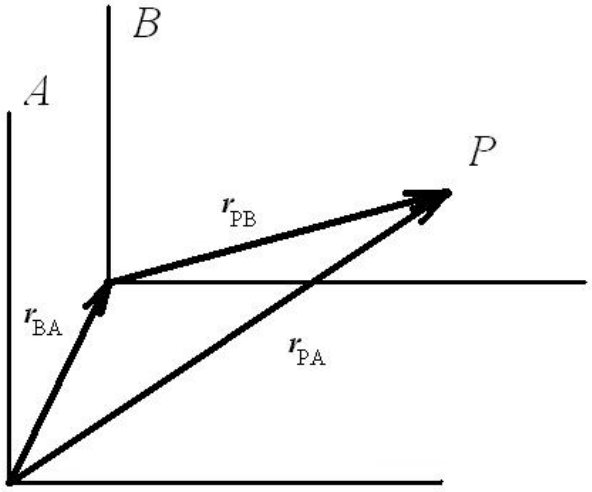
Proof of the Relative Velocity Formula

The figure below represents two coordinate systems *A* and *B*, and a point *P*.



- $\boldsymbol{r}_{PB}$ is a vector representing the position of *P* relative to the system *B*
- $\boldsymbol{r}_{PA}$ represents the position of *P* relative to the system *A*
- $\boldsymbol{r}_{BA}$ is the position of system *B* relative to the system *A*

[Each vector has the form $\boldsymbol{r} = (x, y)$]

We see from the figure that

$$\boldsymbol{r}_{PA} = \boldsymbol{r}_{PB} + \boldsymbol{r}_{BA}$$

If we use the definition of instantaneous velocity, $\mathbf{v} = \lim_{\Delta t \to 0} \frac{\Delta \mathbf{r}}{\Delta t}$, to each term, then we obtained the desired result:

$$\boldsymbol{v}_{PA} = \boldsymbol{v}_{PB} + \boldsymbol{v}_{BA}$$