## Finding Torque

Description: Find the torque due to a force applied at the origin about different axes of rotation. Hints illustrate the use of various approaches to finding torque.

A force $\vec{F}$ of magnitude $F$ making an angle $\theta$ with the $x$ axis is applied to a particle located along axis of rotation A, at Cartesian coordinates $(0,0)$ in the figure. The vector $\vec{F}$ lies in the xy plane, and the four axes of rotation $\mathrm{A}, \mathrm{B}, \mathrm{C}$, and D all lie perpendicular to the $x y$ plane.


A particle is located at a vector position $\vec{r}$ with respect to an axis of rotation (thus $\vec{r}$ points from the axis to the point at which the particle is located). The magnitude of the torque $\tau$ about this axis due to a force $\vec{F}$ acting on the particle is given by

$$
\tau=r F \sin (\alpha)=(\text { moment arm }) \cdot F=r F_{\perp}
$$

where $\alpha$ is the angle between $\vec{r}$ and $\vec{F}, r$ is the magnitude of $\vec{r}, F$ is the magnitude of $\vec{F}$, the component of $\vec{r}$ that is perpendicualr to $\vec{F}$ is the moment arm, and $F_{\perp}$ is the component of the force that is perpendicular to $\vec{r}$.

Sign convention: You will need to determine the sign by analyzing the direction of the rotation that the torque would tend to produce. Recall that negative torque about an axis corresponds to clockwise rotation.
In this problem, you must express the angle $\alpha$ in the above equation in terms of $\theta, \phi$, and/or $\pi$ when entering your answers. Keep in mind that $\pi=180$ degrees and $(\pi / 2)=90$ degrees .

## Part A

What is the torque $\tau_{\mathrm{A}}$ about axis A due to the force $\vec{F} ?$
Express the torque about axis $\boldsymbol{A}$ at Cartesian coordinates $(0,0)$.

Hints (1)

ANSWER:

$$
\tau_{\mathrm{A}}=0
$$

Also accepted: zero

## Part B

What is the torque $\tau_{\mathrm{B}}$ about axis B due to the force $\vec{F} ?$ ( B is the point at Cartesian coordinates $(0, b)$, located a distance $b$ from the origin along the $y$ axis.)

Express the torque about axis $\mathbf{B}$ in terms of $F, \theta, \phi, \pi$, and/or other given coordinate data.

Hints (2)

ANSWER:
$\tau_{\mathrm{B}}=b F \sin \left(\frac{\pi}{2}+\theta\right)$

Also accepted: $b F \cos (\theta)$

## Part C

What is the torque $\tau_{\mathrm{C}}$ about axis C due to $\vec{F} ?(\mathrm{C}$ is the point at Cartesian coordinates $(c, 0)$, a distance $c$ along the $x$ axis.)

Express the torque about axis $\mathbf{C}$ in terms of $F, \theta, \phi, \pi$, and/or other given coordinate data.

## Hints (2)

ANSWER:
$\tau_{\mathrm{C}}=-c F \sin (\theta)$

## Part D

What is the torque $\tau_{\mathrm{D}}$ about axis D due to $\vec{F}$ ? ( D is the point located at a distance $d$ from the origin and making an angle $\phi$ with the $x$ axis.)

Express the torque about axis D in terms of $F, \theta, \phi, \pi$, and/or other given coordinate data.

## ANSWER:

$\tau_{\mathrm{D}}=d F \sin (\phi-\theta)$
Also accepted: $d F \sin (\pi-\phi+\theta)$

You could have found $\tau_{\mathrm{D}}$ by using any of the three equations listed at the top of the page,

$$
\tau=r F \sin (\alpha)=(\text { moment arm }) \cdot F=r F_{\perp},
$$

and making the following associations:

- $r=d$ and $\sin (\alpha)=\sin (\pi-\phi+\theta)=\sin (\phi-\theta)$
- moment arm $=r_{\perp}=d \sin (\phi-\theta)$
- $F_{\perp}=F \sin (\phi-\theta)$.

