrete Nelson's problem. The proposed equation is a Hamiltonian system, which conserves total mass and total energy automatically. Several numerical examples are presented. This is joint work with Professors Wuchen Li (UCLA) and Haomin Zhou (GT).

Measure complexity and Mobius disjointness

Wen Huang
University of Science and Technology of China
China

We will review some progress about Sarnak's Mobius disjointness conjecture by the measure complexity. Some examples including (Quasi)-discrete spectrum systems and skew product maps on torus over a rotation of the circle will be discussed. This is a joint work with Prof. Zhiren Wang and Xiangdong Ye.

Stochastic averaging for multiple scale models driven by fat-tailed noise

Rachel Kuske
Georgia Institute of Technology/University of British Columbia
USA/Canada

Stochastic averaging has a long history for systems with multiple time scales and Gaussian forcing, but far less attention has been paid to problems where the stochastic forcing has infinite variance, such as in Levy processes or alpha-stable noise. Correlated additive and multiplicative (CAM) Gaussian noise, with infinite variance or ``fat tails'' in certain parameter regimes, can arise generically in many models with parametric uncertainty and has received increased attention in the context of atmosphere and ocean dynamics. These applications motivate new reduced models using stochastic averaging for systems with fast processes driven by noise with fat tails. We develop these results for the case of alpha-stable noise, giving explicit results that use the Marcus interpretation, the infinite variance analog to the Stratonovich interpretation. Then we show how reduced models for systems driven by fast linear CAM noise processes can be connected with the stochastic averaging for multiple scales systems driven by alpha-stable processes. We identify the conditions under which the approximation of a CAM noise process is valid in the averaged system, and illustrate methods using effectively equivalent fast, infinite-variance processes. These new types of approximations open the door for stochastic averaging in a wider range of stochastic systems with multiple time scales. This is joint work with Prof. Adam Monahan (U Victoria) and Dr. Will Thompson (UBC/NMi Metrology and Gaming)

Threshold Results for Deterministic and Stochastic Epidemic Models

Michael Li
University of Alberta
Canada

A hallmark of deterministic epidemic models is the threshold result: the disease dies out if the basic reproduction number R_0 is less than 1, and the disease persists if R_0 is greater than 1. Standard stochastic epidemic models, on the other hand, predicts that the disease always dies out with probability 1 as $t\to\infty.$ Such a difference in the predicted long-time behaviours by deterministic and stochastic models was also present in chemical kinetic dynamics and known as Keizer's Paradox. We incorporated stochastic disease incidences into a standard continuous time Markov chain model of SIS type for the transmission of an infectious disease, and considered the associated master equation. The disease-free state is no longer absorbing and a positive stationary distribution (PSD) exists and is globally asymptotically stable. By examining the profile of the PSD as the population size N increases, we prove that, in a Limit Threshold Theorem, the sharp threshold result of deterministic epidemic models can be obtained from a class of stochastic models as $N\to\infty$, in the sense of a "thermodynamic limit".

Polynomial convergence rate to nonequilibrium steady-state

Yao Li
University of Massachusetts Amherst
USA

In this talk I will present my recent result about the ergodic properties of nonequilibrium steady-state (NESS) for a stochastic energy exchange model. The energy exchange model is reduced from a billiards-like deterministic particle system that models the microscopic heat conduction in a 1D chain. By developing a technique called the induced chain method, we proved the existence, uniqueness, polynomial speed of convergence to the NESS, and polynomial speed of mixing for the stochastic energy exchange model. All of these are consistent with the numerical simulation results of the original deterministic billiards-like system.

Discrete-time approach to stochastic parameterization of chaotic dynamics

Kevin Lin
University of Arizona
USA

Many dynamical systems of interest in science and engineering are too complex or computationally expensive to fully resolve, even though only a relatively small subset of the degrees of freedom are observable or of direct interest. In these situations, it is useful to have low-dimensional models that can predict the evolution of the variables of interest without reference to the remaining degrees of freedom, and reproduce their statistics at an acceptable cost. This talk concerns a discrete-time, parametric approach to the problem of constructing reduced models from data. I will discuss some of the theoretical and practical issues that arise, including the representation of memory and noise effects. The method is illustrated using the Kuramoto-Sivashinsky equation, a prototypical model of spatiotemporal chaos. Time-permitting, I will also discuss a connection between this method and the Mori-Zwanzig formalism of nonequilibrium statistical mechanics. This is joint work with Alexandre Chorin and Fei Lu.

Emergent "energy" landscape in stochastic dynamics

Hong Qian
University of Washington
USA

Complex dynamics of interacting populations of intrinsically stochastic individuals can be mathematically represented by a discrete-state, continuous-time Markov jump process. T. G. Kurtz's theorem establishes a relation between this stochastic process, in the limit of a system’s size tending infinity, and the traditional dynamical systems based on ODEs. We apply this theory to several problems in current cell biology in terms of the biochemical constituents, and illustrate the emergent notions of epigenetic phenotypes and their switching, and relation to the classical idea of phase transition. We show the existence of an emergent nonequilibrium landscape, as a generalization of J. W. Gibbs’ energy function, for nearly any complex dynamics. We discover a rather surprising underlying mathematical structure, with geometric and thermodynamic implications.

Transition fronts in random Fisher-KPP equations

Wenxian Shen
Auburn University
USA

The current talk is concerned with transition fronts in random Fisher-KPP equations. I will first introduce the concept of random transition fronts or random traveling waves in general reaction diffusion equations in random media. I will then discuss the existence and stability of random transition fronts in Fisher-KPP equations with randomness.

Front propagation through fluctuating environments

Zhongwei Shen
University of Alberta
Canada

This talk is an introduction to the mathematical theory of front propagation phenomena in fluctuating environments with the focus on the spreading speeds. I will first introduce some backgrounds and classical results in homogeneous environments. It is followed by the presentation of some developments in heterogeneous but deterministic environments. Finally, I will introduce the problem in random/stochastic environments by presenting some mathematical models and related mathematical problems.

Continuity of entropy map for non-uniformly hyperbolic systems

Shirou Wang
Chinese Academy of Sciences/University of Alberta
China/Canada

For a continuous transformation f on a compact manifold M, the entropy map of f is defined by the metric entropy on the set of all f-invariant measures and it is generally not continuous. However, it is still worth our effort to investigate the upper semi-continuity of it since, for instance, it implies the existence of invariant measures of maximal entropy. In this talk I will talk about the upper semi-continuity of entropy map for non-uniformly hyperbolic systems. We prove that for C1 non-uniformly hyperbolic systems with domination, the entropy map is upper semi-continuous.

A numerical algorithm for calculating the rate of exponential forgetting in HMM

Felix Ye
University of Washington
USA

We consider a finite state hidden Markov model (HMM) with multidimensional observations. Under some mild assumptions, the prediction filter forget almost surely the initial condition exponentially fast. However, it is very difficult to calculate this asymptotic rate of exponential loss of memory analytically. We restate this problem in the setting of random dynamical system and use the Lyapunov exponents of the induced random dynamical system defined in the projective space $\mathbb{R}^{n-1}$ to approximate the convergence rate. Finally, we propose a stable numerical algorithm to calculate the rate of exponential forgetting semi-analytically. The numerical simulation result and comparison with current upper bound in literature will be shown in the presentation.

Degeneracy, complexity, and robustness of bio-systems

Yingfei Yi
University of Alberta
Canada

There has been recent emphasis on degeneracy as a feature of structural complexity due to the empirical observations of degenerate properties in known complex systems. The notion of degeneracy was first introduced for neural system as the ability of elements that are structurally different to perform the same function. Degeneracy is known to have close ties with structural complexity and robustness of a neural system. It is already observed for neural systems that high degeneracy not only yields high robustness, but also it is accompanied by an increase in the system complexity. In this talk, we will introduce the notions of degeneracy and structural complexity for a biosystem modeled by a differential equation. We will also discuss their connections with the robustness of the system.

Random dynamical systems and applications

Lai-sang Young
Courant Institute of Mathematical Sciences/New York University
USA

After a brief review of random dynamical systems theory, I will discuss some results having to do with Lyapunov exponents, entropy and fractal dimension, with their applications.

List of participants

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